MATEC Web of Conferences **86**, 01019 (2016) DOI: 10.1051/matecconf/20168601019
IPICSE-2016

Mathematical simulation of two-phase gas-liquid flow for jet aeration structures

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Abstract. The problem of the motion of an aerated submerged jet into stationary fluid mass is investigated. The equations of balance of mass, momentum and forces of interface interaction are solved numerically by finite-difference method. Jet propagation path and maximum depth of submergence of the air bubbles are calculated for various Froude numbers. The results are compared with available experimental data.

1 Introduction

The polluted state of natural and artificial water bodies, settling ponds, and sludge ponds in the biological and chemical industries represents an increasing danger for the environment. In connection with this the acute need arises to construct highly productive treatment and aeration plants. The artificial aeration method used at them by means of an aerated jet of water is rather simple to realize and is quite effective for enriching fluid masses with dissolved oxygen. By virtue of this the problem of the propagation of an aerated submerged jet is extremely important and urgent.

2 Formulation of the problem and numerical procedure

Let us examine on the basis of the method of integral relations the general statement of the problem of propagation of a submerged jet containing uniformly distributed air bubbles and issuing with velocity U_0 from a round pipe with diameter D_0 into a fluid mass at angle θ_0 ($0 \le \theta_0 \le 90^0$) to its surface (Fig. 1). Here we will assume that the air bubbles in the investigated jet have the same size and a regular spherical shape, the radius of curvature for the path of the central line of the jet is much greater than the diameter of the bubbles, and phenomena of disintegration, merger, and formation of new bubbles and phase transitions are absent. We introduce curvilinear coordinates s, n, θ, where s is the coordinate along the central line of the jet; n is the normal to it; θ is the angle between the surface of the water body and the central line of the jet. We will consider that the axial component of the velocity in each cross section is a constant, i.e., $U = U(s)$. Then the equations of balance of mass and momentum for a control volume of the aerated jet in accordance with the method of integral relations can be written as follows:

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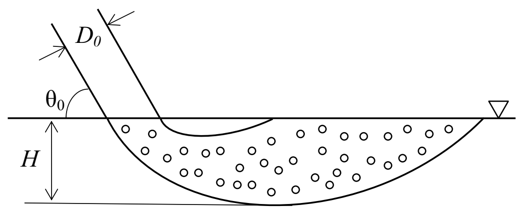


Fig. 1. Schematic of the problem.

$$\frac{d}{ds}[\rho_L U_L(1-\alpha)A] = E \tag{1}$$

$$\frac{d}{ds}\left[\rho_L U_L^2(1-\alpha)A\right] + \frac{d}{ds}\left[\rho_B \alpha U_B^2 A\right] = (\rho_L - \rho_B)\,\alpha A g \sin\theta \tag{2}$$

$$\frac{d}{ds}\left[\rho_B U_B \alpha A\right] + \rho_B V_B \alpha D = 0 \tag{3}$$

$$A\left[\rho_L(1-\alpha)U_L^2 + \rho_B \alpha U_B^2\right]\frac{d\theta}{ds} = \alpha(\rho_L - \rho_B)Ag\cos\theta + \rho_B \alpha D V_B^2 \tag{4}$$

where V is the transverse velocity component; D is the jet diameter; p is the density; A is the cross-sectional area of the jet; α is a parameter characterizing the ratio of the volume of air to a unit of the mixture; g is the acceleration of gravity; the indices L and B pertain respectively to the liquid and bubble phases. The quantity E characterizes the ejection properties of the jet during its propagation and is proportional to the perimeter of the local cross section of the jet, the velocity, and the density. The equations of motion of the bubbles can be obtained from an examination of the balance of forces of interface interaction, including friction (Stokes force), buoyant, and gravitational forces and the force related to interaction of the apparent additional masses. Writing these relations in projections onto the axial and transverse directions, we will have

$$(\rho_B + k\rho_L)U_B\frac{dU_B}{ds} - k\rho_L U_B\frac{dU_L}{ds} = \frac{3}{4}c_{Ds}\rho_L(U_L - U_B)\left|U_L - U_B\right|\frac{1}{d_B} + (\rho_L - \rho_B)g\sin\theta \tag{5}$$

$$\rho_B U_B\frac{dV_B}{ds} = -\frac{3}{4}c_{Dn}V_B\left|V_B\right|\frac{1}{d_B} + (\rho_L - \rho_B)g\cos\theta - k\rho_L U_B\frac{dV_B}{ds} \tag{6}$$

Here c_{Ds}, c_{Dn} are the drag coefficients of the bubbles in the axial and transverse directions; d_B is the diameter of the bubbles; k is the coefficient of the apparent additional mass. Supplementing these equations with relations for determining the Cartesian coordinates x_c, y_c of the central line of the jet

$$\frac{dx_c}{ds} = \cos\theta, \qquad \frac{dy_c}{ds} = \sin\theta \tag{7}$$

we obtain a closed system of equations (1)-(7) for the unknowns U_B, V_B, θ, α, D, x_c, y_c.

Changing to dimensionless variables, having determined them by means of the initial values D_0, α_0, U_{L0} and V_{B0}

$$\xi = \frac{s}{D_0}, \quad \tilde{D} = \frac{D}{D_0}, \quad \tilde{A} = \frac{A}{A_0}, \quad \tilde{U}_L = \frac{U_L}{U_{L0}}, \quad \tilde{U}_B = \frac{U_B}{U_{B0}}$$

$$\tilde{E} = \frac{E}{U_{L0} D_0}, \quad \tilde{V}_B = \frac{V_B}{V_{B0}}, \quad \tilde{\alpha} = \frac{\alpha}{\alpha_0}, \quad \tilde{x}_c = \frac{x_c}{D_0}, \quad \tilde{y}_c = \frac{y_c}{D_0}.$$

we can write system (1)-(7) in the form of eight ordinary differential equations (omitting the tilde sign):

$$\frac{1}{A}\frac{dA}{d\xi} - \frac{\alpha_0}{1-\alpha\alpha_0}\frac{d\alpha}{d\xi} = \frac{4}{\pi}\frac{E}{U_L A(1-\alpha\alpha_0)} - \frac{1}{U_L}\frac{dU_L}{d\xi} \tag{8}$$

$$U_B \alpha \rho_R U_R^2 \alpha_0 \frac{dU_B}{d\xi} + U_L (1-\alpha\alpha_0)\frac{dU_L}{d\xi} =$$
$$= -\frac{4}{\pi}\frac{U_L}{A}E + \frac{4}{\pi}\frac{U_B}{D}\rho_R U_R^2 \alpha_0 V_B \alpha + (1-\rho_R)\alpha\alpha_0 \frac{1}{\mathrm{Fr}}\sin\theta \tag{9}$$

$$\left[(1-\alpha\alpha_0)U_L^2 + \rho_R \alpha\alpha_0 U_B^2 U_R^2\right]\frac{d\theta}{d\xi} = (1-\rho_R)\alpha\alpha_0 \frac{1}{\mathrm{Fr}}\cos\theta + \frac{4}{\pi}\frac{\rho_R \alpha\alpha_0 V_B^2 U_R^2}{\sqrt{A}} \tag{10}$$

$$\frac{1}{A}\frac{dA}{d\xi} + \frac{1}{\alpha}\frac{d\alpha}{d\xi} = -\frac{4}{\pi}\frac{V_B}{U_B D} - \frac{1}{U_B}\frac{dU_B}{d\xi} \tag{11}$$

$$(k+\rho_R)U_B U_R^2 \frac{dU_B}{d\xi} - kU_B U_R \frac{dU_L}{d\xi} =$$
$$= \frac{3}{4}c_{Ds}(U_L - U_B U_R)\left|U_L - U_B U_R\right|\frac{1}{d_R} + (1-\rho_R)\frac{1}{\mathrm{Fr}}\sin\theta \tag{12}$$

$$U_B \frac{dV_B}{d\xi} = \frac{1}{k+\rho_R}\left[-\frac{3}{4}c_{Dn} V_B \left|V_B\right|\frac{1}{d_R} + (1-\rho_R)\frac{1}{\mathrm{Fr}\,\mathrm{U}_\mathrm{R}^2}\cos\theta\right] \tag{13}$$

$$\frac{dx_c}{ds} = \cos\theta, \qquad \frac{dy_c}{ds} = \sin\theta \tag{14}$$

where $Fr = U_{L0}^2 / gD_0$, $U_R = U_{B0}/U_{L0}$, $d_R = d_B / D_0$, $\rho_R = \rho_B / \rho_L$.

The Cauchy problem is examined for system (8)-(14). We will seek the solution of the initial problem for various values of the Froude numbers Fr, initial concentration α_0 and slope angle of the θ_0 ,considering $\rho_R = 0.00129$ and $d_R = 0.01$ to be constant. The system of equations (8)-(14) was integrated numerically by the Runge-Kutta method. The integration step was 0.01-0.05.

3 Numerical results

Let us examine the main properties of the solutions obtained. One of the most important characteristics is the depth H of development in the water body, as which the maximum depth of submergence of the air bubbles is taken here. Figure 2,a shows the dependences of H referred to the initial jet diameter D_0 on the Fr number for slope angles of the jet to the surface of the water body $\theta_0 = 90^0$ and fixed values $\alpha_0 = 0.1 - 0.6$. For a small value $\alpha_0 = 0.1$ the depth H rapidly increases with the Froude number and for large Fr numbers approaches the limiting value $H \approx 60$. This value of H corresponds to Abramovich's analytical calculation [1] for an unaerated jet, and at this depth the axial velocity of the jet is about 10% of the initial value. However, with such a small initial value α_0 the air concentration at large values of H is quite small. With an increase of α_0 the depth H increases noticeably more slowly, but the character of the dependence is preserved qualitatively.

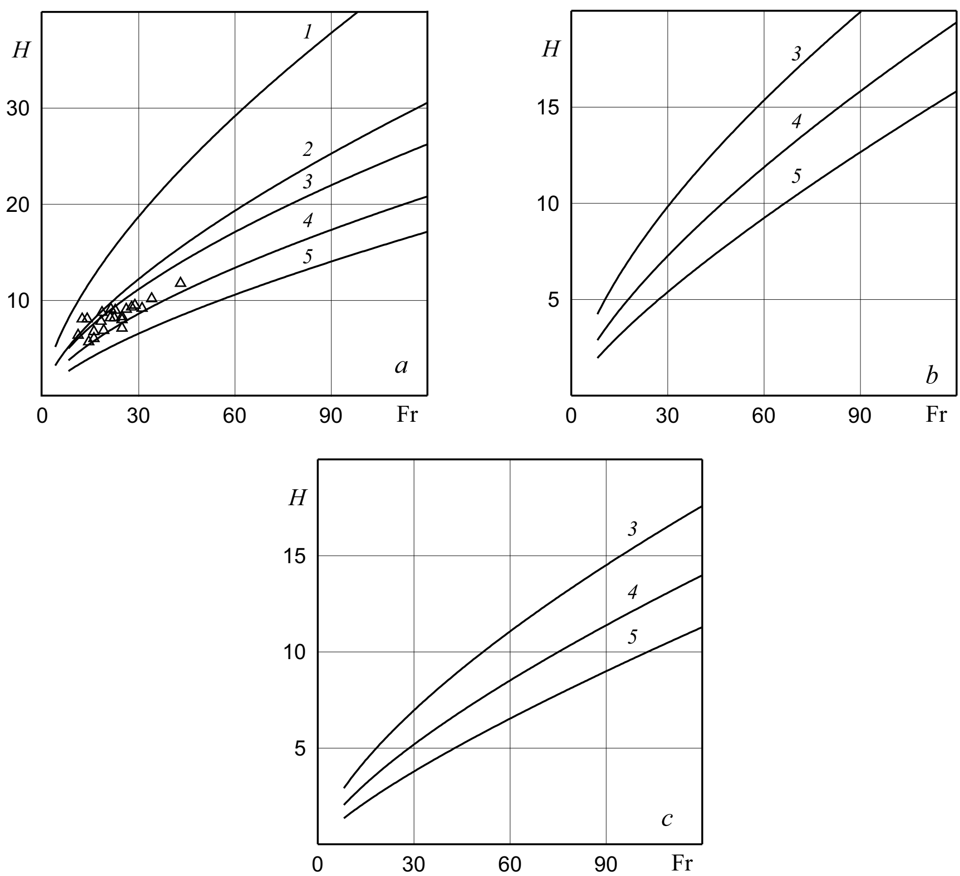


Fig. 2. Dependence of the depth H on the Froude number Fr for slope angles of the jet $\theta_0 = 90^0, 60^0, 45^0$ (a-c) for α_0 = 0.1, 0.3, 0.4, 0.5, 0.6 (curves 1-6).

Let us compare the obtained functional dependences $H = H(Fr)$ for $\theta_0 = 90^0$ with the experimental data [2-4]. Hydraulic investigations of the propagation of an aerated jet were conducted on a laboratory device in Moscow State University of Civil Engineering. The working process in it is organized in the following way. Two coaxially swirled flows rotating in opposite directions are formed by means of spiral swirlers. A vacuum occurs on the axis of the flow, promoting sucking of air from the atmosphere into the flow. As a result of interaction of the swirled flows, the air is broken into small bubbles and the intensity of swirling of the interacting flows rapidly decreases. Thus, at the exit of the aerator nozzle there is an axial flow containing air bubbles rather uniformly distributed over the cross section of the jet. The main parameters of the experimental device have the values: $D_0 = 0.126, 0.168$ and 0.168 m; water discharge up to 0.1 m^3/sec; $\alpha_0 = 0.43 - 0.58$.

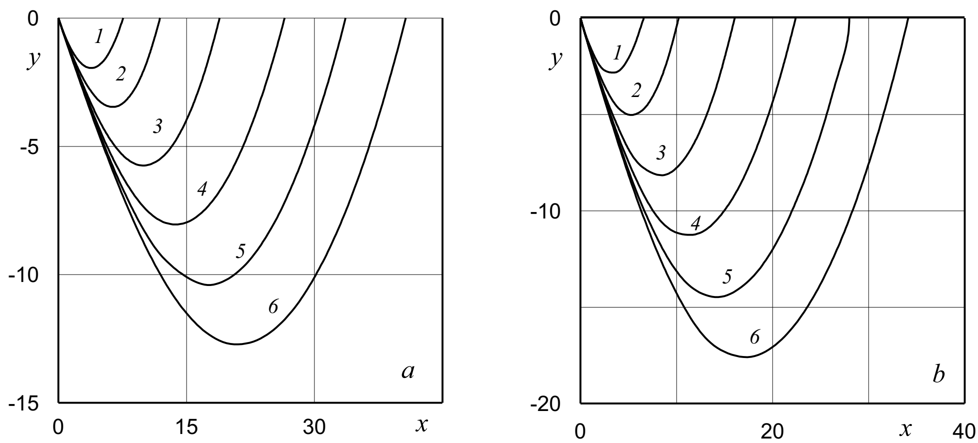


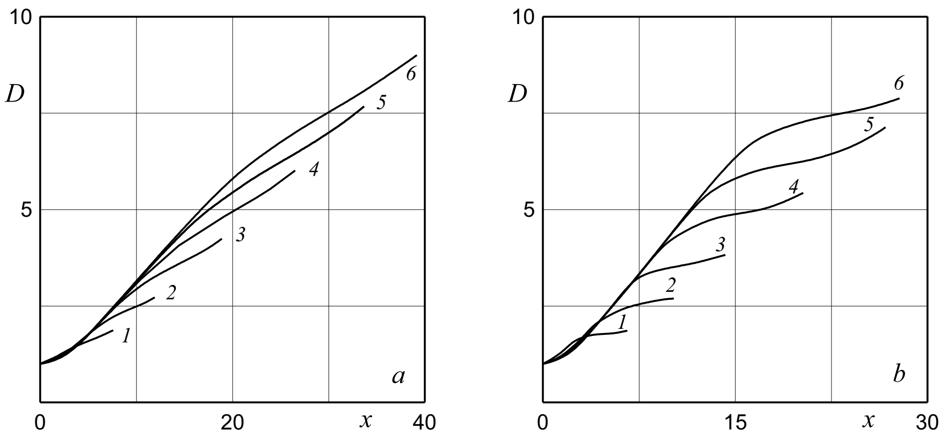
Fig. 3. Paths of the axial line of the propagation of a jet with slope angle $\theta_0 = 45^0$ (a) and $\theta_0 = 60^0$ (b) for *Fr* = 8.16, 16.33, 32.65, 54.4, 85.03, 122.45 (curves 1-6).

The results of these investigations characterizing the maximum depth of submergence H of the air bubbles for $\theta_0 = 90^0$ are plotted as asterisks in Fig. 2,a. It is seen that the experimental points in this case agree rather well with the theoretical curves and that the mathematical model reflects the essence of the phenomenon.

Figure 2,b,c shows the analogous dependences of H on the *Fr* number for slope angles of the jet $\theta_0 = 60^0$ and $\theta_0 = 45^0$ respectively. The character of these curves remains qualitatively the same as in the preceding case. Here the value of *H* for the same values of α_0 and *Fr* and a smaller value of θ_0 becomes somewhat smaller. However, the slope of the jet makes it possible to increase the surface area of aeration.

Figure 3,a,b show the calculated paths of the axial line of the jet during its propagation with slope angles to the surface of the water body $\theta_0 = 45^0$ and $\theta_0 = 60^0$ respectively, for the value $\alpha_0 = 0.5$ and various Froude numbers. The Cartesian coordinates *x*, *y* are read from the point of the surface of the water body to which the aerated jet is delivered and are referred to the outlet diameter D_0. The maximum depth to which the air bubbles are transported is clearly seen in Fig. 3,a,b. On the other hand, by means of these dependences we can estimate the distance from the site of installing the aerator at which effective mixing of air bubbles with the aerated mass occurs. Comparing the dependences in Fig. 3,a and 3,b we see that for the same values of the Froude number and a large slope angle ($\theta_0 = 60^0$)

the depth of the bubble zone is greater but the surface area is smaller. To all appearances, the optimal variant here is the installation of the aerator with an angle of delivery of the jet



$\theta_0 = 45^0$ to the surface of the water mass.

Fig. 4. Change in the diameter of the jet during its propagation with slope angle $\theta_0 = 45^0$ (a) and $\theta_0 = 60^0$ (b) for *Fr* = 8.16, 16.33, 32.65, 54.4, 85.03, 122.45 (curves 1-6).

Figure 4,a,b shows the calculated change in the relative diameter of the jet during its propagation for angles $\theta_0 = 45^0$ and $\theta_0 = 60^0$ for the same values of the Froude numbers as in Fig. 3. It is seen that with the exception of the initial and end segments the change in the diameter is a linear function of x, which agrees with Abramovich's theory.

Conclusions

Thus the investigations permit calculating the main parameters of the process of propagation of the aerated jet, and the results obtained can be used when designing various types of jet aeration systems.

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