Optimization of Temperature Distributions in Critical Cross-sections of Load-bearing Structures of Measurement Optical Systems of Autonomous Objects

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Abstract. Problem of the automatic thermogradient stabilization of the constructive elements sizes is discussed. The actual problem is the determination of the control algorithm controlled heat sources, providing them with the minimum possible number of minimal deviation from the desired temperature in a given cross-section design. The proposed procedure for solving optimal control of temperature distribution in the numerical design of its implementation, allows obtaining results that are suitable for approximate implementation in on-Board computers, Autonomous systems and significantly reduce thermal deformation component of the measurement error.

1 Problem statement

Reduction thermodeformation load-bearing structures, which are placed on heat-producing equipment, greatly reduces the accuracy of optical measurements because of thermal deformation of bearing structures violate the optical configuration of measuring devices [1]. In this case, the current task is the determination of the control algorithm controlled heat sources, providing them with the minimum possible number of minimum deviations from the desired temperature in a given cross section of the structure. The choice of section is determined by a complex solution of thermal deformation and optomechanical tasks for the corresponding layout of equipment on the design.

2 Numerical calculations

In [1] for the supporting structure in the form of a rectangular prism is justified for two nonsmooth boundary value problems of optimal control, which, taking into account the specifics of operation of optical equipment placed on the supporting structure, can be modified to a spatially one-dimensional nonsmooth boundary value problems for semi-

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infinite optimization, for example on the line in a given section $l_y^* \in l_y$, $l_y^* = const$; $l_z^* \in l_z$, $l_z^* = const$; in relative (l_x, l_y, l_z) Cartesian coordinates

1. Maximum accuracy task - the task of achieving the minimum result of ($\varphi = \varphi_k$) the deviation $J_{\min} = \min\limits_{U^{(1)}(\varphi)} J[\Theta(l_x, l_y^*, l_z^*, \varphi_k)]$ of temperature $T(l_x, l_y^*, l_z^*, \varphi_k)$ from a specified base $T_{des} = const$ to the specified section $L(l_x, l_y^*, l_z^*)$

2. Maximum performance task – goals, which embed the minimum time $\varphi_k^{(\min)} = \min\limits_{U^{(2)}(\varphi)} \varphi_k$ resulting temperature distribution $T(l_x, l_y^*, l_z^*, \varphi_k^{(\min)})$ inside the specified region ε_{des}, $J \le \varepsilon_{des}$.

$J = \left\| \Theta(l_x, l_y^*, l_z^*, \varphi_k) \right\|_{L_\infty[l_x \in L]}$. $\varphi_k = \dfrac{\lambda \tau_k}{C x^2_{\max}}$ – the relative end time of the process, λ, C - thermal conductivity and specific heat of the material of construction.

The solution of these problems is reduced to the determination of the optimum law of change of vector control $U^{(\beta\beta)}(\varphi, l_x, l_y, l_z) = [q_x^{(\alpha\alpha)}(\varphi, l_y, l_z), q_y^{(\alpha\alpha)}(\varphi, l_x, l_z), q_z^{(\alpha\alpha)}(\varphi, l_x, l_y)]$, $\beta = 1{,}2$, providing $J^{opt}(q_{x(opt)}^{(\alpha\alpha)}, q_{y(opt)}^{(\alpha\alpha)}, q_{z(opt)}^{(\alpha\alpha)}) = \min\limits_{q_x^{(\alpha\alpha)}, q_y^{(\alpha\alpha)}, q_z^{(\alpha\alpha)}} J$ at the specified value $\varphi = \varphi_k$ – maximum accuracy task(β=1) or $\min\limits_{q_x^{(\alpha)}, q_y^{(\alpha)}, q_z^{(\alpha)}} \varphi_k$ at the specified value $J = J_{des}$ – maximum performance task (β=2) in terms of restrictions on the allowable temperature $\Theta_d = const$.

$$\Theta(\rho, \varphi) \le \Theta_d, \varphi \in (0, \infty), \rho \in \overline{\Omega}_\rho \quad (1)$$

and resource control:

$$\begin{aligned} &\left| q_x^{(\alpha\alpha)}(\varphi, l_y, l_z) \right| \le 1 \;; \alpha = 3.4; l_x = 0; l_x = 1; \\ &\left| q_y^{(\alpha\alpha)}(\varphi, l_x, l_z) \right| \le 1 ; \alpha = 5.6; l_y = 0; l_y = l_{y_{\max}}; \\ &\left| q_z^{(\alpha\alpha)}(\varphi, l_x, l_y) \right| \le 1 ; \alpha = 1.2; l_z = 0; l_z = l_{z_{\max}}; \end{aligned} \quad (2)$$

when exposed to a control object of the internal and external disturbances. In this example (Fig. 1) the temperature of the inner surface of the outer shell of the supporting structure has a constant time value T_{out_i}. The temperature of the outer shell affects the temperature of the faces of the plate by radiant heat transfer. From opposite sides of the supporting structure has a rectangular area, S1 and S2, which, through radiant heat transfer affects the temperature of the environment. On the lateral surfaces of construction installed heat-emitting devices, occasionally included in the work and controlled heat sources 1,2,3,4,5,6 attached to the surface areas S1 and S2 on opposite faces $\alpha = 3$ and $\alpha = 4$ of the supporting construction.

A valid control $\left| U^{(\beta\beta)}(\varphi, l_x, l_y, l_z) \right| \le 1$ in this example can be represented in a multiplicative form: $q_x^{(\alpha\alpha)}(\varphi, l_y, l_z) = U_x^{(\alpha\alpha)}(\varphi) V_x^{(\alpha\alpha)}(l_y, l_z)$, $\alpha = 3.4$; $\alpha = 1{,}2{,}5{,}6$, $q_x^{(\alpha\alpha)}(\varphi, l_x, l_y) = q^{(\alpha\alpha)}(\varphi, l_x, l_y) \equiv 0$, fixed in advance the spatial distribution $V_x^{(\alpha\alpha)}(l_y, l_z)$ of the controllable heat sources on two opposite faces $\alpha = 3{,}4$ with a specified coordinate location $V_x^{(\alpha\alpha)}, \alpha = 3.4$. Under these conditions, the optimal control of autonomous $\overline{U}^{(\beta\alpha)}(\varphi)$ groups of heat sources each α-th face:

$$\overline{U}^{\beta\alpha}(\varphi) = \left[U_x^{(\beta 3)}(\varphi), U_x^{(\beta 4)}(\varphi), U_y^{(\beta 5)}(\varphi), U_y^{(\beta 6)}(\varphi), U_z^{(\beta 1)}(\varphi), U_z^{(\beta 2)}(\varphi)\right] \quad (3)$$

$$U_{opt}^{(1\alpha)}(\varphi) = \arg \min_{\overline{U}^{(1\alpha)}(\varphi)} J \; ; \; U_{opt}^{(2\alpha)}(\varphi) = \arg \min_{\overline{U}^{(2\alpha)}(\varphi)} \varphi_k \; ; \quad (4)$$

in the problem of maximum precision (β=1) and task performance (β=2) using the method of moments [2,3] it is possible to parameterize and to reduce to the problem of finding a finite number of parameters $\Delta_\delta^{(i)}$, δ=1,2, *i;* *i*=1,2,..*J*; where $\Delta_\delta^{(i)}$ the duration δ–th interval of constancy of the total number i, where each vector $\overline{U}^{(\beta\alpha)}(\varphi)$ component alternately takes its maximum value at the level of the constraints (2).

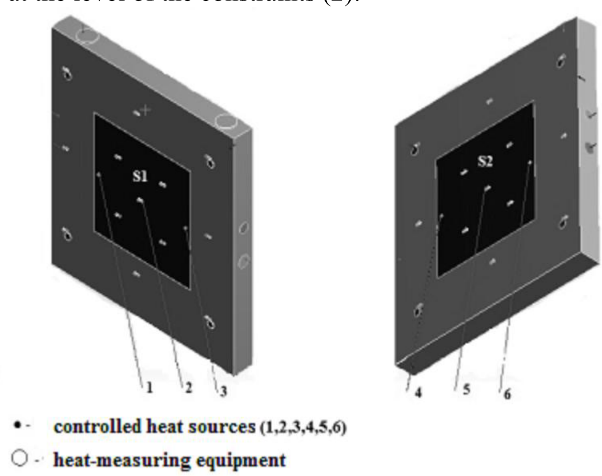

Fig. 1. The location of the heat sources on the supporting structure (example).

The duration $\Delta_\delta^{(i)}$ and the desired number of intervals *i*, is determined by the alternance method [3] from the system of transcendental equations:

$$\Theta(\rho_k, \varphi_k)\big|_{\rho_k \in L_k} = \pm\varepsilon^{(i)}, grad\Theta(\rho_k, \varphi_k)\big|_{\rho_k \in L_k} = 0 \quad (5)$$

where $\Theta(\rho_k, \varphi_k)\big|_{\rho_k \in L_k}$ - numerical solution of boundary value problems in the final time φ_k points $\rho_k(l_x^{(k)}, l_y^*, l_z^*) \in L_k(l_x^{(k)}, l_y^*, l_z^*, \varphi_k) \subset L(l_x, l_y^*, l_z^*)$ of the considered line section $L(l_x, l_y^*, l_z^*)$, which constitute a countable set of limit points, which is the ratio (5).

The procedure of solving the transcendental system (5) is a specially designed software package [5] based on the numerical solution of boundary value problems in ANSYS. Figure 2 shows the resulting deviations from the set temperature to the responsible of the line-section $L(l_x, l_y^*, l_z^*)$ $l_y^* = 0.5$, $l_z^* = 0.5$ load-bearing structure for optimal accuracy (β=1) 1 and 2-interval control. The maximum on this line $L(l_x, l_y^*, l_z^*)$ the temperature deviation does not exceed 4° when exposed to the most severe variant of the heat load.

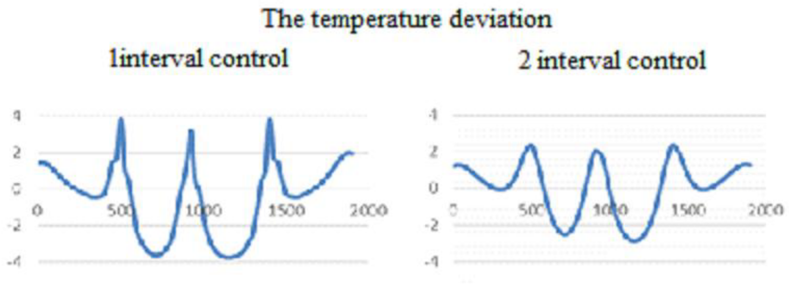

Fig. 2. Chart of deviation from specified temperature on the section line $L(l_x, l_y^*, l_z^*)$, $l_y^* = 0.5$, $l_z^* = 0.5$

3 Conclusions

The proposed procedure for solving optimal control of temperature distribution in the numerical design of its implementation, for example in the ANSYS software environment allows to obtain results that are suitable for approximate implementation in on-board computers, Autonomous systems and significantly reduce thermal deformation component of the measurement error.

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