Reliability analysis of scarifying tooth for trailed scarifier

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Abstract. In this paper are estimated the reliability indicators of scarifying tooth for trailed scarifier. Taking into account the constructive and technological factors, as well as the duration of exploitation, the determined values of Anderson-Darling statistics indicates that the Weibull distribution permits to model with maximum accuracy the lifetime of such attachment. The location parameter is estimated using the correlation coefficient method and minimum lifetimes are determined experimentally. Analyzing the obtained parameters, it can be shown that it can be used the normal distribution for reliability estimation. In this respects, it was applied the point and confidence interval estimation of the average and standard deviation lifetime of scarifying teeth of trailed scarifier.

1 Constructive characteristic of the scarifying teeth

A scarifier attachment is designed to break up compacted material, asphalt and rocky subgrade to facilitate grading [1]. The scarifier is a rear mounted tool used for tearing up asphalt, hard-pack or other obstacles to aid the grading operation [2].

The scarifiers are terracing equipment used at deep loosening of hard and very hard soils, of rocks relatively weak (which cannot be loosened directly with another equipment, type bucket or blade), for loosening of soils with medium hardness, with boulders and roots, or for removing of the old pavements [3].

From constructive point of view, scarifiers can be [3]:

- Trailed by a crawler tractor or other machine for construction;
- Mounted at the front of a tractor or as attachment on a construction machine (bulldozer, motor grader etc.); in the case of motor grader, the scarifiers can be mounted in front of or between the axles.

The active component of the scarifiers (tooth) can be of different types (figure 1), depending on the actual working conditions [3]: (a) straight teeth; (b), (c) curved teeth; (d), (e), (f) teeth of special construction (d. - with heel; e - without heel; f - with removable tip). The teeth can be mounted rigid or articulated on a beam (frame).

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In figure 1 is presented the common scarifying teeth used as auxiliary attachment for bulldozer for execution and rehabilitation of forest roads. The main characteristics of the attachment according to the tensile force of the basic machine are shown in table 1.

Fig. 1. Constructive shapes of scarifying teeth.

Table 1. Scarifier characteristics.

Parameters	Traction force [kN]					
	30	45	60	100	150	250
Numbers of teeth	3÷5	3÷5	3÷5	1÷5	1÷3	1÷3
Horizontal rotation angle [deg]	-	-	-	10÷15	10÷15	10÷15
The length of the tooth [cm]	45÷80	50÷80	60÷100	80÷140	80÷150	100÷190
The maximum depth of cut [cm]	20÷50	40÷60	35÷45	40÷100	50÷120	60÷150
Cutting angle [deg]	40÷60	40÷60	35÷45	40÷80	40÷80	40÷80
The weight of equipment [t]	0.07÷0.19	0.12÷0.20	0.16÷0.21	0.18÷0.21	0.17÷0.24	0.20÷0.27

Compared to usual activities, in the case of rehabilitation and maintenance of forest roads, the scarifying teeth gauge dimensions are lower. In this manner, in Figure 2 it were examined a 3D model of trailed scarifier that has constructive characteristics able to support the traction force of 100 kN (table 1).

Fig. 2. 3D model of the auxiliary attachment.

2 Nomenclature

$\mathcal{W}(x, \beta, \eta, \gamma)$ = three-parameter Weibull distribution;

$\mathcal{W}(x, \beta, \eta)$ = two-parameter Weibull distribution;

β = shape parameter of Weibull distribution;

η = scale parameter of Weibull distribution;

γ = location parameter of Weibull distribution;

$\mathcal{N}(x, \mu, \sigma)$ = normal distribution (Gauss-Laplace);

μ = mean;

σ = standard deviation;

L = lower limit of confidence interval;

U = upper limit of confidence interval;

$\rho(X, Y)$ = correlation coefficient of random variables X and Y ;

CL = confidence limits;

$1 - \alpha$ = confidence level;

ML = maximum likelihood;

$\hat{x}$ = estimated value;

$v(r, n) = \text{random variable } \frac{\hat{\beta}}{\beta}$;

$k(r, n) = \text{random variable } \hat{\beta} \cdot \ln \frac{\hat{\eta}}{\eta}$;

m = sample average;

s = standard deviation of the sample;

$t_{p,v}$ = the quantile p of Student's t -distribution, with v degrees of freedom;

$\chi^2_{p,v}$ = the quantile p of Chi-squared distribution, with v degrees of freedom;

$F_{p;m,q}$ = the quantile p of Fisher-Snedecor distribution, with m and p degrees of freedom.

3 Reliability analyses of the scarifying teeth

In order to analyze the reliability of scarifying tooth, it was applied the Anderson-Darling test, considering the four statistical distributions: Weibull distribution, log-normal distribution, exponential distribution and normal distribution.

The calculation consists in applying statistical relationship [4]:

$$A_n^2 = n \cdot \int_{-\infty}^{\infty} \frac{[Q(y) - F(y)]^2}{F(y) \cdot [1 - F(y)]} \cdot dF(y)$$
 (1)

The obtained results are a measure of the discrepancy between the empirical distribution, sampling values and the considered distribution.

Anderson-Darling test statistic results as:

$$A_n^2 = - \sum_{i=1}^n \frac{2 \cdot i}{n} \cdot \{ \ln F(y_{(i)}) + \ln [1 - F(y_{(n+1-i)})] \} - n$$
 (2)

considering the calculus relation of empirical distribution:

$$Q[y_{(i)}] = \begin{cases} 0, & \text{if } Y < y_{(1)} \\ \frac{i}{n}, & \text{if } y_{(i)} < Y < y_{(i+1)}, \quad i = \overline{1, n-1} \\ 1, & \text{if } Y > y_{(i)} \end{cases}$$
 (3)

The lifetime estimation of analyzed auxiliary attachment it was achieved by applying the finite element method (FEA) and 3D modeling was simulated using Ansys Workbench program. Based on the results, the determined values of Anderson-Darling statistics (table 2) indicate that the Weibull distribution permits the modeling with maximum accuracy of lifetime of such attachment.

Table 2. Anderson-Darling statistics.

Distribution	Anderson-Darling statistic values, A_n^2
Weibull	2.095
Log-normal	2.113
Exponential	3.391
Normal	2.106

The statistical model is validated applying the Anderson-Darling test that permits to compare the date set for considered distributions (figure 3).

Fig. 3. Probability plot of Anderson-Darling test for scarifying tooth.

Also, it is estimated the reliability of trailed scarifier teeth using three-parameters Weibull distribution.

The location parameter (or failure free life) γ is estimated by the correlation coefficient method and minimum lifetimes are determined experimentally (table 3). The value of location parameter indicates the minimum lifetime value of each subassembly estimated under real operating conditions.

Table 3. Minimum lifetime estimation.

Parameter	Estimated value
Correlation coefficient (ρ)	0.99818
Location parameter ($\hat{\gamma}$)	$5.813 \cdot 10^7$

The estimation method consists of determining the location parameter values, γ , under maximum conditions of correlation coefficient values, [5-6]:

$$\rho(Y, C) = \frac{\sum_{i=1}^n \ln(y_i - \gamma) \cdot c_i - \frac{\sum_{i=1}^n \ln(y_i - \gamma) \cdot \sum_{i=1}^n c_i}{n}}{\left\{ \left[\sum_{i=1}^n \ln^2(y_i - \gamma) - \frac{[\sum_{i=1}^n \ln(y_i - \gamma)]^2}{n} \right] \cdot \left[\sum_{i=1}^n c_i^2 - \frac{(\sum_{i=1}^n c_i)^2}{n} \right] \right\}^{1/2}} \tag{4}$$

where the values of deterioration probabilities are determined using the approximate calculation relationship, [4]:

$$c_i = \frac{1}{1 + \frac{n-i+1}{i} \cdot F_{0.50;2 \cdot (n-i+1);2 \cdot i}}, i = \overline{1, n} \tag{5}$$

The results were obtained by using an application in Mathcad 14 software. Knowing the values of the location parameter, by means of a transformation of random variable of the form:

$$X = Y - \gamma \tag{6}$$

the obtained values are two-parameter Weibull distributed (table 4).

Table 4. Lifetime estimation of scarifying tooth.

Issue	Lifetime [cycle], $x_i, i = \overline{1, n}$
1	$4.1872 \cdot 10^7$
2	$5.8872 \cdot 10^7$
3	$7.4872 \cdot 10^7$
4	$9.1872 \cdot 10^7$
5	$1.0887 \cdot 10^8$
6	$1.2487 \cdot 10^8$

The point estimate method used is maximum likelihood. In this case, the estimated values of the shape and scale parameters of the two-parameter Weibull distribution are obtained as a solution of the equations system [6-7]:

$$\begin{cases} \frac{1}{\hat{\beta}} + \frac{1}{n} \cdot \sum_{i=1}^n \ln x_i - \frac{\sum_{i=1}^n x_i^{\hat{\beta}} \cdot \ln x_i}{\sum_{i=1}^n x_i^{\hat{\beta}}} = 0 \\ \hat{\eta}^{\hat{\beta}} = \frac{1}{n} \cdot \sum_{i=1}^n x_i^{\hat{\beta}} \end{cases} \tag{7}$$

Because the sample size is very small, $n = 6$, the asymptotically properties of maximum likelihood estimators cannot be applied in this case, the estimations with confidence intervals were calculated on the basis of two random variables, independent of the sample size (n) and the type of test ($n = r$, for the complete test), [8-11]:

$$\begin{cases} v(r, n) = \frac{\hat{\beta}}{\beta} \\ k(r, n) = \hat{\beta} \cdot \ln \left(\frac{\hat{\eta}}{\eta} \right) \end{cases} \tag{8}$$

Thus, for the shape parameter:
 - corresponding to a confidence level $1-\alpha$, it is obtained as the solution of the equation of the probability:

$$Pr \left[v_{\alpha/2}(n, n) \leq \frac{\hat{\beta}}{\beta} \leq v_{1-\alpha/2}(n, n) \right] = 1 - \alpha \tag{9}$$

and it has the form:

$$\frac{\hat{\beta}}{v_{1-\alpha/2}(n, n)} \leq \beta \leq \frac{\hat{\beta}}{v_{\alpha/2}(n, n)} \tag{10}$$

- unbiased mean estimate:

$$\hat{\beta} = \frac{\hat{\beta}}{\bar{v}(n,n)} \tag{11}$$

- unbiased median estimate:

$$\hat{\beta}_{Me} = \frac{\hat{\beta}}{v_{0.50}(n,n)} \tag{12}$$

For scale parameter:

- corresponding to a confidence level $1-\alpha$, it is obtained as the solution of the equation of the probability:

$$Pr \left[k_{\alpha/2}(n,n) \leq \hat{\beta} \cdot \ln \frac{\hat{\eta}}{\eta} \leq k_{1-\alpha/2}(n,n) \right] = 1 - \alpha \tag{13}$$

and it has the form:

$$\hat{\eta} \cdot \left[e^{\frac{-k_{1-\alpha/2}(n,n)}{\hat{\beta}}} \right] \leq \eta \leq \hat{\eta} \cdot \left[e^{\frac{-k_{\alpha/2}(n,n)}{\hat{\beta}}} \right] \tag{14}$$

- unbiased mean estimate:

$$\hat{\eta} = \hat{\eta} \cdot e^{\frac{-\bar{k}(n,n)}{\hat{\beta}}} \tag{15}$$

- unbiased median estimate:

$$\hat{\eta}_{Me} = \hat{\eta} \cdot e^{\frac{-k_{0.50}(n,n)}{\hat{\beta}}} \tag{16}$$

The point estimate and with confidence intervals for Weibull distribution parameters it was determined based on experimental results (table 5).

The confidence level used is $1-\alpha = 0.95$ and the corresponding values of the random variables $v(r,n)$ and $k(r,n)$ were taken from [6], (table 6).

Table 5. Parametric estimation with 95% confidence interval – Weibull distribution.

Shape parameter	$\hat{\beta}$	$\hat{\beta}$	$\hat{\beta}_{Me}$	$\hat{\beta}_L$	$\hat{\beta}_U$
	3.3322	2.4905	2.8026	1.1322	5.2526
Scale parameter	$\hat{\eta}$	$\hat{\eta}$	$\hat{\eta}_{Me}$	$\hat{\eta}_L$	$\hat{\eta}_U$
	$9.344 \cdot 10^7$	$9.457 \cdot 10^7$	$9.456 \cdot 10^7$	$6.502 \cdot 10^7$	$1.381 \cdot 10^8$

Table 6. Corresponding values of the random variables.

$v(r,n)$	$k(r,n)$
$\bar{v}(6,6) = 1.338$	$\bar{k}(6,6) = -0.0402$
$v_{0.50}(6,6) = 2.802$	$k_{0.50}(6,6) = -0.0399$
$v_{0.975}(6,6) = 5.252$	$k_{0.975}(6,6) = 1.208$
$v_{0.025}(6,6) = 1.132$	$k_{0.025}(6,6) = -1.301$

High values of shape parameters, $\beta = 3.3322$, indicates that it can be used the normal distribution for reliability modeling. The same conclusion can be reached from Anderson-Darling statistics analysis, as shown in Figure 3.

Point estimate of the normal distribution parameters is performed using the equations:

$$\begin{cases} \hat{\mu} = \frac{\sum_{i=1}^n x_i}{n} \\ \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^n (x_i - \hat{\mu})^2}{n - 1}} \end{cases} \tag{17}$$

where μ represents the mean of operating time and σ is standard deviation of operating time.

The calculation of confidence intervals for the parameters provides also valuable information on the two reliability indicators.

The calculation formulas used are:

- corresponding to a confidence level $1-\alpha$, for the mean is:

$$\hat{\mu} + t_{\frac{\alpha}{2},n-1} \cdot \hat{\sigma}/\sqrt{n} \leq \mu \leq \hat{\mu} + t_{1-\frac{\alpha}{2},n-1} \cdot \hat{\sigma}/\sqrt{n} \tag{18}$$

- corresponding to a confidence level $1-\alpha$, for the standard deviation is:

$$\sqrt{\frac{(n-1) \cdot \hat{\sigma}^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}} \leq \sigma \leq \sqrt{\frac{(n-1) \cdot \hat{\sigma}^2}{\chi^2_{\frac{\alpha}{2},n-1}}} \tag{19}$$

The point estimate and with confidence intervals for normal distribution parameters it was determined based on experimental results and there are shown in table 7.

Table 7. Parametric estimation with 95% confidence interval - normal distribution.

Mean	$\hat{\mu}$	$\hat{\mu}_L$	$\hat{\mu}_U$
	$1.4167 \cdot 10^8$	$1.0902 \cdot 10^8$	$1.7432 \cdot 10^8$
Standard deviation	$\hat{\sigma}$	$\hat{\sigma}_L$	$\hat{\sigma}_U$
	$3.1111 \cdot 10^7$	$1.9419 \cdot 10^7$	$7.6302 \cdot 10^7$

4 Results and discussions

The reliability of scarifying teeth can be modeled by using the Weibull distribution model. In table 8 are presented the analytical expressions of reliability functions and the values of the main reliability indicators that are estimated based on experimental results collected from the operation of these attachments (figure 4).

Table 8. The main reliability indicators.

Scarifying tooth for trailed scarifier	$F(t)$	μ	σ
	$1 - e^{-\left(\frac{x-5.813 \cdot 10^7}{9.344 \cdot 10^7}\right)^{3.3322}}$	$1.416 \cdot 10^8$	$3.111 \cdot 10^7$

Fig. 4. Reliability distribution overview plot for scarifying tooth.

5 Conclusions

The study conducted was focused on the reliability analysis of the scarifying teeth of scarifier trailed in real rehabilitation and maintenance conditions of forest roads. This analysis allows the following conclusions:

- Modeling of reliability can be achieved with a good accuracy using three-parameters Weibull distribution;
- The value of the minimum lifetime estimated for actual operating conditions is $5.813 \cdot 10^7$ (table 3);
- High values of shape parameter, allow us to use normal distribution for reliability estimation. This has led to the possibility of punctual estimation and confidence interval of the average lifetime of scarifying teeth of trailed scarifier and to estimate the standard deviation of their lifetime.
- Knowledge of parameter values of reliability statistical model and the values of the main indicators of reliability, enabling a much better approach to preventive maintenance activities and realistic planning them.

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