The problems of calculating the load-bearing structures made of light steel thin-walled profiles

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Abstract. The article presents the results of a study of bearing capacity of thin-walled cold-formed steel beam of the guide profile. Such profiles have a small thickness and complex cross-sectional shape. Bending deformation develops in the cross-sectional plane under the influence of loads in beam. In addition, deformation of constrained torsion and warping arise. These deformations influence the stress distribution at the points of the cross-section of the beam and thereby determine its load-bearing capacity.

1 Introduction

Today in Russia buildings with load-bearing frame of thin-walled cold-formed sections are widely used [1]. They have a number of technological and operational advantages (lightness, high speed of erection, etc.) [2].

As practice shows, often due to errors in the design of buildings of light steel thin-walled structures (LSTS), frequent cases of emergency [3]. Sustaining the work of such structures is a major problem to the widespread introduction of LSTS in massive construction. Obviously, the thin-walled structures at the design stage requires a thorough analysis of their bearing capacity. Not paying attention to the growing popularity of such structures and a large amount of research on all LSTS stage of development, today in Russia there are significant deficiencies in the prescriptive, methodological and computational databases of design of LSTS which does not make it easy and clear to make their calculation.

Theory of calculation based on the hypothesis of plane sections, are not applicable to the thin-walled bars, due to their small thickness and mismatch the centers of gravity and bending. To solve engineering problems of calculation of elements of thin-walled structures can distinguish two groups of ways to calculate: based on the modeling of shells and rods. The first group of methods is associated with the representation of a thin-walled rod in the form of a shell and further numerical calculation, as a rule, using the finite element method, in the program complexes SCAD, Lira, SOFiSTiK etc. Such methods of calculation are accurate enough, but very time-consuming in the engineering design activities especially in terms of the complex structure analysis. In the second group of methods can be

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distinguished analytical and numerical methods for calculation of thin rods associated with the introduction of additional "seventh" degree of freedom and the corresponding power factor – bimoment [2, 4, 5].

Study of the bearing capacity, deformability and manufacturing technology of steel structures of cold-formed sections are devoted to [6-21]. In [6, 11, 12] are considered the possibility of accounting the seventh degree of freedom in the creation of a thin-walled finite element.

2 Statement of the research problem

Let us analyze the degree of influence of component of warping on the stress-strain state of the thin-walled bar.

We will lead the calculation for cantilever balcony beam with cross section GP 254-1.5 [22] like for beam having strain of warping.
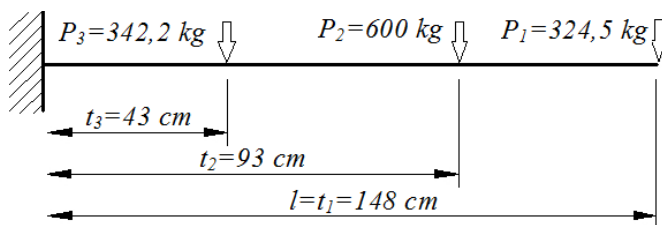

Fig. 1. Design scheme.

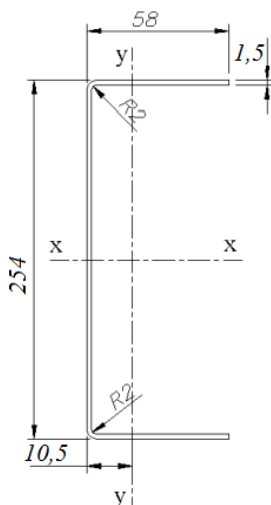### 2.1 Calculation of the strength of solid section bent elements

Fig. 2. Cross section [22].

Geometric characteristics in calculation are taken the lower of two: by assortment [22] and by calculation in the program TONUS (PC SCAD Office) – table 1.

Table 1. Geometric characteristics of cross section GP 254-1.5.

	Parameter	Value according to STO[22]	Value according to TONUS	Units
A	Cross sectional area	5.46	5.458	cm^2
I_t	Moment of inertia at free torsion	-	0.041	cm^4
I_w	Sectorial moment of inertia	-	1669.71	cm^6
W_{u+}	The maximum moment of resistance about an axis U	37.99	36.8	cm^3
W_{v+}	The maximum moment of resistance about an axis V	3.28	2.973	cm^3
I_u	The maximum moment of inertia	484.35	466.44	cm^4
I_v	The mininmum moment of inertia	16.53	14.34	cm^4
Y_b	Coordinate of the center of the bend on the x axis	-	-1.672	cm
Z_b	Coordinate of the center of gravity in the x direction	1.05	0.906	cm

Flexural-torsional characteristics (it is a physical constant of cross section and mathematically representing a characteristic number of differential equation of equilibrium 4th order) [23] amount to:

$$k = \sqrt{\frac{GJ_t}{EJ_\omega}} = 0.0030775 cm^{-1} \tag{1}$$

Upon application of a uniformly distributed load and bringing it up to the maximum (destructive) value due to a sufficiently high ductility of the top shelf, which will be significantly deformed, the profile loses its original geometric shape. Load, in most cases being a "stamping" that is more rigid than the section by its nature, will not have tightly fit to the shelf. To indirect accounting for this geometric nonlinearity we will apply the load is not uniform along the shelf, but on the law of the triangle, which is a fairly common method. Then, the resultant load vector will pass through the center of gravity of the load diagram, lying in the intersection of the medians of the triangle, that is, at a distance (b/3) from the wall edge [4].

As can be seen from Fig. 3 full eccentricity of load application will consist of the eccentricity, due to the mismatch between the center of gravity and the center of the

bending cross section of α_x, (equal Y_b- coordinate of shear center y-axis) and the direct eccentricity of application of the load equal:

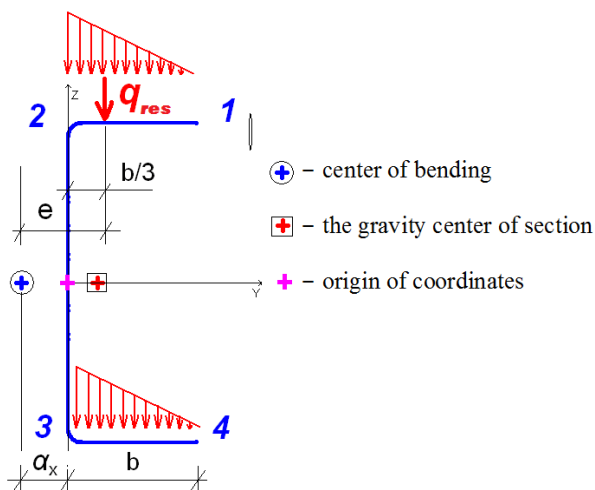$$e = \alpha_x + \frac{b}{3} = 1.672 + \frac{5.8}{3} = 3.605 \, cm \qquad (2)$$

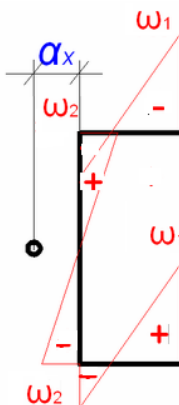**Fig. 3.** Scheme of axes and load application, points numbering of σ_y.

Fig. 4. Diagram of sectoral coordinates [2].

Diagram of sectoral coordinates of section is shown in Fig. 4. The maximum sectoral coordinates are necessary for detecting the stress from bimoment; they comprise:

$$\omega_1 = (\alpha_x - b)\frac{h}{2} = (1.672 - 5.8)\frac{25.34}{2} = -52.30 \, cm^2 \qquad (3)$$

$$\omega_2 = \alpha_x \frac{h}{2} = 21.19 \, cm^2 \qquad (4)$$

According to p. 8.2.1. SR 16.13330-2011[24], calculation of the strength of the beams should be carried out under the action of moments in the two principal planes (and bimoment available) according to the formula

$$\frac{M_x}{I_{xn}R_y\gamma_c}y \pm \frac{M_y}{I_{xn}R_y\gamma_c}x \pm \frac{B_\omega}{I_\omega R_y\gamma_c} \leq 1 \qquad (5)$$

where x and y – the distance from the main axis to the point of the section under consideration; while the action in the wall of the beam moment and shear forces.

The bending moment in the reference section:

$$M = -P_1\,t_1 - P_2\,t_2 - P_3\,t_3 \qquad (6)$$

$$M = -(324.5 \cdot 1.48 + 600 \cdot 0.93 + 342.2 \cdot 0.43) =$$
$$= 480.3 + 558.0 + 147.4 = 118540 \; kg \cdot cm^2 \qquad (7)$$

where t – the distance from the end of beam to the point of load application,
P – applied load

Assuming that the bending occurs only in one plane, taking $M_y = 0$

The formula for the bimoment in the case of a cantilever beam, according to the technical literature [2]:

$$B = -\frac{1}{k}\frac{Pe}{ch(kl)}\big[sh(k(l-t)) - sh(kl)\big] \qquad (8)$$

$$B_1 = 162077.9 \; kg \cdot cm^2 \qquad (9)$$

$$B_2 = 191558.4 \; kg \cdot cm^2 \qquad (10)$$

$$B_3 = 51702.35 \; kg \cdot cm^2 \qquad (11)$$

Bimoment in the reference section obtain as a superposition of moments of individual forces:

$$B = B_1 + B_2 + B_3 \qquad (12)$$

$$B = 162077.9 + 191558.4 + 51702.35 = 405339 \; kg \cdot cm^2 \qquad (13)$$

The maximum normal stress $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ occurring in the most tense cross-section, located in the middle of the span, will be different in the four characteristic points (1, 2, 3 and 4, respectively) of the cross-section will be:

$$\sigma_1 = \sigma_{x1} + \sigma_{\omega1} = -\frac{M_x}{W_x} + \frac{B_\omega}{I_\omega}\omega_1 \qquad (14)$$

$$\sigma_1 = \frac{118540}{36.8} - \frac{405339}{1669.71} \cdot 52.30 = +3221.2 - 12696.4 = -9475.2 \; kg/cm^2 \qquad (15)$$

Similarly, we calculate the value of the normal stress $\sigma_2, \sigma_3, \sigma_4$:

$$\sigma_2 = -8365.3 \; kg/cm^2 \qquad (16)$$

$$\sigma_3 = -8365.3 \; kg/cm^2 \qquad (17)$$

$$\sigma_4 = -9472.2 \; kg/cm^2 \qquad (18)$$

Allowable stress:

$$\sigma_{per} = R_y \cdot \gamma_c = 3200 \cdot 0.9 = 2880 \; kg/cm^2 \qquad (19)$$

2.2 The calculation using the finite element method (analysis of stress-strain state)

According to p. 8.2.1. SR 16.13330-2011 [24], values of stress in the wall of the beam must be checked according to the following formula in the two principal planes of bending:

$$\frac{0.87}{R_y\gamma_c}\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \leq 1 \qquad (20)$$

$$\frac{\tau_{xy}}{R_y \gamma_c} \leq 1 \tag{21}$$

where σ_x – normal stress in the median plane of the wall, parallel to the longitudinal axis of the beam;

σ_y – same, perpendicular to the longitudinal axis of the beam;

τ_{xy} – shear stress in the wall.

$$\tau_{xy} = \frac{Q \cdot S}{I \cdot t_w} \tag{22}$$

Stress σ_x and σ_y, taken in formula (20) with their signs and τ_{xy}, should be determined in the same point of the beam wall.

For this, we use the finite element method in a static statement and solve the problem in the software package SCAD Office.
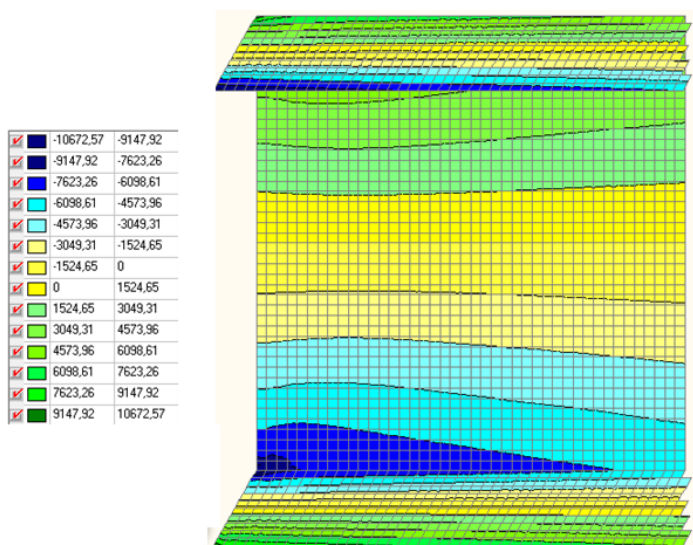

Fig. 5. Normal stress σ_y. Minimum value –*10672.57 kg/cm²*, Maximum value *10672.57 kg/cm²*
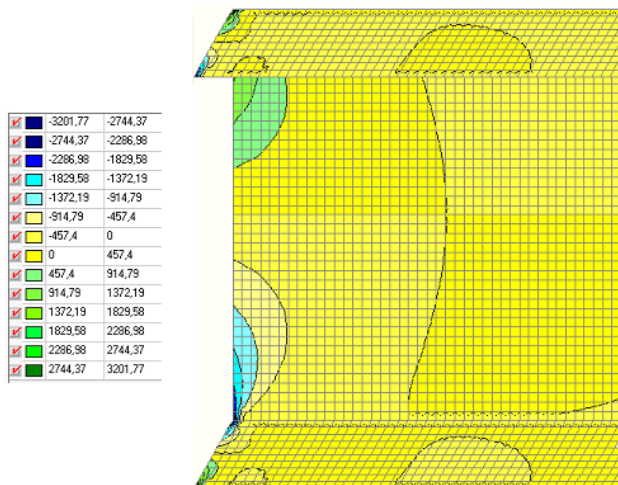

Fig. 6. Normal stress σ_x. Minimum value –*3201.77 kg/cm²*, Maximum value *3201.77 kg/cm²*
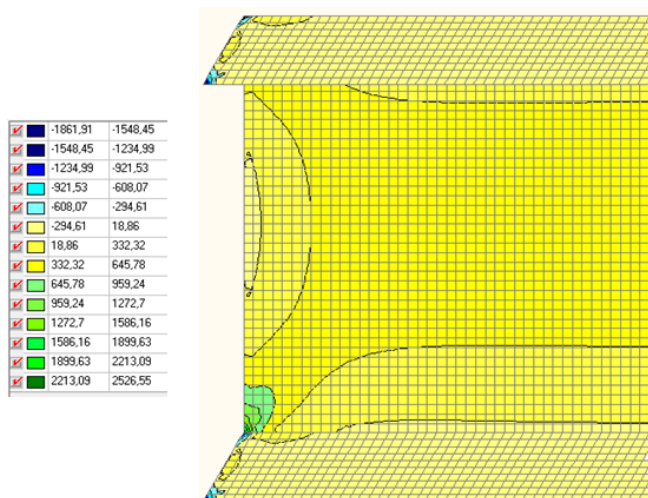
Fig. 7. Shear stress τ_{xy}. Minimum value *–1861.91 kg/cm²*, Maximum value *2526.55 kg/cm²*

For this, we use the finite element method in a static statement and solve the problem in the software package SCAD Office.

$$\frac{0.87}{3200 \cdot 0.9}\sqrt{10672^2 + 3202^2 - 10672 \cdot 3202 + 3 \cdot 1862^2} =$$

$$= \frac{0.87}{3200 \cdot 0.9} \cdot 10010 = 3.02 \, kg/cm^2 > 1 \tag{23}$$

3 Conclusions

According to the calculation results, we can conclude that the curvature of the cross section of the thin-walled cold-formed beams significantly affects their stress-strain state. Contribution of bimoment's component is an essential.

In the analysis of the stress-strain state revealed that the strength condition is not met, under a given load the beam of will lose the bearing capacity.

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