

# Semianalytical analysis of shear walls with the use of discrete-continual finite element method. Part 2: Numerical examples, future development

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**Abstract.** The distinctive paper is devoted to the two-dimensional semi-analytical solution of boundary problems of analysis of shear walls with the use of discrete-continual finite element method (DCFEM). This approach allows obtaining the exact analytical solution in one direction (so-called “basic” direction), also decrease the size of the problem to one-dimensional common finite element analysis. Two numerical examples of structural analysis with the use of DCFEM are considered, conventional finite element method (FEM) is used for verification purposes. The presented examples show some of the advantages of the suggested approach to semianalytical analysis of the shear wall. Future development of DCFEM, particularly associated with multigrid approach, is under consideration as well.

## 1 Numerical examples

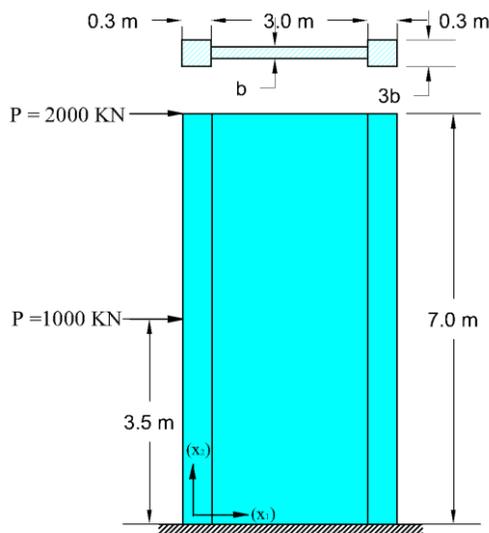
Numerous examples have been considered for illustrating the efficiency of discrete-continual finite element method (DCFEM) [1-8] and two of them are presented below. Corresponding software has been developed with the use of Intel Parallel Studio XE [9] (FORTRAN programming language [10]). Conventional finite element method (FEM) has been used for verification purposes.

### 1.1 Semianalytical solution of the shear wall subjected to horizontal forces

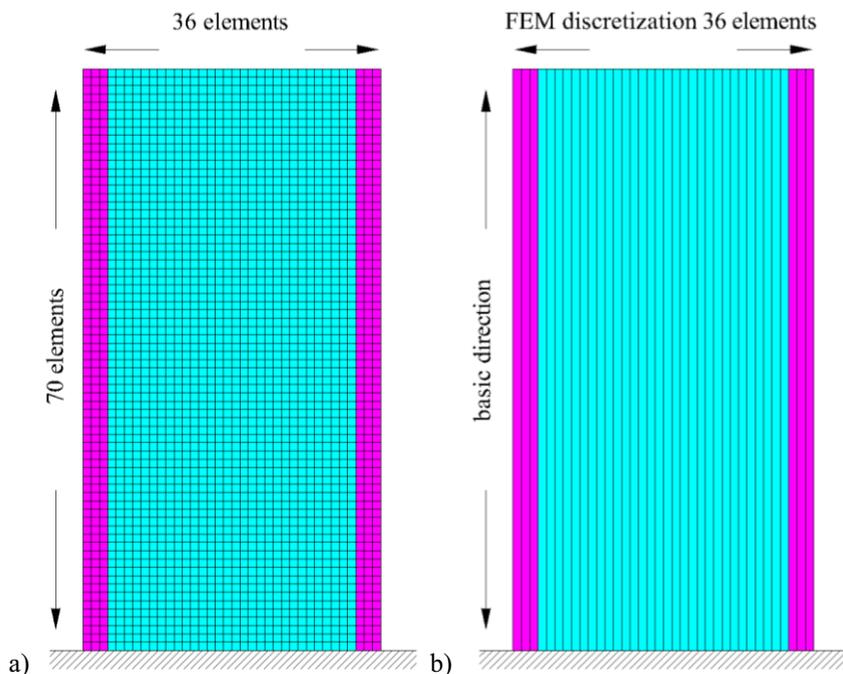
The end-column rectangular 3.6×7.0 m concrete shear wall, subjected to two horizontal forces, is considered (Fig. 1). Module of elasticity and Poisson ratio of the wall are equal to  $E_{wall} = 26500$  Mpa and  $\nu = 0.3$  respectively [11]. For the simulation of module of elasticity of the end column in two-dimensional model, the module of elasticity of the wall is multiplied to the ratio of the thickness of the column with respect to the wall.

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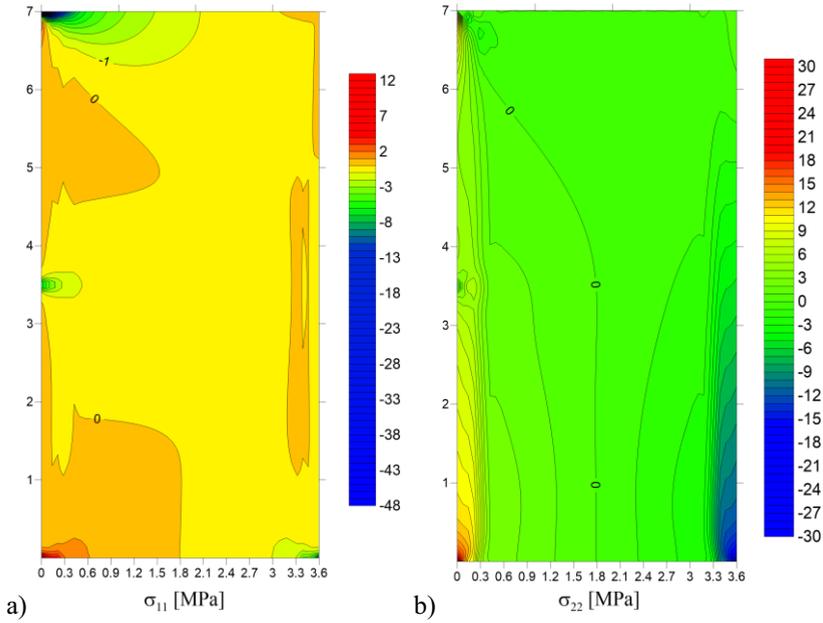


**Fig. 1.** End-column rectangular concrete shear wall subjected to two horizontal forces.

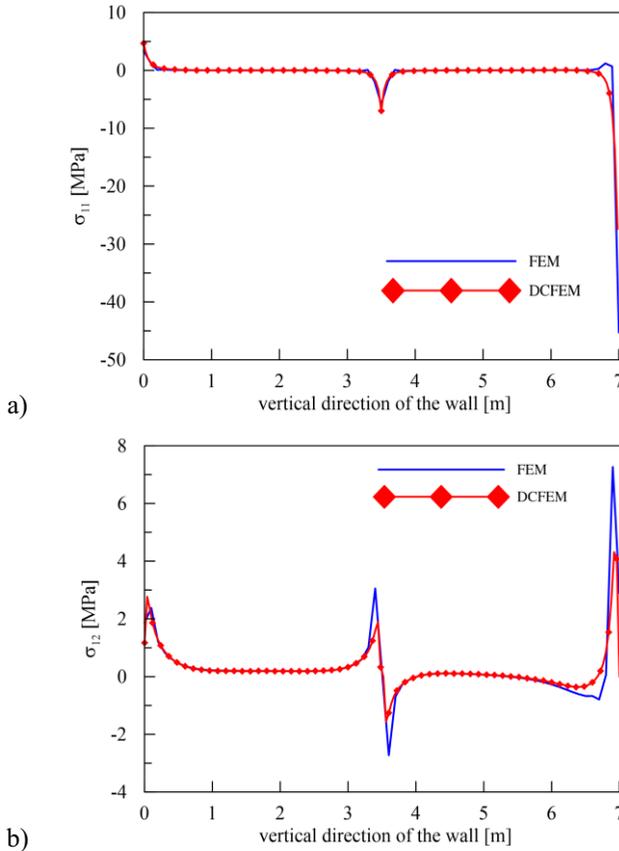


**Fig. 2.** Discretization of the shear wall with FEM (a – 2520 elements) and DCFEM (b).

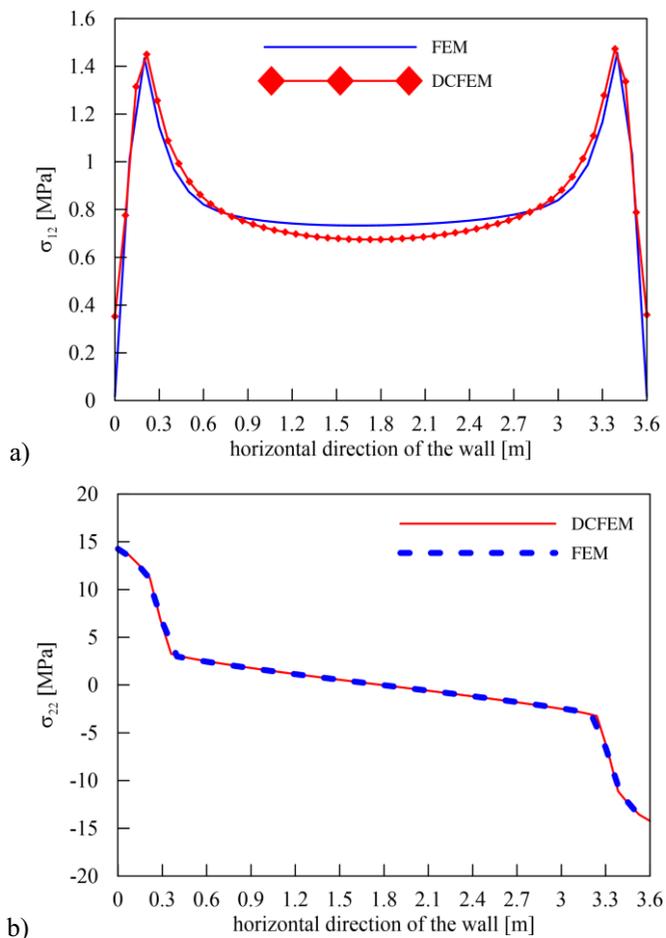
Considering the Fig. 1, we have  $E_{column} = 3E_{wall}$  for the column module. The approximation models of the shear wall within FEM and DCFEM are shown in Fig. 2. With respect to the length of the wall, the analytical solution is constructed along the vertical direction. The two-dimensional stress distribution in the shear wall is shown in Fig. 3. The results of DCFEM solution and comparison with the FEM solution (ANSYS Mechanical [12-14]) along the vertical length of the wall for  $x_1 = 0.1$  m and along the horizontal length of the wall for  $x_2 = 0.3$  m are represented in the Fig. 4 and Fig. 5, respectively.



**Fig. 3.** Two-dimensional stress distribution in the shear wall (Mpa): a)  $\sigma_{11}$ ; b)  $\sigma_{22}$ .



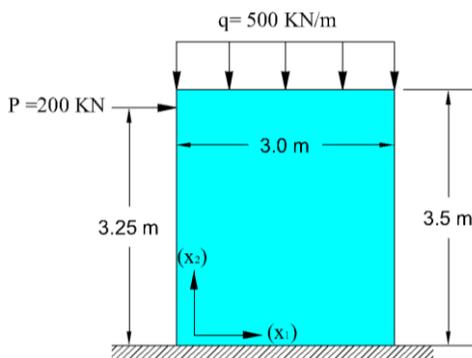
**Fig. 4.** DCFEM and FEM solutions along vertical length,  $x_1 = 0.1$  m (Mpa): a)  $\sigma_{11}$ ; b)  $\sigma_{12}$ .



**Fig. 5.** DCFEM and FEM solutions along vertical length,  $x_2 = 0.3$  m (Mpa): a)  $\sigma_{12}$ ; b)  $\sigma_{22}$ .

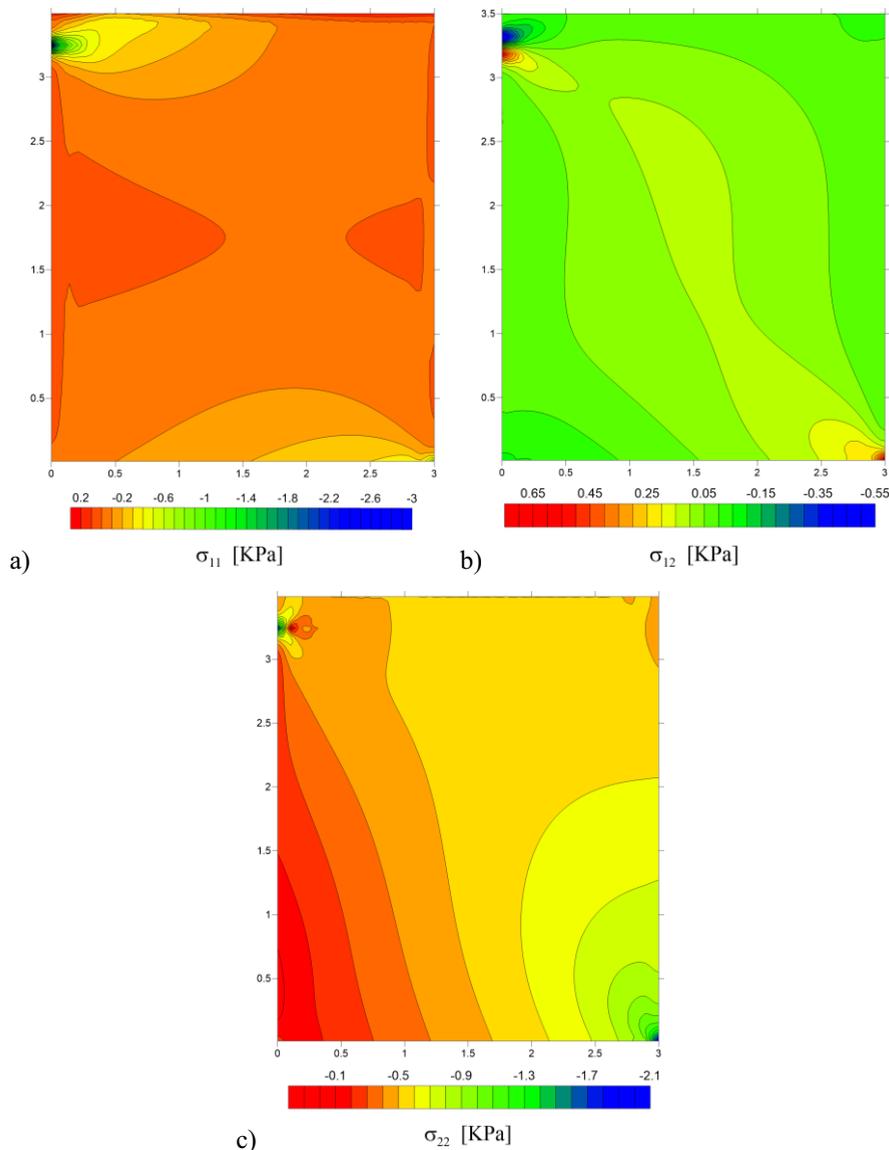
## 1.2 Semianalytical solution of the shear wall subjected to vertical distributed load and horizontal force

The simple rectangular  $3.0 \times 3.5$  m concrete shear wall subjected to a horizontal force and distributed load on the top, is considered (Fig. 6).



**Fig. 6.** Simple concrete shear wall, subjected to a horizontal force and distributed load.

The shear wall has constant thickness 0.15 m, its module of elasticity is equal to  $E_{wall} = 26500$  Mpa, Poisson ratio is equal to  $\nu = 0.3$ . 4200 eight noded quadrilateral elements is used for the FEM solution, while the higher-precision DCFEM solution requires only 60 discrete-continual finite elements. The two-dimensional stress distribution in the shear wall is shown in Fig. 7.



**Fig. 7.** Two-dimensional stress distribution in the shear wall (Kpa): a)  $\sigma_{11}$ ; b)  $\sigma_{12}$ ; c)  $\sigma_{22}$ .

The results of DCFEM solution and corresponding comparison with the FEM solution (ANSYS Mechanical [12-14]) along the vertical length of the wall for  $x_1 = 0.05$  m and along the horizontal length of the wall for  $x_2 = 0.2$  m are represented in the Fig. 9 and Fig. 10 respectively.

## 2 Future development of discrete-continual finite element method

Vital tendency of future development of DCFEM is associated with the use of multigrid approach for semianalytical structural analysis. Since the pioneering work of R.P. Fedorenko [15], multigrid literature has grown at an astonishing rate.

Multigrid methods are motivated by the observation that standard iterative methods like Gauss-Siedel, damped Jacobi and block-Jacobi are effective in reducing the high frequency error, but are not effective in reducing the low frequency content of the error. These standard solvers are called smoothers as they render error smooth by reducing high frequency content of the error. The in-effectiveness of standard iterative methods can be overcome by projecting the solution onto a coarse space, where the low frequency error becomes a high frequency error. Thus, the reason for using multigrid methods is to accelerate convergence.

Multigrid methods are among the most efficient iterative algorithms for solving the linear algebraic systems associated with elliptic partial differential equations [16-23]. As is known, the algorithm requires a series of problems be “solved” on a hierarchy of grids with different mesh sizes. For many problems, it is possible to prove that its execution time is asymptotically optimal. The niche of multigrid algorithms is large-scale problems where this asymptotic performance is critical. The need for high-resolution partial differential equations (PDE) simulations has motivated the parallelization of multigrid algorithms. However, these algorithms require to handle the operations on several coarser levels where the communication costs are higher than the computation costs.

Multigrid algorithms can be classified into two categories: geometric and algebraic. The primary difference is that algorithms of the former type use an underlying mesh for constructing coarser multigrid levels (coarsening), and algorithms of the latter type use the entries of the fine-grid matrix for coarsening in a black-box fashion. Algebraic multigrid (AMG) methods [24-27] are gaining prominence due to their generality and the ability to deal with unstructured meshes. Geometric multigrid methods are less general but have low overhead, are quite fast, and are easy to parallelize (at least for structured grids) [28-30].

The main steps in developing multigrid methods include: coarsening the fine grid (corresponding fine grid operator can be generated by finite element application), choosing grid transfer operators to move between meshes (i.e., the restriction and prolongation operators), determining the coarse mesh discretization matrices, and finally developing appropriate smoothers. Developing effective multigrid methods often boils down to striking a good balance between setup times, convergence rates, and cost per iteration. These features in turn depend on operator complexity, coarsening rates, and smoother effectiveness.

## 3 Discussion of the results. Conclusions

The presented examples show some of the advantages of the suggested approach to semi-analytical analysis of the shear wall. Discrete-continual finite element method is rather efficient for evaluation of so-called boundary effects (such as, for instance, the stress field near the concentrated force). It and also has completely computer-oriented algorithm, computational stability, optimal conditionality of resultant system and applicable for the various loads at an arbitrary point or a region of the wall. The structural discontinuities in the analytical direction can be taken into account, by addition of new appropriate boundary condition at the relevant section. Discrete-continual finite element method allows increasing the accuracy of the solution (especially in solution of high-rise shear walls) and provides reduction of the computational efforts [1-8, 31].

The Reported study was Funded by Government Program of the Russian Federation “Development of science and technology” (2013-2020) with in Program of Fundamental Researches of Ministry of Construction, Housing and Utilities of the Russian Federation and Russian Academy of Architecture and Construction Sciences, the Research Project 7.1.1.

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