ANALYTICAL ESTIMATION OF THE DISTRIBUTION OF THERMAL NEUTRON FLOW

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Abstract. The neutron’s evaluation of impact on the internal reflector as an example of the plate reactor in single-group approximation. As a result there were obtained wave equations analytical solutions and graphic solution of the formulated problem.

1 Introduction

The distribution of critical size or fuel critical concentration (mass) is the main task of the nuclear reactors theory. Consideration of the critical problem was limited of classical forms reactors (sphere, infinite cylinder and plate) without neutron reflector and with it.

Estimation of internal reflector considerate only limiting case of its diffusivity: for vacuum and black body (the diffusion equation is eliminates).

More than half a century of nuclear reactor operating experience identified the reasonability using internal and external reflectors [1, 6], also it’s shown by previously researches [2, 3].

E.g., in research reactor IRT-T using not on only external but also internal reflectors can lead to increase of the average density of the neutron flow [4]. Also we choosing beryllium as the material of the reflector cause it increase of the average density of the neutron flow. In general case flow distribution is carried out by numerical methods in multi-group approximation.

2 Mathematical problem and study technique

The distribution of the neutron flux in the core, the inner and outer reflectors in the single-group approximation is usually described by the following wave equations:

\[
\frac{d^2 \Phi_1}{dx^2} + \chi_1 \cdot \Phi_1 = 0, \tag{1}
\]

\[
\frac{d^2 \Phi_2}{dx^2} - \chi_2 \cdot \Phi_2 = 0, \tag{2}
\]
Here $\chi_1$, $\chi_2$, $\chi_3$ are the material parameters of the core and reflectors.

Figure 1 is showing schematic distribution of thermal neutron flux in single-group approximation.

Using the following conditions we will find the equations solution (1), (2), (3):

1. $x = 0$, $\nabla \phi(x, 0) = 0$,  

2. $x = \frac{H_1}{2}$, $\phi\left(\frac{H_1}{2}\right) = \phi_1\left(\frac{H_1}{2}\right)$,  

3. $x = \frac{H_1}{2}$, $D_2 \nabla \phi_2\left(\frac{H_1}{2}\right) = D_1 \nabla \phi_1\left(\frac{H_1}{2}\right)$,  

4. $x = \frac{H_2}{2}$, $\phi\left(\frac{H_2}{2}\right) = \phi_3\left(\frac{H_2}{2}\right)$,  

5. $x = \frac{H_2}{2}$, $D_2 \nabla \phi_2\left(\frac{H_2}{2}\right) = D_3 \nabla \phi_3\left(\frac{H_2}{2}\right)$,  

6. $x = \frac{H_1}{2} = \frac{H_2}{2} + \delta \phi$, $\phi_3\left(\frac{H_2}{2}\right) = 0$.  

Here $D_i$ is diffusion coefficient.

Application of the external reflector reduces loss of neutrons and saves core equal to effective additive [3].

$$\delta \phi = \frac{1}{\chi_1} \arctg \left( \frac{D_1 \chi_1}{D_3 \chi_3} \theta (\chi_1, T) \right).$$

Now introducing an effective boundary condition:
7. \[ x = \frac{H_{\text{i}}}{2} + \delta_{\phi}, \quad \Phi \left( \frac{H_{\text{i}}}{2} + \delta_{\phi} \right) = 0. \] (10)

Solutions of wave equations (1), (2) are following:
- core in infinite plate form
  \[ \Phi_{1}(x) = c_{1} \cdot \sin(x) + c_{2} \cdot \cos(x). \] (11)
- reflector in finite plate form
  \[ \Phi_{2}(x) = c_{3} \cdot \sinh(x) + c_{4} \cdot \cosh(x). \] (12)

After mathematical simplifications, finding the constant of integration it was obtained the critical equation for core in infinite plate form with an internal reflector:
\[ D_{2} \cdot \chi_{2} \cdot \text{th} \left( \frac{\chi_{2} \cdot H_{1}}{2} \right) = -D_{1} \cdot \chi_{1} \cdot \text{ctg} \left( X_{KP} \right), \] (13)

where:
\[ X_{KP} = \chi_{1} \cdot \left( \frac{H_{1}}{2} - \frac{H_{1}}{2} \right). \] (14)

3 Experimental results

Knowing the thickness of external reflector and material parameters, you can define a critical value of core size.

![Graph showing neutron flow distribution](image)

**Fig. 2.** The distribution of neutron flow for a core and for an internal reflector in single-group approximation.

Reflectors efficiency can be evaluated not only for saving active zone but also by the average thermal neutron flux ratio change in the core to its maximum value. For the reactor
without reflector this value [5, 6] equals 0.637. If we have a beryllium reflector – it will be 0.939. In this research it’s equal 0.82.

4 Conclusion

Research’s result can be explained by the fact that single-group approximation isn’t take into account the contribution to the accumulation of thermal neutrons in the reflector due to the slowing down of fast neutrons.

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