Reliability of timber structure bolt connection subjected to double unequal shears with thick plates as outer members

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Abstract. For reliability check of timber structure bolt connections subjected to double shear with thick plates as outer members and inner timber member is nowadays possible to use equations which were derived by Johansen and which are also used in Eurocode standards. These equations assume that the loads are the same on both sides. But it may often be a case when the loads are not the same. The paper deals with reliability in ultimate limit state of timber structure bolted connections subjected to unequal double shear with thick plates as outer members and inner timber member. There are derived equations for calculation of the reliability of this connection type in the ultimate limit state.

1 Introduction

Structure engineers can use nowadays for reliability check of timber structure bolt connections subjected to double shear with thick plates as outer members and inner timber member Eurocode standards [1]. These formulas were derived by Johansen in [2] and can be found also for example in [3].

In [4] there were derived formulas for design load-carrying capacity of bolt connections subjected to double unequal shear with thick plates as outer members and inner timber member. Example of such connection is showed in Fig. 1. We assume that \( F_{d1} > F_{d2} \). The formulas are listed below as Eq. 1, 2, and 3. Each formula corresponds to a particular mode of failure. In formula (1), the member for characteristic withdrawal capacity of the bolt is omitted.

The case when the thin plates as outer members are used can occur very often but this case is not the subject to this paper. This case is described in [5-7].
Mode of the failure a)

\[ R_{d1} = \frac{k_{mod}}{\gamma_M} \left( 2.3 \cdot \sqrt{M_{y,Rk} \cdot f_{h,2,k} \cdot d} \right) \]  \hspace{1cm} (1)

Mode of the failure b)

\[ R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot t_2 - R_{d2} \]  \hspace{1cm} (2)

Mode of the failure c)

\[ R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \left( \left[ 2 \cdot \left( \frac{t_2}{d} \right)^2 + \frac{4 \cdot M_{y,k}}{f_{h,2,k} \cdot d} + \frac{2 \cdot \gamma_M \cdot R_{d2} \cdot t_2}{k_{mod} \cdot f_{h,2,k} \cdot d} \right] - t_2 \right) - R_{d2} \]  \hspace{1cm} (3)

2 Reliability of bolted connection subjected to unequal double shear according to Figure 1

We can assess the reliability in ultimate limit state according to Eq. 4

\[ \frac{F_{d1}}{R_{d1}} \leq 1 \]  \hspace{1cm} (4)

For determination of reliability is needed to define the way of loading. For mode of failure “a)” the design load carrying capacity \( R_{d1} \) is independent on the \( R_{d2} \) so the way of loading is unimportant. But for the modes of failure “b)” and “c)” the situation is different. Let’s assume the connection according the Fig. 1 for: \( f_{h,2,k} = 26.174 \text{MPa}, t_2 = 60 \text{mm}, d = 16 \text{mm}, k_{mod} = 0.8, \gamma_M = 1.3, f_{u,k} = 360 \text{MPa}. \)
In Fig. 2 the dependence $R_{d1}$ on $R_{d2}$ is shown for the mode of failure “b)” which is described by Eq.2.

![Fig. 2](image)

**Fig. 2** Assessment of the connection reliability for mode of failure “b)” – Eq. 2.

In Fig. 3 the dependence $R_{d1}$ on $R_{d2}$ is shown for the mode of failure “c)” which is described by Eq.3.

![Fig. 3](image)

**Fig. 3** Assessment of the connection reliability for mode of failure “c)” – Eq. 3.
The design loads of the connection $F_{d1}$ and $F_{d2}$ according to Fig. 1 can be drawn as a point in Fig. 2 and Fig. 3. Because we assume in accordance with 1 that $F_{d1} > F_{d2}$, the points representing the loads of the connection will be located always above the line with slope of 45°. If the point with coordinates $[X= F_{d2}; Y= F_{d1}]$ is located under the red curve in hatch area, the connection is satisfied in terms of the ultimate limit state. So the red curve represents the reaching of the ultimate limit state. But for the assessment of the connection reliability is advisable to derive a formula, which would give us a percentage measure of the reliability connection. This percentage would tell us “how far” the connection design is to the ultimate limit state. This is the reason why we need to define the way of the loading as it was mentioned in the previous. In the following the two ways “A” and “B” of loading are considered.

3 Reliability for the way of the loading ”A”

In this way of loading the design load $F_{d2}$ stays constant and the design load $F_{d1}$ is increasing only. Then we can define $R_{d2} = F_{d2}$ and substitute it into the Eq. 2 and 3 which we calculate the design load-carrying capacity $R_{d1}$ from. The reliability condition of the connection in terms of the ultimate limit state is then:

$$\frac{F_{d1}}{R_{d1}} \leq 1 \quad (5)$$

The $R_{d1}$ is taken as the minimum value got from Eq. 1,2 and 3

4 Reliability for the way of the loading ”B”

In this way of loading, the both design loads are increasing and the ratio $F_{d1}/F_{d2}$ remains constant. This way of loading is possible to expect in practice most often.

In accordance with Fig. 2 and Fig. 3, the assessment of the ultimate limit state can be expressed as:

$$\frac{a}{b} = \frac{F_{d1}}{R_{d1}} < 1 \quad (6)$$

Let’s define a ratio $n$:

$$n = \frac{F_{d2}}{F_{d1}} = \frac{R_{d2}}{R_{d1}} < 1 \Rightarrow R_{d2} = n \cdot R_{d1} \quad (7)$$

When we substitute Eq.7 into Eq. 2 (which describes the failure mode “b”) and make some mathematical simplifications, we will get the design load-carrying capacity $R_{d1}$ depending on $n$:

$$R_{d1} = \frac{k_{mod} \cdot t_2 \cdot f_{h2,k} \cdot d}{\gamma_M \cdot (1 + n)} \quad (8)$$

And similarly, when we substitute Eq.7 into Eq. 3 (which describes the failure mode “c”), we will get the design load-carrying capacity $R_{d1}$ depending on $n$: 
\( R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \sqrt{\frac{t_2^2 \cdot 2 \cdot (1 + n^2) + (1 + n)^2 \cdot \frac{8 \cdot M_{yk}}{f_{h,2,k} \cdot d} + t_2 \cdot (n - 1)}{(1 + n)^2}} \) (9)

5 Summary

The formulas for assessment of reliability in terms of ultimate limit state of bolted connection subjected to unequal double shear with thick plates as outer members were derived.

References

2. K.W. Johansen, Theory of timber connections. International Association of Bridge and Structural Engineering. 9, 249-262 (1949)