

# Reliability of buildings with internal network

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**Annotation.** During the course of performance, reliability of a building with internal network located in Western Administrative District of Moscow was examined and estimated. Reliability means probability of no failure in the initial time.

## 1 Introduction

Reliability value makes not as many as one [1, 3, 6] and is recommended as 0.99865 in building systems with normal level of reliability. Earlier [4], calculation of reliability of monolithic multi-storey buildings using logical-and-probabilistic methods was provided. Thus, formulas are derived for the frame-type, flexible rods.

Slightly different approach is required to assess reliability of buildings with internal network, but the main provisions of reliability calculation remain unchanged. According to [3], the main property that determines reliability of a building is faultless of its performance. Reliability means probability of non-occurrence of the first limit state of load-bearing structures. Let us select strength of normal sections of all possible states of the first group. Other possible states of the first group are not considered as far as they are improbable and constructively impossible. For example: at maximum flexibility of columns in this building being equal to seven their loss of stability is possible only after normal cross-sectional destruction [9].

Thus, according to the accepted concept, reliability means probability of no failure in the initial time. We consider the first group of limiting states, for which required value of reliability makes 0.99865 ( $3\sigma$ ). We consider that the main power factors of flexible systems are bending moments, and bending moments and longitudinal forces are the main power factors of beam columns.

Within the concept adopted, failure does not occur in the context of deflections, crack opening width exceeding permissible standard values, which are mainly determined by psychological, performance or other factors. For example, when designing columns for buildings only the first group of limiting states is mainly considered.

In this paper, during reliability calculation, variability of mechanical properties is only taken into account, because it is the most studied and categorized by the relevant reference documentation. We shall also take into account that average values of geometric parameters (their mathematical expectations) are inserted into SNiP and Code Design formulas, so reliability is evenly affected by them.

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## 2 Subject and methods of research

Three-section building is considered. We assume that reliability of the building will be determined by the minimum reliability of one of three sections. Load bearing capacity of each section is provided by six-storey (including basement) frame. Extreme poles of these frames are masonry separation walls. Middle poles are columns. Basement columns are made of reinforced concrete. Columns of the building superstructure are masonry. Basement girders are made of steel, superstructure girders are made of reinforced concrete.

In addition to its main task (flooring), floor slab serves as horizontal brace. Building reliability calculations take into account sequential operation of frames and floor slabs. Therefore, failure (event being opposite to reliability) occurs upon floor slab or frame failure.

Linearization method was chosen to calculate reliability of floor slabs and girders. This method involves formula relationship of random arguments function, and this dependence should be differentiable.

Monte Carlo method was used to evaluate columns reliability. Versatility and high accuracy are the main advantages of Monte Carlo method. The disadvantages are sometimes encountered procedural difficulties in calculation of small values, as well as relatively high labor intensity when debugging programs requiring special care and rigor of their developers.

Let us calculate reliability of girder section by means of linearization method. Bending moment  $M_x$  is selected as girder reliability criterion. Transverse force in this case is structurally taken into account. The girder span makes 5.405 m.

Concrete resistance and resistance of stretched and compressed reinforcement are accept as random variables:  $\tilde{R}_b$ ,  $\tilde{R}_s$  and  $\tilde{R}_{sc}$ . As a result of surveys, we know average resistance and reinforcement variation coefficients:  $R_{sm} = 384 \text{ MPa}$ ,  $V_s = 0.07$ .

## 3 Research data

Statistic data on concrete are indicated in Table 1.

**Table 1.** Results of testing by means of "SCHMIDT" hammer, GOST 22690-88.

1	2	3	4	5	Mean strength, MPa		Coef. of variation	Concr. grade, MPa	Test location
					instrume n	actual			
41.56	36.52	44.35			40.8	37.4	0.097	31.4	Above 2-nd floor
42.14	45.23				43.7	42.2	0.050	38.7	Above 3-rd floor
43.03	40.66	27.37	39.7	47.0	39.6	35.4	0.186	24.6	Above 4-th floor
3802	51.3	50.69	38.1		44.5	43.7	0.168	31.1	Above 5-th floor
40.26	50.42	38.02			42.9	40.9	0.154		Above 5-th floor

Average acquired concrete strength variation coefficient ( $V_b$ ) on girders makes 0.164. Average girder concrete grade makes 26.9 MPa. Average cube strength makes  $R_m - 40 \text{ MPa}$ .

Average prism strength is defined according to the following formula (1):

$$R_{bm} = R_m \cdot (0.77 - 0.001R_m) = 40 \cdot (0.77 - 0.001 \cdot 40) = 29.20 \text{ MPa} \quad (1)$$

Design strength of reinforcement and concrete:  $R_s = 340 \text{ MPa}$ ;  $R_b = 29.8 \cdot (1 - 3 \cdot 0.164) = 15.1 \text{ MPa}$ .

It is deduced from experiments, that the girder is reinforced by 2Ø25A-III (positive reinforcement) and 2Ø10A-III (negative reinforcement). The instrument has shown reinforcement diameter and 12 mm, but, as a reserve, we have taken diameter of 10 mm. Reinforcement coefficients of positive and negative reinforcement are correspondingly equal to:  $\mu = 0.010957$  and  $\mu' = 0.0017531$ , i.e. double reinforcement case is considered. Coefficient  $C\mu$ :  $C\mu = \mu' / \mu = 0.16$ .

Resisting moment  $M_x$ :

$$M_x = R_b b h_0^2 \alpha_m + R_{sc} A_s' (h_0 - a') \quad (2)$$

$$\alpha_m = \xi(1 - 0.5\xi) \quad (3)$$

$$\xi = (R_s A_s - R_{sc} A_s') / R_b \quad (4)$$

$$\xi_R = 0.8 / (1 + \varepsilon_{s,el} / \varepsilon_b) = 0.8 / [1 + (340/200000)/0.0035] = 0.538 \quad (5)$$

Let us transform expressions (2) – (4) into one, as well as divide left and right parts of equation (2) by  $b h_0^2$ :

$$m_Y = (R_s \mu - R_{sc} \mu') [1 - 0.5 \times (R_s \mu - R_{sc} \mu') / R_b] \quad (6)$$

Further, let us take partial derivatives on  $R_s$ ,  $R_{sc}$  and  $R_b$ . Taking into account that  $(h_0 - a') / h_0 = 0.9$ , we will get the following result

$$\begin{aligned} \partial m / \partial R_s &= \mu [1 - (R_{sm} \mu - R_{scm} \mu') / R_{bm}] = 0.01096 \times \\ &[1 - 384 \times (0.01096 - 0.001753) / 29.8] = 0.00966 \end{aligned} \quad (7)$$

$$\begin{aligned} \partial m / \partial R_{sc} &= -\mu' [1 - (R_{sm} \mu - R_{scm} \mu') / R_{bm} - 0.9] = -0.001753 \times \\ &[1 - 384 \times (0.01096 - 0.001753) / 29.8 + 0.9] = -0.003123 \end{aligned} \quad (8)$$

$$\begin{aligned} \partial m / \partial R_b &= 0.5 \times (R_{sm} \mu - R_{scm} \mu')^2 / R_{bm}^2 = 0.5 \times 384 \times \\ &(0.01096 - 0.001753)^2 / 29.8^2 = 0.000002 \end{aligned} \quad (9)$$

Let us define mean square deviations of reinforcement and concrete resistance,  $MPa$ :

$$\sigma_{R_s} = M_s \times K_{sm} = 0.07 \times 384 = 26.88 \quad \sigma_{R_b} = M_b \times K_{bm} = 0.164 \times 29.8 = 4.89 \quad (10)$$

Section load bearing capacity under average values of material resistance  $m_Y$  and mean square deviation of load bearing capacity  $m_Y$  thus will be defined according to formulae (6) and (11). In formulae (6) and (11),  $R_{sm}$  shall be inserted instead of  $R_s$  and  $R_{sc}$ , and  $R_{bm}$  shall be inserted instead of  $R_b$ .

$$\sigma_Y^2 = \sum_{i=1}^n \left( \frac{\partial \varphi}{\partial x_i} \right)_m^2 \sigma_{x_i}^2 + 2 \sum_{i < j}^n \left( \frac{\partial \varphi}{\partial x_i} \right)_m \left( \frac{\partial \varphi}{\partial x_j} \right)_m r_{ij} \sigma_{x_i} \sigma_{x_j} \quad (11)$$

Where  $r_{ij}$ ,  $\sigma_{x_i}$ ,  $\sigma_{x_j}$  – are correspondingly correlation coefficient and mean square deviations of random variables. In our case, correlation coefficient  $r_{ij}$  is equal to zero for random independent variables (concrete and reinforcement resistance).

$$\sigma_Y^2 = 0.00966^2 \times 26.88^2 + (-0.003123)^2 \times 26.88^2 + 0.000002^2 \times 4.887^2 = 0.745, \sigma_Y = 0.86$$

$$m_Y = 384 \times (0.01096 - 0.001753) \times [1 - 0.5 \times 384 \times (0.01096 - 0.001753) / 29.8] = 3.33$$

Thus, we will receive  $m_Y = 3.33$  MPa,  $\sigma_Y = 0.86$  MPa. Let us define load bearing capacity  $m_0$  with probability of three standards (0.99865):

$$m_0 = m_Y - 3\sigma_Y = 3.33 - 3 \times 0.86 = 0.75 \text{ MPa}$$

With load of 10.0 kPa on a floor slab we acquire relative moment equal to 1.61 MPa. Reliability of a girder under load of 10.0 kPa makes:  $P[(3.33 - 1.61) / 0.86 = 2.29] = 0.97725$ . Acquired value of reliability has probability of less than  $3\sigma$ , thus load shall be limited down to 8.70 kPa. At floor load of 8.70 kPa, the girder moment makes 0.75 MPa and the girder reliability under this load makes:  $P[(3.33 - 0.75) / 0.86 = 3.0] = 0.99865$ . Acquired reliability value makes probability of  $3\sigma$ . In this case, the girder reliability is comparable with requirements of GOST 27751-2014.

Reliability of concrete columns is defined by means of Monte Carlo method.

The columns are solid-cast. Section – 0.80×0.60 m. Effective depth,  $h_0$ : 0.80 – 0.05 = 0.75 m. Concrete strength of columns corresponds to B25 grade. The columns are reinforced symmetrically along the contour of 8Ø25:  $A_{s,tot} = 39.27 \text{ cm}^2$ . Here,  $A_s' = A_s = 14.726 \text{ cm}^2$ .

Reinforced concrete column slenderness ratio makes:

$$\lambda = l_0 / h = 3.3 / 0.8 = 4.13 \tag{12}$$

The first group of limiting states is the most important within calculations of beam columns. We assume that failure of an element does not occur if the following three conditions are met:

$$\begin{cases} N \leq \tilde{N}_c & (13) \\ \tilde{N} \leq \tilde{N}_{cr} & (14) \\ Ne \leq \tilde{M} & (15) \end{cases}$$

In formulas (13) – (15)  $N$  – axial force, deduced from static analysis;  $\tilde{N}_c$  – axial force, perceived by the element during direct compression and deduced from probability calculation;  $\tilde{N}$  – accident force, determined as a result of probability calculation;  $\tilde{N}_{cr}$  – axial buckling load (Euler's force), also defined by means of probability calculation;  $\tilde{M}$  – resisting moment, deduced from probability calculation.

Symbol “~” (twiddle) over variable value shows its random nature.

Let us mark the events, indicated by conditions (13) – (15), by the letters  $A, B, C$ . Then, no failure operation of an element shall be expressed through conditional probability formula:

$$P(ABC) = P(A) P(B/A) P(C/AB) \tag{16}$$

Or

$$P(C/AB) = P(ABC) / [P(A) P(B/A)] \tag{17}$$

Probability calculations were carried out according to (17), Concrete grade and slenderness were used as initial data;  $\alpha_s = R_s A_s / (R_b b h_0) = 3400 \times 14.726 / (148 \times 60 \times 75) = 0.075$ ;

$C_\mu = \mu' / \mu = 1$ ;  $\xi / \xi_R$ ;  $\gamma_{bi}$  ( $i = 1, 2, 3, 4$ );  $\varphi_i$ ; Henceforward, SNiP 52-01-2003 symbols are applied.

Using general case formulae (13) – (15), we shall carry out deterministic calculation of load bearing capacity under calculated material parameters.

According to random values distribution laws and correlation dependencies, we shall specify sets of realizations  $\tilde{R}_s, \tilde{R}_b, \tilde{E}_b$ .

Using each set of random value realizations,  $\tilde{R}_s, \tilde{R}_b, \tilde{E}_b$  we shall perform computations under (10)–(12), adopting that  $b=h_0=1$ . As a result of one computation:  $\tilde{v} = N/\tilde{N}, \tilde{u} = N/\tilde{N}_{cr}, \tilde{w} = \tilde{M}/M$ .

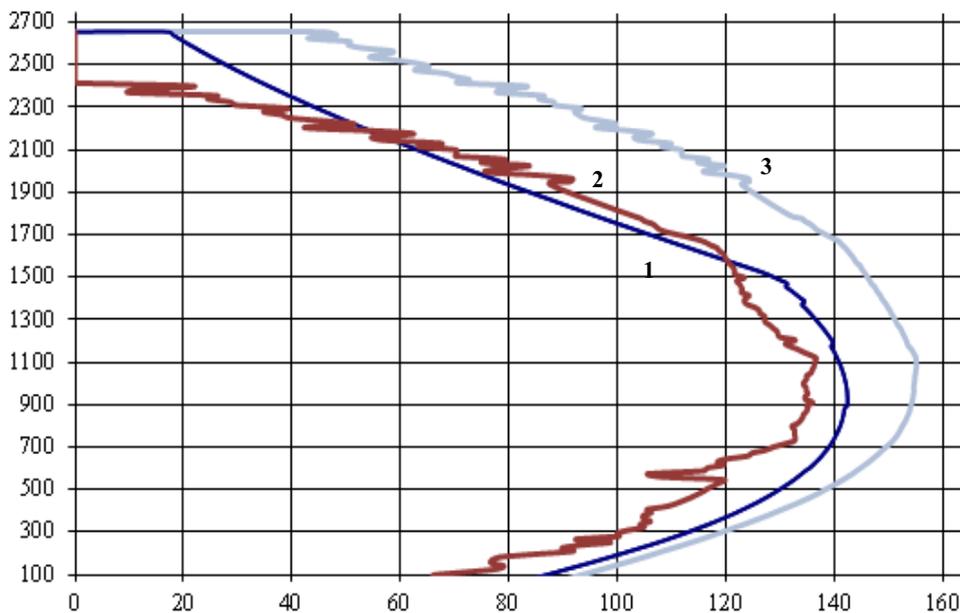
Now, we shall fit acquired results  $\tilde{v}, \tilde{u}, \tilde{w}$  with Pearson curves. Probability density is acquired  $f(\tilde{v}), f(\tilde{u}), f(\tilde{w})$ .

According to (17) we shall calculate  $P(C/AB)$ .

Diagrams shown on Fig. 1 were plotted for flexibility elements 4, 13 according to results of calculations. Maximum load duration coefficient was specified as a reserve for durability and reliability:  $\varphi_l = 2..$  Results of deterministic calculations according to SNiP 52-01-2003 under calculated resistance values are shown with full lines. Results of probability calculations with probability of 0.99865 or  $3\sigma$  (three standards) are shown with dotted lines. Reliability diagrams are plotted for B25 grade concrete and reinforcement – A-III.

## 4 Evaluation

Based on diagram 2 on Figure 1, reliability of reinforced concrete columns within the range 1600 kN to 2100 kN is just over  $3\sigma$  and maximum reliability value makes 0.9998. With forces indicated, two materials - concrete and reinforcement - are operating simultaneously. “Bad” concrete (design resistance  $R_b$ ) is compensated by “good” reinforcement (for e.g., average resistance  $R_{sm}$ ) and vice versa.



**Fig. 1.** Load bearing capacity of reinforced concrete columns, B25 grade concrete, A400 (A-III) grade reinforcement: 1 – deterministic calculations; 2 – bearing capacity diagram with probability of

0.99865 (integrating according to Pearson curve); 3 – bearing capacity diagram with probability of 3σ (3 is deducted from average value 3σ).

The column reliability quickly decreases with increase of axial force  $N$ . It is affected by the influence of initial concrete modulus of elasticity taking into account the column stability. This is the case of concrete failure, i.e. when structural reliability is mainly determined by one material - concrete. Concrete is known to have higher strength variation than reinforcement, so these elements have the lowest reliability while remaining factors remain the same.

Reinforcement is the main operating element in case of large eccentricities, so reliability is rather lower.

Thus, the highest reliability for the columns considered is seen in cross sections, which provide uniform operation of reinforcement and concrete.

At floor load of 8.70 kPa, the column section reliability at the top edge of foundation ( $N = 1099.3 \text{ kH}$ ,  $M = 24.8 \text{ kH}\times\text{M}$ ) makes  $0.9^6713 \approx 1$

Let us estimate reliability of separation partition of the first floor of the masonry. Separation partitions are the vertical load bearing frame elements. Calculated separation partition dimensions: depth of cross section,  $h = 0.51 \text{ m}$ ; width of cross-section,  $b = 1.70 \text{ m}$ ; insertion depth of the girder into the masonry makes 200 mm. Masonry resistance with M100 bricks and M4 mortar makes 0.9 MPa. Masonry elastic response  $\omega$  is determined via Table 15\* [8]:  $\alpha = 500$ .

Load transfer from the crossbars to the masonry is done through the spreading beam. 0.20 m of the girder is supported by the masonry. Stress block under the girder is quadrangular.

Load bearing capacity of the separation partition is carried out using the formula (18):

$$N \leq N_0 = m_g \varphi_1 R A_c \omega \quad (18)$$

Where  $m_g$  – coefficient taking into account prolonged stress effect,  $m_g = 1$ ;  $\varphi_1 = (\varphi + \varphi_c)/2$ ;  $\varphi$  – buckling coefficient;  $\varphi_c$  – buckling coefficient for compressed part of section, determined by actual height of element  $H$  acc. to Table 18 [8] within bending moment plane;  $A$  – element sectional area;  $A_c = A(1 - 2e_0/h)$  – area of compressed part of section at rectangular stress diagram, determined taking into consideration that its gravity center coincides with point of application of calculated axial force  $N$ ; area border position  $A_c$  is determined if first moment of this moment relatively to its gravity center equals to zero.

Masonry flexibility:  $\lambda_{h_i} = 3.3/0.51 = 6.471$  and  $\lambda_{h_c} = 3.3/0.12 = 27.5$ . Calculated parameters:  $m_g = 1$ ;  $e_0 = (51/2 - 20 \times 1/2) = 15.5$ ;  $\lambda_{h_i} = 3.3/0.51 = 6.471$  and  $\lambda_{h_c} = 3.3/0.2 = 16.5$ ;  $\varphi = 0.89$ ,  $\varphi_c = 0.35$ ;  $\varphi_1 = (0.89 + 0.35)/2 = 0.62$ ;  $\omega = 1 + e_0/h = 1 + 15.5/51 = 1.3 \leq 1.45$ ;  $A_c = 51 \times 170 \times (1 - 2 \times 15.5/51) = 3400$ .

Axial force being under the influence of external load and own weight of the masonry will make:

$$N = [18.00 \times 0.51 \times (3.3 \times 4)] \times 1.0 + 8.70 \times 3.1 \times 6/2 \times 4 + 6.00 \times 3.1 \times 6/2 = 500.62 \text{ kN}$$

Load bearing capacity of the masonry:  $N_0 = 1 \times 0.62 \times 0.09 \times 3400 \times 1.3 = 256.12 \text{ kN}$ . Load bearing capacity of the masonry under this supporting method is insufficient to accommodate floor slab load, which makes 8.70 kPa.

Reliability of separation partition of the first floor of the masonry makes 0.5. Upper floors were not considered, because maximum axial forces are acting on lower floors. Thus, frame reliability with load of 8.70 kPa, will make:  $0.5 \times 0.99999713 \times 0.99865 = 0.499324857$ .

Post footing reliability is determined by reliability of the slab and the column footing. The slab in this case is the flexural member with single reinforcement. Post footing slab

failure is determined as event  $Q_1$ . As for the column footing, we will consider possibility of destruction caused by eccentric compression (event  $Q_2$ , selection of vertical reinforcement area) and oblique section bending (event  $Q_3$ , selection of horizontal reinforcement area).

Reliability of column footing shall be determined as follows:

$$R_{2,3} = R_2 \times R_3 = 0.99903 \times 0.99918 = 0.99821 \quad (19)$$

Foundation reliability will be provided, if slab and column footing will not fail at the same time:  $R_f = R_{2,3} \times R_1 = 0.99821 \times 0.99886 = 0.99707$ .

Correspondingly, foundation failure can be calculated as follows:  $1 - 0.99707 = 0.00293$ .

Thus, the building (system) reliability,  $R_c$ , under 1-5 floor slab load of 8.70 kPa will make

$$R_c = 1 - Q_f \times Q_p \times Q_p = 1 - (0.00293 + 0.5 + 0) = 0.49707 \quad (20)$$

Thus we have abnormally low reliability value. Total load acting on the floor slabs shall be lowered down to 5.70 kPa or separation walls of floors 1–3 shall be reinforced.

Low reliability of the building is confirmed by presence of vertical and horizontal cracks in external walls of brick masonry. The crack width reaches 0.005 m. Cracks can be seen across full height and width of exterior walls. Actual building reliability is slightly higher, since actual load in separate premises is lower, than 8.70 kPa. With load on floors making 5.70 kPa, the building reliability rises up to 0.9987.

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