

# Concrete strength evaluation using fracture mechanics technology

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**Abstract.** This paper presents the physical phenomenon of concrete fracture process by static uniaxial compression. The analysis of strength characteristics of concrete is carried out using fracture mechanics technology, and what is more, the actual operating conditions of structures are evaluated. The expediency of using energy approach and kinetic theory is given to consider material fracture processes with a large number of defects.

## 1 Introduction

Nowadays, concrete and reinforced concrete are widely spread among building materials. What is more, their application is increasing. This conclusion is based on the analysis of spatial arrangement of buildings and contemporary construction. It is reflected in a considerable increase in the number of storeys, construction sites and the use of frame - monolithic structure of buildings that occur mainly in residential construction.

Moreover, the analysis of concrete properties reduces to determining the strength characteristics affecting its durability. The main concrete property is durability, that is, the capacity of concrete to resist to different external effects. But not only strength affects durability, it is also necessary to consider such characteristics as porosity, crack strength that characterizes the properties of the material itself as well as the intensity of the external load.

When we consider concrete fracture process the most effective is the application of the concept and theory of fracture mechanics. According to the proposed method the destruction of any kind of material is the result of development of the already existing defects in the body material what is suitable for concrete and takes into account the heterogeneity of its structure. After all, the concrete already has a significant number of micro- and macroscopic defects the formation of which is in accordance with the technological features of production. The control over the process of structure formation is possible only after the receipt of finished products.

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## 2 Main Part

Thus, existing defects when subjected to external forces increase in size, develop into macro defects and reach critical disclosure in the form of main cracks. That is, they divide the material into parts destroying it. Stress redistribution, when disclosing defects, leads to the beginning of a chain reaction, and is brittle fracture of concrete.

The fracture process itself can be divided into three stages, discussed in the works [1,2], where the energy concept is applied using kinetic theory of fracture mechanics.

The energy values of crack formation, allocated in the destruction, define three stages of the considered process:

1. characterizing the accumulation of micro damages;
2. the period of preparation due to the formation of numerous microcracks;
3. the period of rapid material destruction.

At the first stage the energy amount of the material deformation is negligible and practically has no effect on the structure and characteristics of the material.

At the second stage there is more energy amount of destruction than at the first one. It results in emergence of variously oriented microcracks affecting the physical and mechanical properties of concrete. At this stage there is a destruction of the weakest links of concrete structures which are located in the places of low-strength inclusions or with a maximum stress concentration.

At the third stage, with increase in load the amount of the released destruction energy far exceeds the capacity of the first two stages. The release of such amount of energy can be justified by the process of further stress redistribution and its increase to such a critical value where the increase in length and width of cracks leads to their coalescence. Further formation and increase processes of large, main cracks absorb all the energy supplied to material and result in loss of stability and its division into parts, that is, to the destruction of material.

It should be noted that the preparation of fracture even under a static compression is extended in time and can be an indicator of durability while the process of "destroying" occurs like an avalanche. The most important influence on the duration of the preparatory destruction process has the rate of load application.

The use of energy concept makes it possible to trace the connection of the characteristics, crack strength with the proposed parameters of its evaluation.

Based on the Griffith's theory of equilibrium cracks, we can assume the development of volume damage as flat crack separation, that is, independent growth of its length and width is allowed.

The energy balance characteristic describing cracks development can be presented as:

$$\sum_{i=1}^{i=k} \frac{\pi l_i \sigma_i^2 \Delta l_i \Delta \delta_i}{2E} (1 - \mu^2) = 4\nu \sum_{i=1}^{i=k} \Delta l_i \Delta \delta_i, \quad (1)$$

where  $\sum \Delta l_i \Delta \delta_i$  - the overall length and width of gaps formed in the concrete sample, m;

$\nu$  - concrete surface energy, J / m<sup>2</sup>;

$E$  - concrete modulus of elasticity, MPa;

$\sigma_i$  - working stress in  $i$ -th crack, Pa.

We simplify the formula (1) by introducing the averaged length and width of the crack, so the action of stress  $\sigma$  will cause the increase of the averaged crack at the specified length and width:

$$\frac{\pi\sigma^2 L\Delta L\Delta\delta}{2E}(1-\mu^2) = 4v\Delta L\Delta\delta \quad (2)$$

We replace the product  $\Delta L\Delta\delta=S$ . The second member of equation (2) expresses the fracture or crack formation energy:

$$A = \frac{\pi\sigma^2 LS}{8E}(1-\mu^2) \quad (3)$$

Due to this, it is necessary to investigate the behavior of the formed crack depending on the stress increase in the concrete element. Wherefore we use Zhurkov's transformed equation of kinetic theory, presented by K.I. Kuznetsova [6]:

$$\frac{dl}{dt} = a_0 e^{\frac{\alpha\sigma}{R}} \quad (4)$$

where  $a_0$  – characteristic reflecting the tendency of the material to crack formation under certain conditions of external actions, m / s;  $\alpha$  – characteristic reflecting the intensity of crack formation processes.

Based on the load standard conditions with linear variation of load application rate, we present the following equation:

$$\frac{d\sigma}{dt} = K \quad (5)$$

Substituting this expression into (5) we obtain:

$$\frac{dl}{d\sigma} = \frac{dl}{dt} \cdot \frac{dt}{d\sigma} = \frac{a_p}{K} \exp\left(\alpha \frac{\sigma}{R_p}\right) \quad (6)$$

We replace  $\beta=a_p/K$ , then the expression (6) takes the form

$$\frac{dl}{d\sigma} = \beta \exp\left(\alpha \frac{\sigma}{R_p}\right) \quad (7)$$

Integrating the dependence (7) we obtain:

$$l = \int_0^{\sigma} \beta \exp\left(\alpha \frac{\sigma}{R_p}\right) d\sigma = \frac{\beta R_p}{\alpha} \exp\left(\alpha \frac{\sigma}{R_p}\right) \Big|_0^{\sigma} = \frac{\beta R_p}{\alpha} \left[ \exp\left(\alpha \frac{\sigma}{R_p}\right) - 1 \right] \quad (8)$$

The equation (8) shows the dependence of the crack formation length from the level of the applied stress. Taking into account the kinetic characteristics of  $\alpha$  and  $\beta$  considering various temperature – humidity conditions of concrete destruction we substitute the dependence into (3) and obtain the expression:

$$A = \frac{\pi\sigma^2 S\beta R_p \left[ \exp\left(\alpha \frac{\sigma}{R_p}\right) - 1 \right]}{8E\alpha} \quad (9)$$

Taking into account the relative level of stresses, the dependence takes the form:

$$A = \frac{\left(\frac{\sigma}{R_p}\right)^2 \pi R_p^2 S \beta R_p \left[ \exp\left(\alpha \frac{\sigma}{R_p}\right) - 1 \right]}{8E\alpha} \quad (10)$$

As discussed above, at the moment of destruction the release of the micro cracks formation energy exceeds to the maximum, which means that  $\sigma = R_p$ , and in consideration of (2) the dependence (10) takes the form:

$$\nu S = \frac{\pi S R_p^3 \beta [\exp(\alpha - 1)]}{8E\alpha} \quad (11)$$

However, to simplify the descriptive part, we use the research results of V.A. Kuzmenko [5] showing that in the case of uniaxial compression the maximum tensile stresses are calculated as "reduced stresses" and are equal to  $\mu \sigma_{com}$ , that is,

$$\sigma_{tens}^{red} = -\mu \sigma_{com} \quad (12)$$

Taking into account the equivalence of "reduced stresses", we substitute the dependence (11) and derive the critical stress, which led to the destruction of the material

$$R_{com} = \sqrt[3]{\frac{8E\nu\alpha}{\pi\mu^3\beta[\exp(\alpha-1)](1-\mu^2)}} \quad (13)$$

The energy concept and the kinetic theory in fracture mechanics are often applied for merit rating of materials that have a significant number of micro- and macroscopic defects in the structure.

### 3 Conclusion

Current trends of the proposed method are the simplicity and accuracy of calculations.

The fracture mechanics methods as science based regulatory framework for evaluation the properties of concrete and reinforced concrete gained widespread with the introduction of GOST 29167-91 in the 1990s [3,9]. That is the result of large-scale experiments completion conducted by RILEM (International Union of Testing and Research Laboratories for Materials and Structures) and the research results of many Russian scientists [4,7,8].

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