

## Effects of intermediary reservoir in a two-stage impedance pump

V. C.-C. Lee<sup>1,a</sup> and J.-C. Chong<sup>1</sup>

<sup>1</sup>*Department of Mechanical Engineering, Faculty of Engineering and Science, Curtin University Malaysia*

**Abstract.** Impedance pump is a simple valve-less pumping mechanism, which transport fluid through the mismatch of impedance in the system. A typical open-loop impedance pump consists of an elastic tube, connected to rigid tubing, where the rigid section is connected to a reservoir. Mismatch in impedance occurs when an asymmetrical periodic excitation is exerted on the elastic tube. Studies showed that sequential excitations on the elastic tube infers higher volumetric efficiency than a single excitation. This work studies the effects of an intermediary reservoir between two excitation points on an elastic tube. This study aims to shed some light on the steady state response and fluid motion within the intermediary reservoir; in which increased volumetric efficiency is demonstrated.

### 1 Introduction

Impedance pump is a simple pumping mechanism, which transport fluid through the mismatch of impedance in the system. It is very simple in design, and offers a promising new technique for producing and amplifying a net flow for both macro- and micro-scale devices [1, 2]. It is a valve-less pump, hence does not require impellers to operate. In addition, it offers a low noise and low energy alternatives to current pumping system. A typical open-loop impedance pump consists of an elastic tube, connected to rigid tubing, where the rigid section is connected to a reservoir. Asymmetrical excitation at a single location of the fluid-filled elastic tube will result in unidirectional flow due to the mismatch in impedance. Such pumping mechanism has shown to be highly sensitive towards the impedance in the tube, the location, and excitation frequency [1, 3-6]. The first demonstration of valve-less pumping through an impedance pump, was demonstrated by Gerhart Liebau in 1954, using an elastic tube connected to reservoirs at different heights [7, 8].

Following Liebau's work in 1954, there have been several researches studying the underlying physics of the system, be it numerical or experimental. The major contributions includes Hickerson in 2005 [5], Loumes in 2007 [9], Rosenfeld in 2010 [10], Meier in 2011 [11], and Lee in 2012 [12]. All these work show significant milestones in the advancement of impedance pump, emphasizing on the exploration and application of impedance pump in different domains. It was in the 2010 that impedance pump showed possibility in flow amplification. Rosenfeld's study on sequential excitations on a single elastic tube showed promising

results where increase in net flow was observed [10]. Similarly, another study by Lee on a two-stage system, integrated from two single-stage systems [13] also showed an improvement in net flow. Difference between both works is that Lee's configuration includes a reservoir between two excitation points, whereas Rosenfeld's does not. There has not be any further study on the effects on the role of reservoir, but it does seem to be a form of stabilizer for both incoming waveform, as described in Ref [12].

In this work, the effects of an intermediary reservoir between two excitation points on an elastic tube is studied. This study aims to shed some light on the steady state response and fluid motion within the intermediary reservoir; in which increased volumetric efficiency is demonstrated.

### 2 Mathematical Modelling

Two mathematical models are presented in this work. Figure 1(a) shows the physical model of a sequential systems without intermediary reservoir; while Figure 1(b) shows the physical model with intermediary reservoir. With reference to both models, two mathematical models are derived. Considering the distance between both excitation points is relatively close, the viscosity effect is not considered in the model. This is due to the effects of tube elasticity dominance over the viscosity, as described in Ref [3]. To simplify the conditions, both excitation points are assumed to have the similar frequencies. First model is constructed based on a two-degree-of freedom (2-DOF) lumped system model, as presented in Equation 1. Due to the nature where flow exists within the intermediary reservoir, an additional degree of freedom is

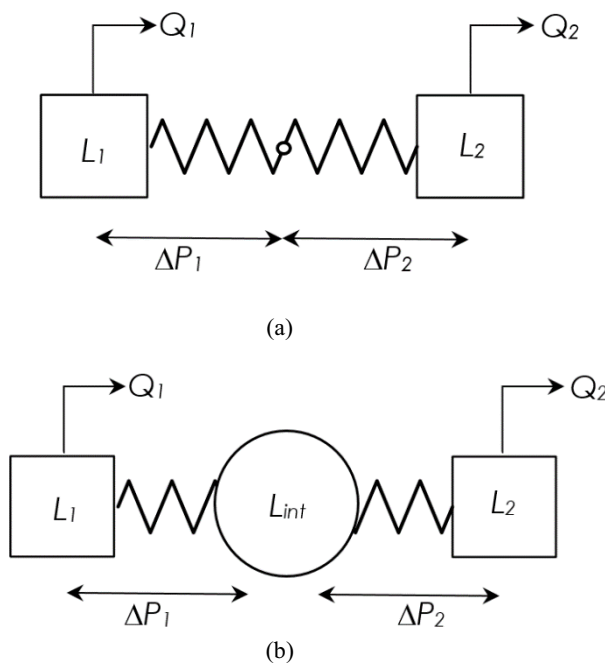
<sup>a</sup> Corresponding author: vincent@curtin.edu.my

assigned for the reservoir. Hence, the second model is constructed based on a three-degree-of-freedom (3-DOF) lumped system model, incorporating the intermediary reservoir as a lumped volume, as presented in Equation 2.

$$\begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} \ddot{Q}_1 \\ \ddot{Q}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} \\ -\frac{1}{C_1} & \frac{1}{C_2} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \omega \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix} \sin(\omega t) \tag{1}$$

$$\begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_{int} & 0 \\ 0 & 0 & L_2 \end{bmatrix} \begin{bmatrix} \ddot{Q}_1 \\ \ddot{Q}_{int} \\ \ddot{Q}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} & 0 \\ -\frac{1}{C_1} & \frac{C_1+C_2}{C_1 C_2} & -\frac{1}{C_2} \\ 0 & -\frac{1}{C_2} & \frac{1}{C_2} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_{int} \\ Q_2 \end{bmatrix} = \omega \begin{bmatrix} \Delta P_1 \\ 0 \\ \Delta P_2 \end{bmatrix} \sin(\omega t) \tag{2}$$

$L_i$  is the fluid inertia,  $C_i$  is the tube elasticity,  $\omega$  is the excitation frequency,  $Q_i$  is the flow rate,  $\Delta P_i$  is the change in pressure, and  $L_{int}$  is the lumped volume of the intermediary reservoir (also represented as a fluid inertia due to its oscillating nature during transient motion). Solving the differential equations provide the steady state response of the systems. For simplicity, all parameters are simulated using a constant 1.



**Figure 1.** Physical model of sequential excitation system (a) without intermediary reservoir, and (b) with intermediary reservoir.

For the system without the intermediary reservoir

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \frac{\omega \begin{bmatrix} 1-L_2 C_1 \omega^2 & 1 \\ 1 & 1-L_1 C_1 \omega^2 \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix}}{L_1 L_2 C_1 \omega^4 - (L_1 + L_2) \omega^2} \sin(\omega t) \tag{3}$$

For the system with the intermediary reservoir

$$\begin{bmatrix} Q_1 \\ Q_{int} \\ Q_2 \end{bmatrix} = \frac{\omega \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ 0 \\ \Delta P_2 \end{bmatrix}}{A} \sin(\omega t)$$

$$a = (-L_{int} L_2) \omega^4 - \left[ L_2 \left( \frac{C_1 + C_2}{C_1 C_2} \right) + \frac{L_{int}}{C_2} \right] \omega^2 + C_2^2 \left( \frac{C_1 C_2 + C_2^2}{C_1 C_2} - 1 \right)$$

$$b = d = -\frac{L_2}{C_1} \omega^2 + \frac{1}{C_1 C_2}$$

$$c = g = \frac{1}{C_1 C_2}$$

$$e = L_1 L_2 \omega^4 + \left( \frac{L_1 C_1 + L_2 C_2}{C_1 C_2} \right) \omega^2 + \frac{1}{C_1 C_2}$$

$$f = h = -\frac{L_1}{C_2} \omega^2 + \frac{1}{C_1 C_2}$$

$$i = (L_{int} L_1) \omega^4 - \left[ L_1 \left( \frac{C_1 + C_2}{C_1 C_2} \right) + \frac{L_{int}}{C_1} \right] \omega^2 + C_1^2 \left( \frac{C_1^2 + C_1 C_2}{C_1 C_2} - 1 \right)$$

$$A = (L_{int} L_1 L_2) \omega^6 + \left[ \frac{(L_{int} L_1 + L_1 L_2) C_1 + (L_{int} L_2 + L_1 L_2) C_2}{C_1 C_2} \right] \omega^4 + \left[ \frac{C_1^2 (-L_1 C_2 + L_2 C_2 - 1) - (C_1 + C_2) (L_1 C_1 + L_2 C_2)}{C_1^2 C_2^2} \right] \omega^2 + \frac{C_2}{C_1 C_2} \tag{4}$$

### 3 Computational Simulation

Computational fluid dynamics (CFD) simulation is conducted to study the fluid motion within the intermediary reservoir. This is to further support the deduction made using the lumped system model, where fluid motion cannot be illustrated mathematically. In accordance to the Ref [13], it was demonstrated that the intermediary reservoir was able to increase the system's volumetric efficiency. However, a longer time was required before a steady state condition was met. Hence, a computational approach is selected to shed some light on the fluid motion.

A 3-dimensional dynamic model is developed using the ANSYS Fluent 14.5. Standard k-ε, with standard wall function, is used as the turbulence model. SIMPLE algorithm is applied for the pressure-velocity coupling. The computational domain is constructed, with emphasize on the intermediary reservoir. Two openings

are modelled at the bottom of the reservoir to serve as inlets for the oscillating flows. Initial conditions are taken from previously obtained steady-state solution [13] to improve the convergence of the transient solution. Boundary conditions are prescribed by a known flow velocity, subject to oscillation, as described in Equation (5).

$$v_i = V \sin(\omega t) \quad (5)$$

## 4 Results and Discussion

With reference to section 2.0 and 3.0, the effects of intermediary reservoir in a two-stage impedance pump are discussed. The discussion is divided into three subsections, (i) on the steady state response, as modelled using the lumped system model, and (ii) fluid motion within the intermediary reservoir, as modelled using the ANSYS Fluent 14.5. Subsection (iii) discusses the functionality of fluid inertia in the intermediary reservoir.

### 4.1 Steady state response

For a sequential system, without the intermediary reservoir, the responses are simply subject to the net pressure change along the tube. From Equation 3, it is shown that the response  $Q_1$  is the summation of net pressure change between  $\Delta P_1$  and impediment from  $L_2$  (fluid inertia from second point of excitation) plus pressure change in second point of excitation. Similar case is shown for response  $Q_2$ . These responses show that for a sequential system, the respective response is significantly determined by the impedances ( $L_i C_i$ ) from neighbouring excitations. Hence, a similar flow pattern is observed for both response, as illustrated in Figure 1.

Inclusion of an intermediary reservoir immediately increases the degree of freedom by one. That is, from previous 2-DOF to a 3-DOF system. This added a whole new definition to the system, where previously single tube elasticity ( $C_1$ ) is now divided into  $C_1$  and  $C_2$ . The intermediary reservoir encourages mass transfer, as the fluid motion is now relatively more active, due to the interference of incoming waves from both excitations and their associated impedances, as described in Equation (4). However, a longer time is required to reach steady state, as mentioned in Ref [12]. It is also showed that the tube elasticity plays a dominant role in ensuring the mass transfer, unlike in the previous system where the tube elasticity only comes as an impediment to the respective excitation points. Hence, a higher volumetric efficiency can be achieved. Figure 2 shows the plots of  $Q_1$  and  $Q_2$  from both models.

Different from a 2-DOF system, both responses differ. Both  $Q_1$  and  $Q_2$  showed to have a higher amplitude, with the inclusion of the intermediary reservoir. The fluid motion within the reservoir is rather a mystery, as in how the amplitude of the flow is enhanced. Hence a computational simulation is performed to support the understanding of underlying flow motion in the reservoir.

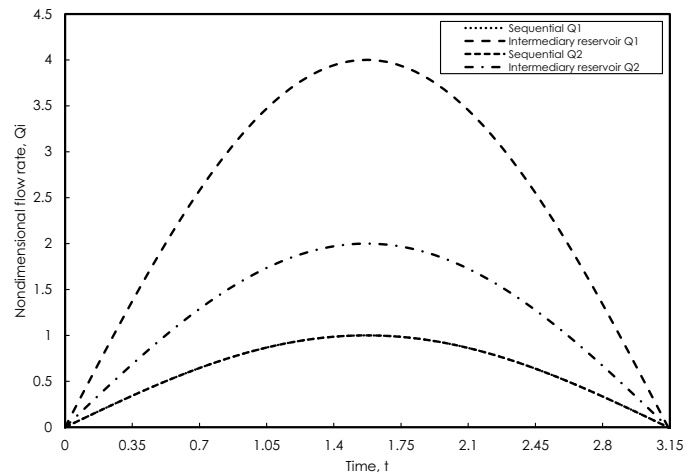
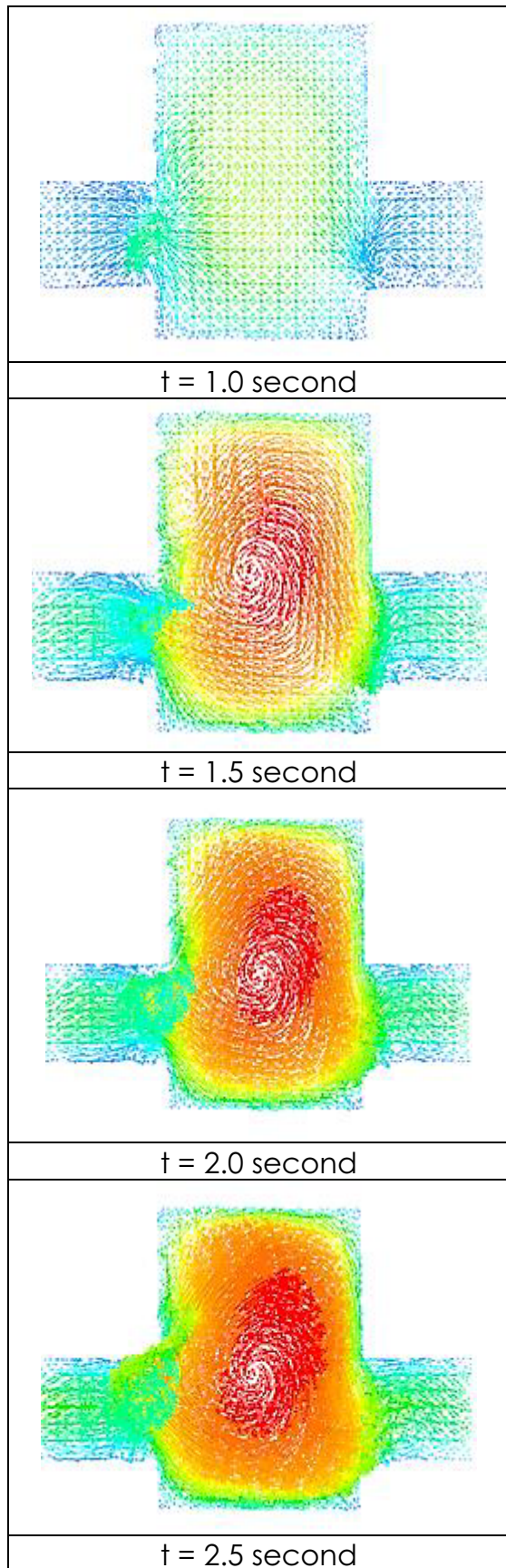


Figure 2. Steady state responses for a sequential and intermediary reservoir system.

### 4.2 Fluid motion

The responses are best described with the illustration of fluid motion. As the incoming wave from  $Q_1$  and  $Q_2$  entering the reservoir, it creates an opportunity for mass transfer. As the excitations continues, the mass transfer increases, where a formation of circular fluid motion is observed. The net flow is then channelled towards the opening on the right. From Figure 3, it can then be deduced response  $Q_1$  serves as a more dominance driving pressure wave than response  $Q_2$ . Denoting pressure wave from  $Q_1$  as  $P_{Q1}$  and pressure wave from  $Q_2$  as  $P_{Q2}$ , the fluid motion can easily be explained. As  $P_{Q1}$  entering the intermediary reservoir, a disturbance is created within the reservoir.

While the incoming pressure wave from the other end  $P_{Q2}$  (response  $Q_2$ ) enters meets  $P_{Q1}$ , an interference of wave is generated. Due to the wall boundary in the reservoir, the flow moves upwards. The flow is then directed downwards due to gravitational pull and effect of atmospheric pressure, hence a circular fluid motion is created. The flow is then channelled to the right end opening, as the flow interfere with the incoming  $P_{Q1}$ . With reference to Figure 3, it is demonstrated that the intensity of the circular flow becomes higher as time passes. This signifies that the velocity of the flow increases with the interference of pressure waves. Hence, explained the underlying phenomenon of increased flow amplitude from earlier section. This also suggest that the intermediary reservoir serves as a driving mechanism for flow enhancement, rather than a simple stabilizer as described in Ref [12].



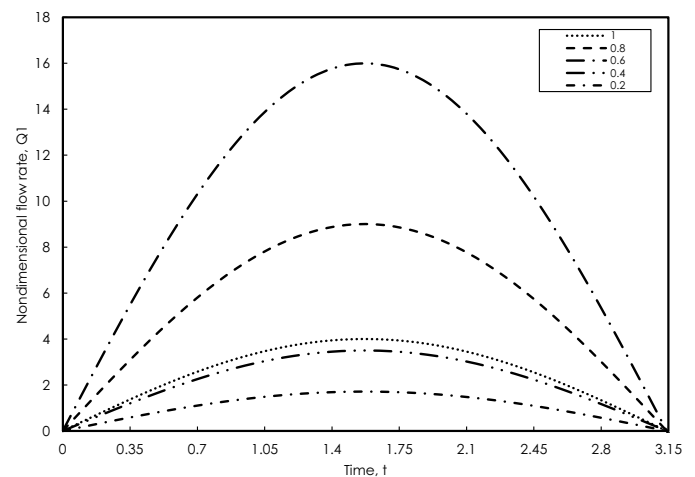
**Figure 3.** Fluid motion generated in intermediary reservoir

### 4.3 Fluid Inertia

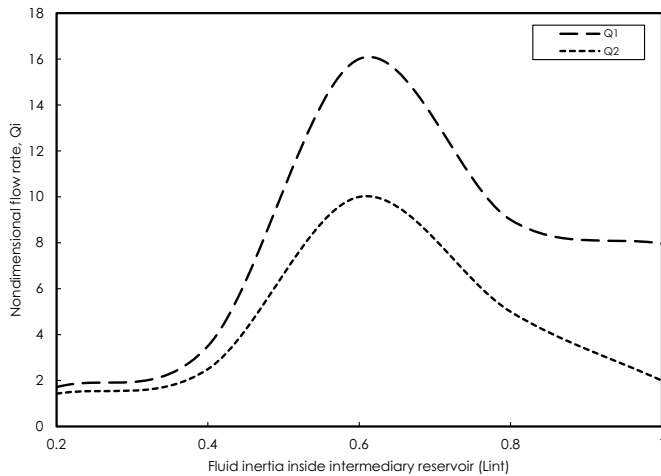
Further study is performed to investigate the functionality of fluid inertia in the intermediary reservoir. A series of fluid inertia is simulated with  $L_{int}$  ranging from 0.2 to 1, with interval of 0.2. It was observed that the highest enhancement in response occurs at  $L_{int}$  of 0.6, with a huge factor of 16, as compared to  $L_{int}$  of 1 (as simulated earlier), which is by factor of 4, as shown in Figure 4. This suggests that  $L_{int}$  of 0.6 could possibly be the resonance of the reservoir. Although the maximum amplitude occurs only for a short instance, it should be emphasized that the plotted figure is only for half an excitation; and that prolonged excitation may cause destruction towards the system.

Both responses  $Q_1$  and  $Q_2$  as a function of  $L_{int}$  are plotted in Figure 5, further validating  $L_{int}$  of 0.6 to be the resonance of the reservoir. From Figure 5, a rather vivid behaviour of the fluid inertia is illustrated, where the fluid inertia is in fact a spring-mass system on its own, with own natural frequency. This also show that having a fluid inertia oscillating at  $L_{int} > 0.6$  is more advisable, as that region will be the isolation region, as opposed to  $L_{int} < 0.6$  which is the amplification region. Oscillation within the isolation region is more advisable, due to its stability and isolation from resonance. This may, however, difficult to be achieved as it is rather difficult to control the oscillation of fluid within the reservoir.

In addition, it is also justified the earlier claim suggesting response  $Q_1$  serves as a more dominance driving pressure wave than response  $Q_2$ . This is evident in Figure 5, where response  $Q_1$  is a magnitude larger than response  $Q_2$ , at  $L_{int} \geq 0.6$ . It may then be deduced that the fluid inertia plays an essential role not only in enhancement in responses, but in ensuring the stability of the system as well.



**Figure 4.** Steady state responses for different fluid inertia.



**Figure 5.** Q1 and Q2 response as a function of fluid inertia.

## 5 Conclusion

A study on the effects of intermediary reservoir for a two-stage impedance pump is conducted to understand the underlying phenomenon for the pump's increased volumetric efficiency. Two approaches are presented in this work, (i) mathematical lumped system modelling, and (ii) computational fluid dynamics simulation. It is demonstrated that the responses increase with the inclusion of an intermediary reservoir, suggested being a driving mechanism for flow enhancement. The finding is further supported with CFD model to understand its fluid motion, which evident towards the increased volumetric efficiency. In addition to that, it is also evident that the fluid inertia in the intermediary reservoir plays an essential role in enhancing responses and ensuring system stability.

## Recommendation for future work

Fluid inertia within the intermediary reservoir appears to behave as a spring-mass system on its own and is expected to have a natural frequency of its own. As the responses are highly dependent on the excitations frequencies, a study on correlating the excitation frequencies and the fluid inertia behaviour within the intermediary reservoir can be further studied in optimizing the responses.

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