

# The Tool Path Planning for Ring Torus Optical Surface Diamond Turning with Parallel 2DOF Fast Tool Servo

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**Abstract.** FTS (Fast Tool Servo) has always been an important method for manufacturing non-axisymmetric optical surface. In this paper, a novel tool path planning method is presented which plans tool path in two coordinate directions of a parallel structure 2 DOFs FTS simultaneously. Comparing with single DOF FTS, this method has significantly improved the ability of producing non-axisymmetric optical surface, such as ring torus surface.

## 1 Introduction

It is a effective and flexible method that turning optical surface using single point diamond tool. High precise geometric tolerance and nano scale surface roughness can be gained by holding workpiece only one time. One of approaches for turning non-axisymmetric optical surface is using diamond turning incorporate with FTS. FTS provide high frequency reciprocate motion along x or (and) z axis of turning lathe synchronize with slow speed feed motion of diamond turning.

Large stroke and high bandwidth are always be pursued and they are main directions which researchers has worked on to improve. Recently, approaches for improving these two performances are centralized in how to innovate or refine driving method, for example, driving by piezoelectric stacks [1][2], magnetostrictive FTS[3], normal stress electromagnetically driven motor[4][5]. The capability of single DOF FTSs introduced above is always been limited by the stroke of actuators. There is another way to improve the performance of FTS. Xiao-qin Zhou from Jilin university provided a new approach that turning non-axisymmetric optical surface by FTS with multi-DOF[6]. In this paper, we present a method that magnificent improve FTS's manufacturing ability by planning tooltip trajectory in both x and z axis of a parallel structure 2DOF FTS simultaneously, achieving high frequency motion in both directions.

In the paper, ring torus optical surface is taken for example. By comparing the tool path planning of 2DOF FTS and single DOF FTS, Conclusion is drawn that 2DOF FTS has remarkable ability of performance improving than single DOF FTS.

## 2 The tool path planning for ring torus surface turning

### 2.1 The equation of ring torus surface

Equation 1 describes ring torus surface in a cylindrical coordinates system. The origin of the coordinates system is coincident with the vertex of the ring torus surface; the z axis is collinear with spindle axis of lathe and the direction is point to cross curve origin. The polar angle's reference direction is collinear with x axis of lathe.  $\theta$  is polar angle and  $\rho$  is radial coordinate

$$z = R_b - \sqrt{\left(R_b - R_c + \sqrt{R_c^2 - r^2 \sin^2 \theta}\right)^2 - r^2 \cos^2 \theta} \quad (1)$$

where:  $R_b = b + a$ ,  $R_c = a$ , a is radius of base curve of ring torus, b is radius of cross curve of ring torus.

### 2.2 The tool path planning for ring torus surface using 2DOF FTS

Ring torus optical surface can be created by using two slow-speed feed motions of diamond turning lathe along x and z axes, and two fast-speed motions of 2DOF FTS along the same axes as well. These motion can be described as equation 2:

$$\begin{cases} r = r_s(t) + r_f(r_s, \theta) \\ z = z_s(r_s) + z_f(r_s, \theta) \end{cases} \quad (2)$$

where subscript  $s$  means slow-speed of turning lathe,  $f$  means fast-speed of 2DOF FTS.

Polar coordinate equation 3 is designed for description the x axis motion of turning lathe and the motion along workpiece radius of 2DOF FTS.

$$\begin{cases} \theta = \omega t \\ r_s = R - vt \\ r_f = A \frac{x_{\max}}{2} \cos(2\theta) \end{cases} \quad (3)$$

where:

$$A = \begin{cases} 1 & r_s > R - S \\ r_s / (R - S) & r_s \leq R - S \end{cases}$$

$\theta$  is polar angle,  $\omega$  is angle velocity of lathe spindle,  $R$  is the maximum radius of workpiece,  $r$  is the distance between tooltip and origin of polar coordinate system,  $S$  is the maximum stroke of 2DOF FTS along one axis,  $x_{\max}$  is the maximum stroke along radius.

The equation that describes the motion of 2DOF FTS along z axis is constructed as equation 4

$$z_s = a + b - \left[ \sqrt{(b+a)^2 - (r_s + AS)^2} + b + \sqrt{a^2 - (r_s - AS)^2} \right] / 2 \quad \dots \dots (4)$$

$$\begin{aligned} z_f &= z - z_s \\ &= \left[ \sqrt{(b+a)^2 - (r_s + AS)^2} + b + \sqrt{a^2 - (r_s - AS)^2} \right] / 2 \\ &\quad - \left[ \sqrt{b + \sqrt{a^2 - (r_s + AS \cos 2\theta)^2} \sin^2 \theta} - (r_s + AS \cos 2\theta)^2 \cos^2 \theta \right] \quad \dots \dots (5) \end{aligned}$$

The simulation of trajectory of tooltip on the 2DOF FTS that constructed in this paper is shown in Fig.1. Ring torus geometry is shown in Fig. 2 and the combination of ring torus geometry and tooltip trajectory curve is shown in Fig. 3. It's obviously that the geometry surface and the curve is matched perfectly.

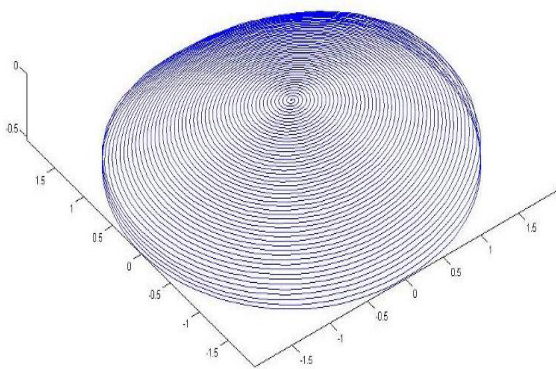


Figure 1. Tool path plan for turning ring torus.

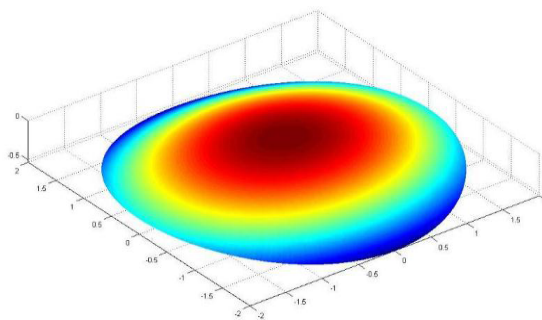


Figure 2. Ring torus geometry.

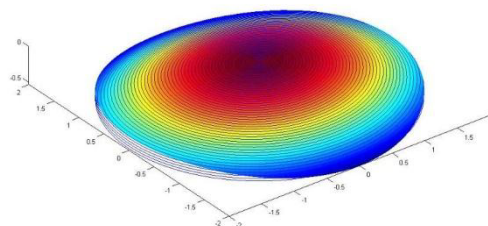


Figure 3. match of ring torus geometry and tool path.

### 3 Performance comparison between single DOF FTS and 2DOF FTS

#### 3.1 Qualitative performance comparison between single DOF FTS and 2DOF FTS

For Qualitative comparison between single DOF FTS and 2DOF FTS, the curves of 2DOF FTS motions along x and z axis are drawn in Fig. 4 and Fig. 5 separately. In these graphs, horizontal coordinate is the angle of turning lathe spindle and vertical coordinate is x or z displacement.

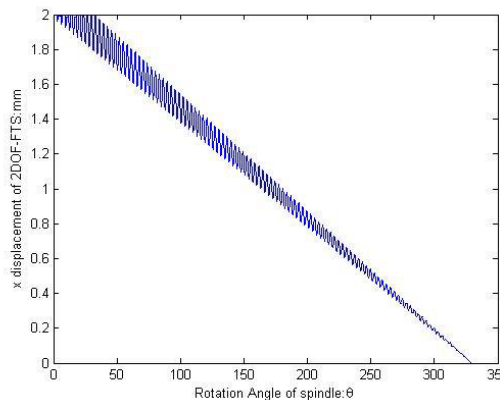
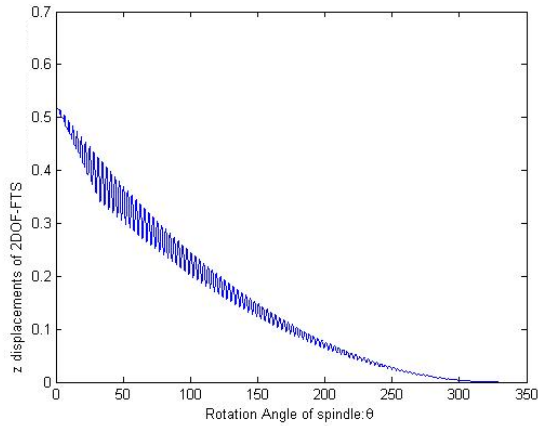


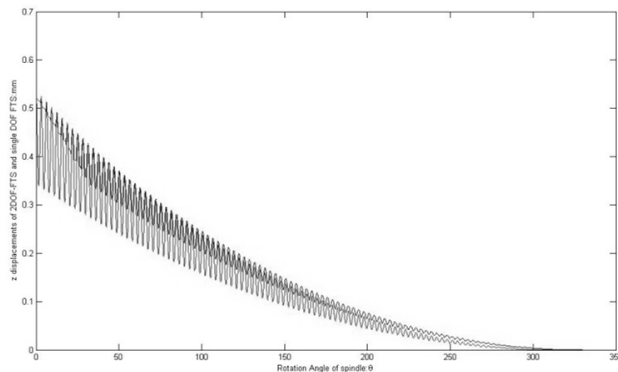
Figure 4. x displacement by angle of spindle.



**Figure 5.** z displacement by angle of spindle.

The high frequency motion component shown in Fig. 4 and 5 are accomplished by 2DOF FTS along x and z axis separately meanwhile turning lathe's x and z slider fulfill the low frequency motion components.

According the Fig. 4 and Fig. 5, the max strokes along x and z axis are not needed at first period of cutting process but after several ones. If we turn the same workpiece by single DOF FTS, the max stroke always happened at the first period as the light curve shown in Fig. 6. The reason is that the tooltip is not always keep in touch with workpiece before the period of max strokes happened if the ring torus fabricated by 2DOF FTS. Before the period during which max stroke happens, the cutting process is not continuous to avoid collision between tooltip and workpiece by swing tooltip out of workpiece outer cycle along x axis. Once tooltip swing away from the workpiece, the motion of 2DOF FTS along z axis will be hold. That is the key why smaller strokes needed by using 2DOF FTS than single DOF FTS. The method is described clearly by the trajectory shown in Fig. 1, Fig. 3 and equation 5.



**Figure 6.** Two kinds of FTS tooltip motion curves along z axis

In order to illustrate the comparing more clearly, both the single DOF FTS's and the 2DOF FTS's tooltip motion curve along z axis of turning lathe is drawn in the same graph as shown in Fig. 6. In Fig. 6, the light curve is single DOF FTS motion trajectory, and the dark curve is 2DOF FTS motion curve. Obviously, about 100% more stroke is required using single DOF FTS than using 2

DOF FTS for fabricating the same ring torus surface in this example.

### 3.2 Quantitative performance comparison between single DOF FTS and 2DOF FTS

According to equation 5, the maximum displacement value of 2 DOF FTS along z axis arise while the angle of turning lathe spindle  $\theta = 0$  and the minimum value arise while  $\theta = \pi/2$ . The maximum and minimum value are expressed as equation 6.

$$\begin{cases} z_0 = a + b - \sqrt{(a+b)^2 - R^2} \\ z_{\pi/2} = a + b - \left[ b + \sqrt{a^2 - (R-2S)^2} \right] \end{cases} \quad (6)$$

where  $z_0$  means the displacement along z axis while  $\theta = 0$  and  $z_{\pi/2}$  means the displacement along z axis while  $\theta = \pi/2$ .

The maximum z displacement can be obtained from equation 6 as equation 7:

$$\Delta z_{\max} = \sqrt{(a+b)^2 - R^2} - \sqrt{a^2 - (R-2S)^2} - b \dots (7)$$

Equation 7 presents the limit of single DOF FTS displacement.

The 2 DOF FTS is going to its limit of displacement when  $\Delta z_{\max} = S$ . Substitute  $\Delta z_{\max} = S$  into the equation 7, an one-Variable Quadratic inequality of S is obtained. Arranging the result, S is expressed as equation 8.

$$S = \frac{m - \sqrt{m^2 - 5n}}{5} \quad (8)$$

where  $m = \sqrt{(a+b)^2 - R^2} - b + 2R$ ,

$$n = \left[ \sqrt{(a+b)^2 - R^2} - b \right]^2 + R^2 - a^2$$

Equation 8 describes the relationship between ring torus with parameters  $a$ ,  $b$ ,  $R$  and the manufacturing ability of 2DOF FTS with determined stroke  $S$ . The maximum stroke required for ring torus with parameters  $a$ ,  $b$ ,  $R$  using 2 DOF FTS is expressed as equation 9.

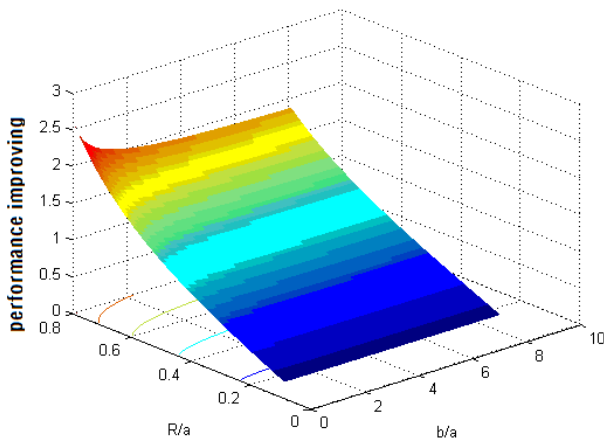
$$\Delta z'_{\max} = \sqrt{(a+b)^2 - R^2} - \sqrt{a^2 - R^2} - b \quad (9)$$

Let  $\alpha = b/a$ ,  $\beta = R/a$  and substitute  $\alpha$ ,  $\beta$  in equation 9, equation 10 is obtained.

$$P_r = \frac{\sqrt{1 - (\beta - 2S/\alpha)^2} - \sqrt{1 - \beta^2}}{S/\alpha} \quad (10)$$

The parameters a, b and R in equation 9 is the absolute dimensions of ring torus surface. However,  $\alpha$  and  $\beta$  are the opposite, they indicate the degree of non-axisymmetric without relationship of the dimensions of ring torus. Hence, equation 10 has more meaning that is

general. The function graph of equation 10 is shown in Fig. 7.



**Figure 7.** Performance improving with 2DOF FTS.

According to the figure 7, as  $\beta$  increases, the performance of 2DOF FTS is significant improved respect to single DOF FTS. As  $\alpha$  decreases, the performance improving of 2DOF FTS is not obvious. during the value of  $\alpha$  change from 0.1 to 0.8 and  $\beta$  change from 0.1 to 8,  $P_r$  is going to change from 0.19 to 2.53 continuously. In other word, the manufacturing ability of 2 DOF FTS is improved about 20% to 250% respect to single DOF FTS. In figure 7, the value range of  $\alpha$  and  $\beta$  are wide. In practical application, these two parameters is located in the middle of surface in figure 7. For example, Radius of curvature of Astigmatism correcting lens is between 50mm to 130mm. Correspondingly, let  $b=100mm$ ,  $a=60mm$  and  $R=30mm$ , the results of equation 7 and equation 9 are  $\Delta z_{\max} = 2.6mm$  and  $\Delta z'_{\max} = 5.2mm$ . That means, the lens can be fabricated using 2 DOF FTS with both stroke of 2.6mm instead of single DOF FTS with stroke of 5.2mm. In this example, the manufacture ability of 2 DOF FTS is 200% times than single DOF FTS.

## 4. Conclusion

In this paper, a novel tool path planning method is presented. The method plans tool path along both x and z axis of single point diamond turning lathe using parallel structure 2 DOF FTS. Take the fabrication of ring torus optical surface for example, the equation of tooltip motion trajectory is given. It is proved that the fabricating ability of FTS will be significant improved using the method presented in this paper by analysing fabrication limits of both single DOF FTS and 2 DOF FTS.

## References

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