

Research of interaction of a natural and forced convection in a vortex chamber of the chemical reactor

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Abstract. In this paper, a chemical reactor for producing refractory metals was considered. A physical and mathematical model of fluid motion and heat transfer in a vortex chamber of the chemical reactor under forced and free convection has been described and simulated. The numerical simulation was carried out in "velocity–pressure" variables by using an alternating direction implicit scheme. The velocity field and the temperature distribution in the reactor were obtained. Parametric studies on effects of the Reynolds, Prandtl and Rossbi criteria on the flow characteristics were also performed. The graphs presented show that natural convection has a significant impact on the hydrodynamics of the flow and intensifies the heat transfer. Reliability of the calculations was verified by comparing the results obtained by another method

1 Problem statement

Modeling of viscous gas dynamics and heat transfer is considered in a vortex chamber that present a cylindrical chamber (See Fig. 1).

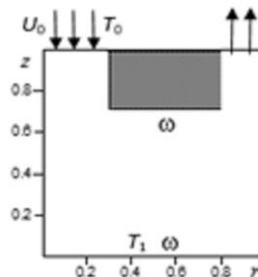


Fig.1. The computational domain of the vortex chamber.

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The carrier gas flow having axial velocity U_0 and temperature T_0 enters from the pipe along the axis above, flows over the rotating disk and exits through the annular channel at the periphery of the top of the vortex chamber. The top and bottom walls of the apparatus are rotating with an angular velocity ω . Part of the bottom wall is maintained at a temperature T_1 , and other walls including a small surface at the periphery of the bottom wall of the chamber are considered as heat-insulated. By virtue of relatively small velocities and temperature differences, gas is assumed incompressible and density change is taken into account only in terms that includes the force of gravity and besides density difference is proportional to the temperature difference with the opposite sign according to the Boussinesq approximation. Subject to the foregoing, non-dimensional transport equations of momentum and heat and continuity equation in cylindrical coordinates in the assumption of axial symmetry in angular direction are follows:

$$\begin{aligned} \frac{\partial u}{\partial \tau} + \frac{\partial u^2}{\partial r} + \frac{\partial uw}{\partial z} - \frac{1}{\text{Re}} \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} \right) &= -\frac{\partial p}{\partial r} + \frac{1}{r \text{Re}} \frac{\partial u}{\partial r} + \frac{v^2 - u^2}{r}; \\ \frac{\partial v}{\partial \tau} + \frac{\partial uv}{\partial r} + \frac{\partial vw}{\partial z} - \frac{1}{\text{Re}} \left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} \right) &= \frac{1}{r \text{Re}} \frac{\partial v}{\partial r} - \frac{2uv}{r}; \\ \frac{\partial w}{\partial \tau} + \frac{\partial uw}{\partial r} + \frac{\partial w^2}{\partial z} - \frac{1}{\text{Re}} \left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} \right) &= -\frac{\partial p}{\partial z} + \frac{1}{r \text{Re}} \frac{\partial w}{\partial r} - \frac{uw}{r} + \frac{Gr}{\text{Re}^2} \vartheta; \\ \frac{\partial \vartheta}{\partial \tau} + \frac{\partial u\vartheta}{\partial r} + \frac{\partial w\vartheta}{\partial z} - \frac{1}{\text{Pr Re}} \left(\frac{\partial^2 \vartheta}{\partial r^2} + \frac{\partial^2 \vartheta}{\partial z^2} \right) &= \frac{1}{\text{Pr Re } r} \frac{\partial \vartheta}{\partial r} - \frac{u\vartheta}{r}; \\ \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} &= 0. \end{aligned} \quad (1)$$

The dimensionless form of equations is obtained by using the following scales: vortex chamber radius R_0 , axial velocity at the input U_0 , density at the input ρ_0 and the maximum temperature difference $T_1 - T_0$.

The system of equations obtained contains criteria of Reynolds, Prandtl and Grashof:

$$\text{Re} = \frac{\rho_0 U_0 R_0}{\mu}; \quad \text{Pr} = \frac{c_p \mu}{\lambda}; \quad \text{Gr} = \frac{g \beta (T_1 - T_0) \rho_0 (R_0)^3}{\mu^2}. \quad (2)$$

The dimensionless temperature is determined as $\vartheta = (T - T_0)/(T_1 - T_0)$. In addition, the following notations are introduced: u is radial velocity component, v is peripheral velocity component and w is axial velocity component.

2 Numerical calculations

The numerical simulation of a set (1) was carried out in “velocity–pressure” variables using method of pressure and velocity correction [1]. One can present a system of equations consisting of the momentum equations and the continuity equation in the vector form as follows:

$$\frac{\partial \mathbf{V}}{\partial t} = -\frac{\nabla p}{\rho} + F(\mathbf{V}); \quad \text{div} \mathbf{V} = 0. \quad (3)$$

One can obtain the two equations using time-splitting method for the momentum equation:

$$\frac{\mathbf{V}^+ - \mathbf{V}^n}{\Delta t} = -\frac{\nabla p^n}{\rho} + F(\mathbf{V}^+, \mathbf{V}^n); \quad (4)$$

$$\frac{\mathbf{V}^{n+1} - \mathbf{V}^+}{\Delta t} = -\frac{\nabla(\delta p)}{\rho}, \quad (5)$$

Here δp is the pressure correction which is equal to the difference between the pressures at new $n+1^{\text{st}}$ layer of time and at the n^{th} layer known, \mathbf{V}^+ is an intermediate grid function. After scalar multiplication of the equation (5) by the gradient and taking into account the solenoidality of velocity on the $n+1^{\text{th}}$ layer, we have obtained the Poisson equation for the calculation of pressure correction as follows:

$$\nabla^2(\delta p) = \rho \frac{\nabla \cdot \mathbf{V}^+}{\Delta t}. \quad (6)$$

One can represent this equation as transient Poisson equation for the convenience of calculation:

$$\frac{\partial(\delta p)}{\partial \tau} - \nabla^2(\delta p) = -\rho \frac{\nabla \cdot \mathbf{V}^+}{\partial t}. \quad (7)$$

In Eq. (8) $\Delta \tau = B \Delta t$ is the time step performing the role of iteration parameter which value is chosen so that ensure the most rapid convergence of problem solution. For the difference scheme to construct, it is used a staggered grid. The set of equations obtained was being solved by using an evolutionary method of relaxation in time. For the each equation in the set obtained to solve, an alternating direction implicit scheme was used. This method is second-order accurate in time and it is unconditionally stable. Convective and diffusion terms in transport equations for a staggered grid are represented by dint of the Exponential Scheme [2] based on the control volume method.

For a unique solution to obtain, the boundary conditions in dimensionless form are set. At the input of chamber we have:

$$\frac{\partial u}{\partial z} = 0, \quad v = 0, \quad w = -1, \quad \vartheta = 0.$$

At the symmetry axis by $r = 0$ we have

$$u = 0, \quad v = 0, \quad \frac{\partial w}{\partial r} = 0, \quad \frac{\partial \vartheta}{\partial r} = 0.$$

At walls of chamber there are no-slip and heat insulation conditions:

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial \vartheta}{\partial n} = 0,$$

with the exception of rotating surfaces, for which no-slip condition cause an emergence of another criterion, the inverse Rossby number $R\omega = R_0\omega/U_0$. The value of peripheral velocity at the rotating surface is $v = R\omega \cdot r$. We have $\vartheta = 1$ at the bottom wall by $r < 0.8$ and $\partial\vartheta/\partial z = 0$ by $0.8 < r < 1$ (heat insulation condition).

At the chamber output, the Neumann conditions are used:

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial z} = \frac{\partial \mathcal{G}}{\partial z} = 0.$$

3 Conclusions

The results of the numerical solution of the set (1) are presented in Fig. 2 and 3, and besides, Fig. 2 illustrates the solution by natural convection not taken into account and Fig. 3 shows the solution with natural convection by the same parameters and criteria of the flow considered. The figures presented show that a natural convection impacts significantly on the swirling flow hydrodynamics. A position of the circulation zone of the gas streamlines changes. The area of large values of peripheral velocity moves in the centerline direction. The temperature increases in a large part of the vortex chamber. This evidences the intensification of heat transfer from the bottom wall of vortex chamber.

For the calculations verification the problem considered was converted to “vortex–stream function” variables and solved by the alternating direction implicit scheme using non-staggered grid. This method gave the same results. In additional, the created model was applied for numerically solving the classical test problem of the velocity distribution in an annular channel and that of a rotating infinite disk in a stationary liquid. The study findings showed a good agreement with the exact solutions.

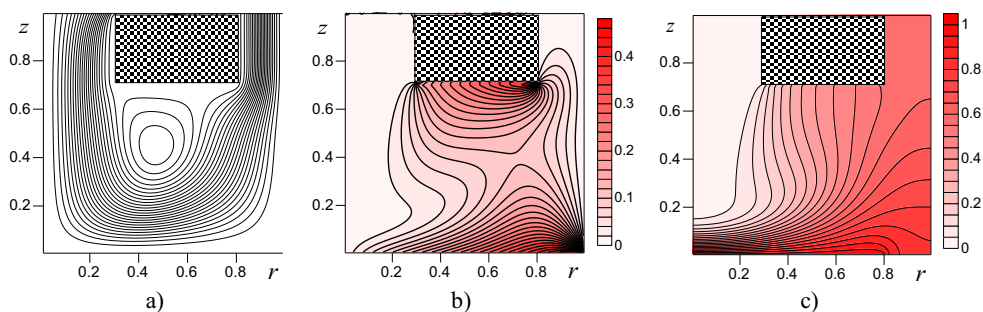


Fig.2. Stream function (a), peripheral velocity (b) and temperature (c) distribution at $Re=50$, $R\omega=0.5$, $Pr=1$, $Gr=0$.

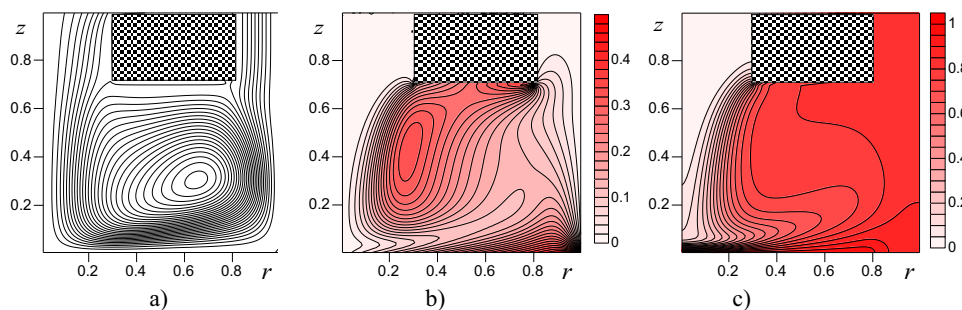


Fig.3. Stream function (a), peripheral velocity (b) and temperature (c) distribution at $Re=50$, $R\omega=0.5$, $Pr=1$, $Gr=10^5$.

References

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