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The investigation of the dynamic behaviour of a complex assembled structure using the frequency response function based substructuring method

Wan Imaan Izhan Wan Iskandar Mirza[1], *Muhamad Norhisham* Abdul Rani[1,*], *Mohd Azmi* Yunus[1], *Liyana* Roslan[2] and *Ahmad Burhani* Ahmad Basri[1]

[1]Dynamics and Control Laboratory, Faculty of Mechanical Engineering, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Malaysia
[2]Faculty of Mechanical Engineering, Universiti Teknologi MARA Kampus Pasir Gudang, 81750 Masai, Johor, Malaysia

Abstract. The frequency response function based substructuring method (FRF-BSM) for modelling and investigating dynamic behaviour of engineering structures has received much attention in recent years among modal analysts. However, the accuracy and efficiency of the predicted dynamic behaviour of the structures via the method is often found to be different from the test data. The discrepancy is believed to be the result of the coupling types used in the modelling. This paper aims to investigate the potential candidates of coupling types for FRF based substructuring in predicting the dynamic behaviour of a complex assembled structure which consists of a large flat span and two simplified aircraft pylons. Modal tests are performed to measure the dynamic behaviour of the assembled structure and its components. The finite element method is used for constructing analytical models of the assembled structure. The FRF-BSM is then used for the assembly of the span and pylons, and also to predict the dynamic behaviour of the assembled structure using rigid and elastic coupling. The comparison of results revealed that elastic coupling has demonstrated better capabilities to represent the bolted joints in the test structure, which may due to the coupling calculated is reasonably representing the stiffness of the bolted joints.

1 Introduction

In the field of structural dynamics, dynamic substructuring method has played an important role and has continued to be a great value since five decades ago. Dynamic substructuring is a process which divides an assembled structure into subsystems which can be analyzed or tested individually and then combining them by an assembly procedure to predict the response of the build-up structure [1-3]. This method has been used to evaluate the dynamic behaviour of a large complex structure and to give possibility to combine the analytical

* Corresponding author: mnarani@salam.uitm.edu.my

with the experimental parts. It also allows combining and sharing subsystems from different project groups. In recent times, de Klerk [4] has summarised the history, review, classification of techniques and has provided a general framework for dynamic substructuring. In general, the dynamic substructuring is divided into two different classes which are the time-domain and frequency-based method [5]. FRF based substructuring (FBS) is the process that predicts the FRF of a combined structure (coupled system) on the basis of free interface FRF of the substructures (uncoupled system). The equations for the FRF"s of the combined system are derived by balancing the forces and enforcing continuity at the interface [6,7]. The FBS represents the dynamic behaviour of the coupled system with FRFs, rather than set of discretized mass and stiffness equations.

By using the FBS method, the FRFs of the subsystem model can be either experimentally measured or analytically derived. This enable to combine the FRF's from the test data with the analytical model in order to predict the FRF of the whole combine system. This is the main advantage of this method. Allen [8-10] and Mayes [11, 12] presented several approaches about coupling the analytical model and some experimental issues related to FBS especially during the measurements of rotational degree of freedoms at the coupling. Neglecting the rotational degree of freedom will affect the FRF of the assembled system, as presented by Manzato recently in [13]. Hence, for this study, the rotational degree of freedom for the subsystems were included to archive a high degree of accuracy.

Nevertheless, when it comes to analytical modelling the dynamic behaviour of a structure with multiple joints, the results calculated from this method may differ from the test data. This is believed to be a result of the inappropriate coupling types used to represent the joints. Many researchers have developed FBS schemes for the particular case studies. However, little attention has been given on the effects of the coupling type used in modelling and predicting the dynamic behaviour of a structure. Nevertheless, modelling bolted joints is becoming very difficult because of the properties of the joint itself which is complex to be understood [14, 15]. In addition, there is no universal method to model dynamics of joints so much so that each application has been investigated on case by case basis [16]. For dynamic substructuring method, the properties of coupling need to be considered and understood in order to obtain a satisfactory level of accuracy in dynamic behaviour prediction. Although each subsystem is modelled flawlessly, the invalid assumption in coupling type to assemble the subsystems may lead to discrepancies in dynamic behaviour prediction of coupled system.

This work is concerned with the study of appropriate coupling types used for multiple bolted joints in a structure. The appropriate coupling type is presented for the investigation of the dynamic behaviour of the structure. Bolted joints have been chosen for this study as these joints are commonly used in most engineering application and are also used by modal analysts in their research related to FBS in [8, 10, 12]. For the dynamic substructuring model, the FRFs of the subsystems were derived based on the analytical model of the subsystems and are assembled together using rigid and elastic type of coupling.

1.1 Theoretical explanation of FRF coupling

Consider a system(S) consists of subsystem (A) and (B) which the DOFs are classified either as internal or coupling DOFs as shown in Figure 1. Both subsystems are independent of one another before the coupling. The c-DOFs must be the same for both subsystems.

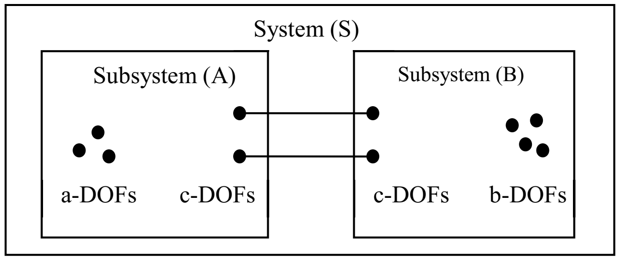

Fig. 1. Schematic diagram of systems (s), subsystem (A) and subsystem (B).

The equation of motion for each subsystem in the frequency domain is:

$$\{X\}_n = [H]_{nn}\{F\}_n \tag{1}$$

where $\{X\}_n$ is the complex displacement, $[H]_{nn}$ is the complex admittance matrix in form of displacement, force $\{F\}_n$ is the applied force vector and the subscript n is the total number of DOF for each subsystem.

The basic governing relationship for subsystem A:

$$\left\{\frac{\{X^A\}_a}{\{X^A\}}\right\}_n = \begin{bmatrix}[H^A]_{aa} & [H^A]_{ac}\\ [H^A]_{ca} & [H^A]_{cc}\end{bmatrix}\left\{\frac{\{F^A\}_a}{\{F^A\}_c}\right\}_n \tag{2}$$

and for subsystem B:

$$\left\{\frac{\{X^B\}_b}{\{X^B\}_c}\right\}_n = \begin{bmatrix}[H^B]_{bb} & [H^B]_{bc}\\ [H^B]_{cb} & [H^B]_{cc}\end{bmatrix}\left\{\frac{\{F^B\}_b}{\{F^B\}_c}\right\}_n \tag{3}$$

For the rigid connections between subsystems A and B, compatibility implies that

$$\{X^A\}_c = \{X^B\}_c = \{X^C\}_c \tag{4}$$

and the force equilibrium requires that

$$\{F^A\}_c = \{F^B\}_c = \{F^C\}_c \tag{5}$$

The FRFs of the system can be defined as

$$\begin{Bmatrix}\{X^s\}_a\\ \{X^s\}_c\\ \{X^s\}_b\end{Bmatrix}_n = \begin{bmatrix}[H^s]_{aa} & [H^s]_{ac} & [H^s]_{ab}\\ [H^s]_{ca} & [H^s]_{cc} & [H^s]_{cb}\\ [H^s]_{ba} & [H^s]_{bc} & [H^s]_{bb}\end{bmatrix}\begin{Bmatrix}\{F^s\}_a\\ \{F^s\}_c\\ \{F^s\}_b\end{Bmatrix}_n \tag{6}$$

The c-DOFs from the partitioned equation of equation (2) and (3) are derived and can be equated and solved for the connection force as

$$\{\tilde{F}^A\}_c = [[H^A]_{cc} + [H^B]_{cc}]^{-1}[[H^B]_{cb}\{F^B\}_c - [H^A]_{ca}\{F^A\}_a + [H^B]_{cc}\{F^S\}_c] \tag{7}$$

with $\{\tilde{F}^A\}_c$ is the internal transmitted force at the interfaces of subsystem A at the coupling. The equation derived above can be used to determine the coupled system FRFs on the basis of the uncoupled FRFs of the individual components. While for the elastic coupling, a joint can be described analytically as follows;

$$\{F\} = [Z]\{X\} \tag{8}$$

where F is the force vector, Z is the impedance matrix at the coupling and X is the displacement vector. The detail derivation of the coupling with joints can be found in [17].

2 Description of the test structure

The structure under the investigation was a mimicked version of an aircraft. It consists of three major substructures which are a unit of a large span plate and two units of a mimicked version of aircraft pylons. The large span substructures was made of an aluminium sheet with 4mm thickness while each of the pylon consists of seven other components which were assembled together using bolted joints. The three major substructures were assembled together using eight bolted joints as shown the Figure 2. To ensure the characteristic of bolted joints are similar to each other, the same amount of torque force has been applied. In this study, the assembled structure was numerically and experimentally investigated.

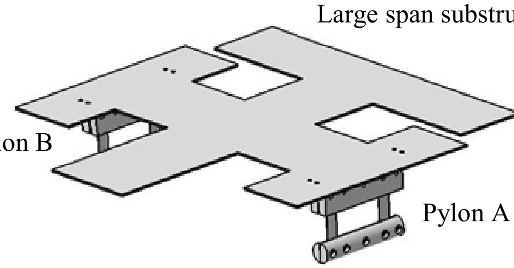

Fig. 2. Assembled structure.

3 FRF based substructuring method

In this investigation, the FRFs of the subsystems were derived from the numerical model. Prior to performing the FRF based substructuring assembly analysis of the assembled structure, each of the subsystem was numerically solved under normal mode solutions. The updated material properties were used to the subsystems to minimize components error. The desired excitation and response points are derived in the form of FRF for each of the coupling points. In general, there are two methods to derive the FRFs for the numerical model as discussed in [18]. The first method is by derivation of the FRFs from the numerical model of the subsystems from the eigenvalues. The second method is by performing an FRF synthesis based on a finite number of the modal parameters of the subsystem. Since the natural frequencies and mode shapes of the components were already obtained from the normal mode solution of the numerical model, the second method was used to obtain the FRFs. For FRF synthesis method, the synthesized FRF matrix $H_{syn}(\omega_k)$ and mode shapes are expressed by:

$$\mathrm{H_{syn}}(\omega_\mathrm{k}) = \sum_{i=1}^{N} \frac{\{\emptyset\}_i\{\emptyset\}_i^T}{\left(\omega_{n_i}^2-\omega_k^2\right)+j2\xi_i\omega_k\omega_{n_i}} \tag{9}$$

where N is the number of calculated modes, $\{\emptyset\}_i$ is the ith mass normalised mode shapes, ω_{n_i} is ith natural frequency and ξ_i is the ih modal damping ratio.

The subsystems with synthesized FRFs were assembled in Virtual.Lab software as shown in Figure 3. The frequency of interest was set between 1-30 Hz with 0.5 Hz frequency step. For this case study, two types of coupling have been used which are the rigid and elastic types of coupling in order to identify which type of coupling is suitable to be used for multiple bolted joints. The FRF at the reference point obtained from the FBS analysis was compared with the measured FRF at the reference point for validation process.

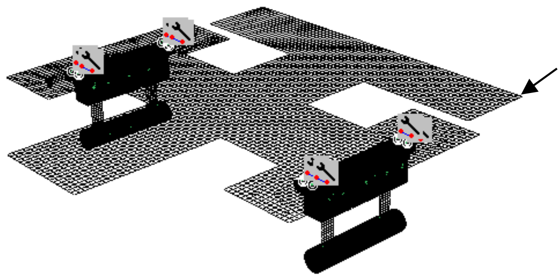

Fig. 3. FBS setup for the assembled structure.

4 Elastic coupling modelling for bolted joints

This way of modelling presented in this section is one of most used in Nastran software, as it is simple but sufficiently accurate. Bolted joints consist of axial, shear and rotational stiffness for the six degrees of freedom (DOF) on each nodes. For the translational DOFs, assume K_1 is the value for the axial stiffness, K_2 and K_3 are the shear stiffness of the bolted joint. The axial stiffness K_1 is obtained from the equation derived based on the Hooke's Law:

$$K_1 = \frac{AE}{L} \tag{10}$$

where A, E and L is the cross-sectional area, Young's modulus and the length of the bolt respectively. The shear stiffness of K_2 and K_3 are calculated using the shear flexibility by Huth formulation:

$$K_2 = K_3 = \frac{1}{c} \tag{11}$$

Which C is the shear flexibility which acquire from equation:

$$C = \left(\frac{t_1 + t_2}{2d}\right)^a \left(\frac{b_1}{t_1 E_1} + \frac{b_2}{t_2 E_2} + \frac{b_1}{2t_1 E} + \frac{b_2}{2t_2 E}\right) \tag{12}$$

Where subscripts t_1 , E_1 and t_2 , E_2 are the thickness and Young's modulus of two plates. b_1 and b_2 are the coefficients depending on the joint plate material, and a is the fastener type coefficient.

For the rotational stiffness of K_5 and $K_{6,}$ the values can be calculated using;

$$K_{5-6} = \frac{1}{\varepsilon}\left(Max(K_2, K_3)\frac{L^2}{4}\right) \tag{13}$$

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Usually, rotational stiffness around the fastener K_4 can be fixed to 100.

5 Test setup of the assembled structure

The structure was tested in free-free boundary conditions by using an impact testing method. To simulate the free-free boundary conditions, the test structure was hanged to the test rig using four nylon strings and springs. From the initial finite element result of the assembled structure, the frequency bandwidth, excitation point and also the mode shapes of the structure has been determined. The frequency under the investigation was between 1-30 Hz where the rigid body modes was not included. An impact hammer was used to excite the structure and 3 rowing accelerometers were used to acquire the dynamic data. The response signals and load were interpreted via LMS SCADAS analysis. The modal parameters from the FRF were extracted from the resonance amplitude of FRF data. The initial results of the finite element model were used for mode pairing purposes.

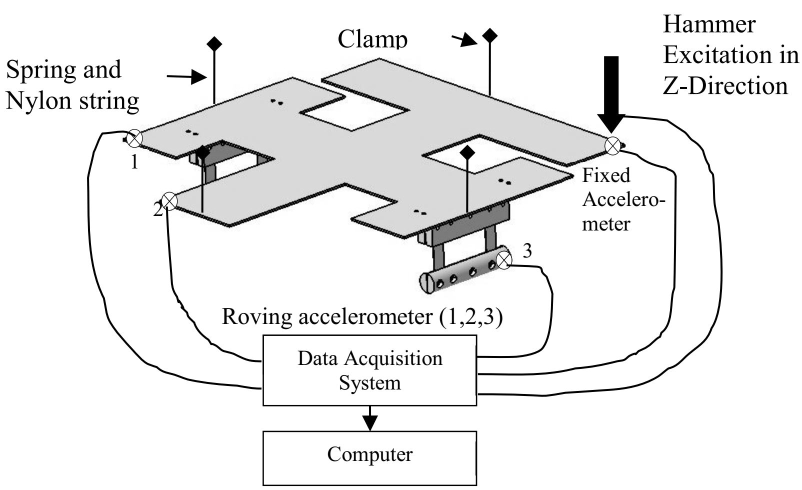

Fig. 4. Test setup for the assembled structure.

6 Results and discussion

Prior to performing the assembly analysis via FBS method for the structure with bolted joints, the dynamic behaviour of each of the subsystem were numerically and experimentally investigated. The measured results were used to reconcile the finite element model of the subsystems in order to minimize the discrepancies within the test data. The percentage error of the predicted result from the finite element model of the subsystems are summarize in Table 1.

Table 1. Natural frequencies and percentage errors of the updated substructures of large flat span, Pylon A and Pylon B.

Mode	Large Span (Hz)	Error with test data	Pylon A (Hz)	Error with test data	Pylon B (Hz)	Error with test data
1	16.33	0.18%	31.14	0.54%	32.05	5.18%
2	17.00	0.59%	37.29	5.72%	38.58	4.21%
3	31.35	0.19%	115.12	0.73%	120.31	1.43%
4	31.63	0.35%	695.01	4.69%	710.39	1.17%
5	37.79	0.19%	715.10	0.50%	733.54	1.81%
		1.50%		12.19%		13.80%

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It can be seen in particular that the total errors for the large span in Table 1 are 1.5 percent, which is the significance achievements in this investigation. This is due to the large flat span was a simple structure without any joints. In contrast, the total percentage errors for both pylon are in high values which are 12.19% and 13.80% for pylon A and pylon B respectively. The high errors were contributed by the second and fourth modes for pylon A while by the first and second modes for the pylon B. It is believed that the source of the discrepancies might as a result of the invalid assumption in finite element modelling of the bolted joints at the subsystems, as discussed in [19, 20]. However, since the numerical results of the two Pylons are above 30 Hz, the finite element models seems to be reasonable as the modes of interest for the assembled system in this study are no more than 30Hz.

On the other hand, the dynamic behaviour of the assembled structure was calculated using the FBS method. For the first investigation of the coupling in FBS, the reconciled model of the subsystems were combine together using rigid type of coupling. The FRF calculated from the use of rigid coupling was compared with the test FRF as presented in Figure 5. The drive point FRF was obtained from both test and finite element model reference point as shown in Figures 3 and 4. Figure 5 shows that amplitude of the rigid coupling FRF is ahead of the resonance peak of the test FRF. This indicated that the numerical model was posed with higher stiffness in comparison with the test structure. This is as the result of the rigid type of coupling at which can be considered as infinitely stiff. Therefore, it may be reasonable to assume that rigid type of coupling seem to lack the capability to bolted joints with an acceptable level of accuracy.

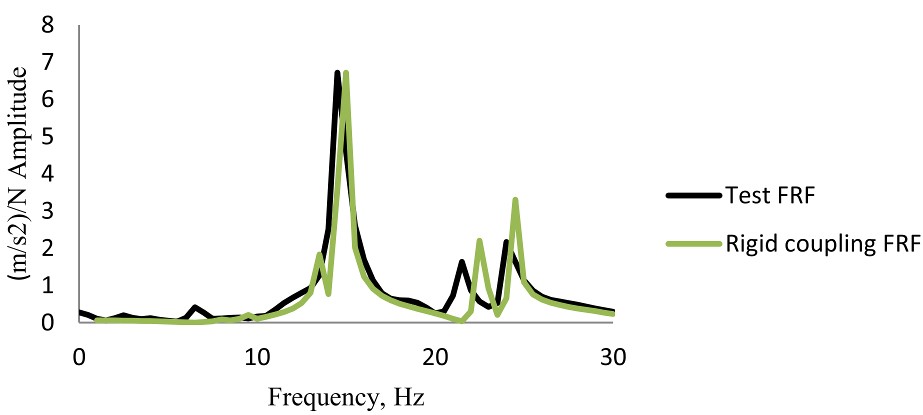

Fig. 5. Comparison of test and rigid coupling FRFs.

In order to reduce the stiffness at the joints, elastic type of coupling has been used to combine the structure. Benefitting from the capability of the elastic type of coupling in predicting the dynamic behaviour of the structure, the stiffness value for all the translations and rotations can be included at the coupling. The stiffness of the coupling are calculated by using the Huth formulation as stated before. The FRF calculated from the using of elastic coupling was compared with the test FRF as presented in Figure 6. The comparison of both FRF seems to be more acceptable compared with the rigid type of coupling. . This may due to the fact that elastic coupling calculated is reasonably representing the stiffness of the bolted joints. Obviously, the elastic type of coupling by using the Huth formulation has better capabilities to represent the bolted joints in the test structure. The comparisons of the FRFs of the measurement, rigid and elastic type of coupling are presented in Figure 7 for better view.

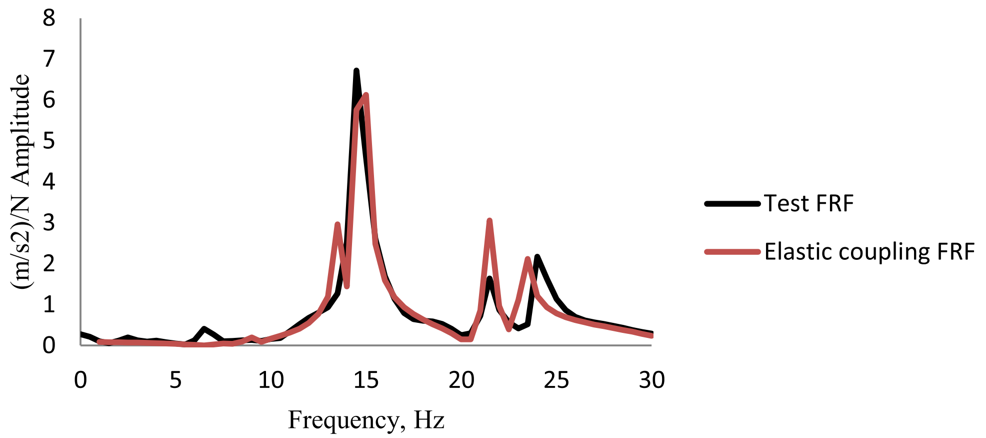

Fig. 6. Comparison of test and elastic coupling FRFs.

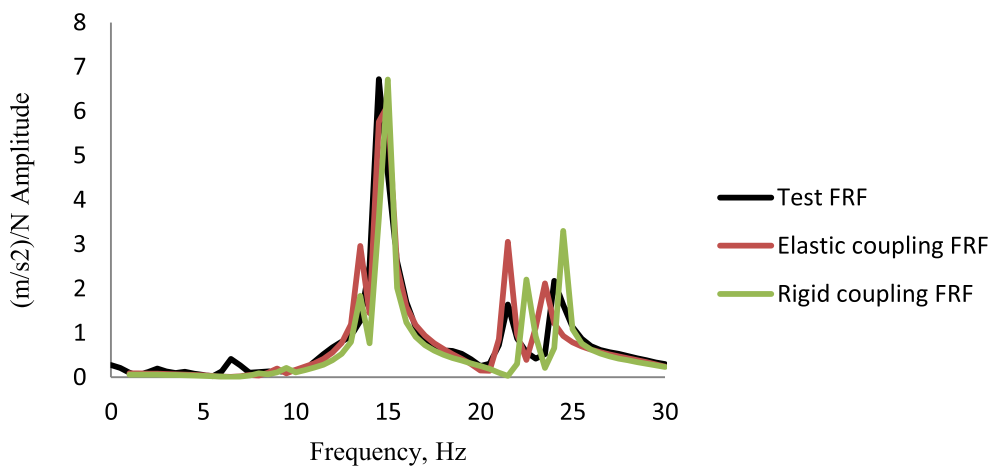

Fig. 7. Comparison of test data, rigid and elastic coupling FRFs.

7 Conclusions

The dynamic behaviour of a simplified model of an aircraft structure with multiple bolted joints was investigated numerically and experimentally. The evaluation of dynamic behaviour of bolted joints via FBS analysis by using two types of coupling which are the rigid type and elastic coupling has been performed by the authors. The test result has been carefully measured and the comparison of results in term of FRFs from the measurement and FBS method is presented. The comparison of results revealed that elastic coupling has demonstrated better capabilities to represent the bolted joints in the test structure. This may due to the fact that elastic coupling calculated is reasonably representing the stiffness of the bolted joints. The study reveals that there are high possibilities to obtain a high degree of accuracy of dynamic behaviour prediction from the use of elastic coupling in combining a numerically derived and experimental model of a structure via FBS method.

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