MATEC Web of Conferences **84**, 00024 (2016) DOI: 10.1051/matecconf/20168400024
International Symposium IPHT 2016

Criterion for dry spot development in isothermal liquid film on a horizontal substrate

Leonid Maltsev[1], and Oleg Kabov[1,2,a]

[1]*Kutateladze Institute of Thermophysics SB RAS, Novosibirsk, Russia*
[2] *National Research Tomsk Polytechnic University, Tomsk, Russia*

Abstract. The paper proposes the criterion for development of dry spots in isothermal liquid films on a horizontal substrate and the formulas for gravity and surface tension forces applied at a given contact angle in the plane of the substrate on the liquid rim element surrounding a dry spot. The balance of these forces determines further pattern of initial small dry spot – whether it will disappear or develop into a large spot.

1 Introduction

Studies of rupture of thin liquid films on solid surfaces are important for modeling of multiphase flows in microfluidic devices, heat exchange systems, mining industry, and for biomedical applications such as dynamics of the tear film in the eye [1]. When the interface approaches the wall, the film can rupture, resulting in the formation of a three-phase contact line. The overall dynamics of many types of multiphase flows encountered in various applications depends on these local phenomena [2]. In cooling systems based on thin-film flows driven by either gravity or shear stresses at the liquid-gas interface, formation of dry spots results in significant reduction of heat flux from the heated wall, as shown experimentally in [3]. Liquid films, flowing under gravity over a localized heater on vertical or inclined flat plates, rupture at sufficiently high heat fluxes generated by the heater [4, 5].

An important aspect of the problem of the liquid film rupture is the substrate wettability effect, [6]. In most theoretical models describing the isothermal rupture of a liquid film, the equilibrium contact angle appears as a major parameter determining the critical film thickness [7]. Experiments on the rupture of the liquid film in the absence of heating qualitatively confirm the dependence of the critical thickness of the film on the contact angle [8]. The equilibrium contact angle is used as basic parameter in many models of rupture of the heated liquid film [9]. However, this is inconsistent with some experimental works [3]. It should be noted that submicron liquid film formed on the interface of dry spots can make a major contribution to the total heat and mass transfer due to intensive evaporation [10, 11, 12]. In some cases, rupture of the film is accompanied by formation of ultrafine residual liquid film, which exists for a very limited time [5,13].

Current research provides a theoretical analysis of the effect of the equilibrium contact angle, surface tension and liquid weight on the critical size of dry spots in isothermal liquid film of a given thickness on a horizontal flat substrate.

 DOI: 10.1051/matecconf/20168400024

2 Analysis of the forces acting on the liquid rim surrounding the dry spot

Let the motionless liquid film rest on a horizontal substrate under the action of gravity. Assume that at some point, a dry spot, having the shape of a circle, appears in the film. The dry spot is usually surrounded by a liquid rim. Figure 1 shows a general view of a dry spot surrounded by a rim, and Fig. 2 presents a cross section of the rim by a plane passing through the center of the dry spot.

Each element of the rim cut off by central angle $\delta\varphi$ is subjected to the action of the gravity and the force caused by surface tension on a curved surface of the rim. Let us write expressions for these forces acting within the plane of the plate. We introduce the following notation: ρ is the fluid density, r_0 is the radius of the dry spot, g is the acceleration of gravity, and h is the thickness of the liquid film.

Gravity causes static pressure in the film:

$$p = \rho g z \tag{1}$$

The total action of pressure forces on the rim element in the plane of the substrate is equal to

$$P = \frac{1}{2}\rho g h^2 r_1 \delta\varphi \tag{2}$$

and is directed towards the center of the rim.

Capillary force N acting on the rim element in the plane of the substrate can be represented in the following form,

$$N = \sigma r_0 \delta\varphi \int_{(L)} (k_1 + k_2) cos\psi dS \tag{3}$$

Here σ is the surface tension of the liquid, k_1 and k_2 are the main curvatures of the rim surface at the points of contour L of the rim cross section.

According to Meunier's theorem [14], the curvature associated with the axial symmetry of the rim, can be written as

$$k_1 = \frac{\cos\psi}{r} \tag{4}$$

where r is the distance from a given point of the contour L to the axis of symmetry of the rim.

The second curvature related to the shape of the contour L, can be expressed as

$$k_2 = \frac{d\vartheta}{dS} \tag{5}$$

Where $\vartheta = \frac{\pi}{2} - \psi$ is the angle between the substrate and the tangent to the contour L.

Consequently,

$$N = \sigma\left[-\int_{(L)} \frac{\cos^2\psi}{r(\psi)} R(\psi) d\psi + \int_{(L)} \sin\vartheta \, d\vartheta\right] r_0 \delta\varphi \tag{6}$$

here $R(\psi)$ is the radius of the local curvature of the contour L. The contribution to the total force from the surface tension at the solid-liquid interface can be shown to be zero.

It is natural to assume that the contour L is a smooth curve with a continuous change (not necessarily monotonic) of the angle ϑ from $-\theta_0$ to 0, where θ_0 is contact angle. Then the second integral in equation (6) does not depend on the shape of L and is equal to $(1- \cos\theta_0)$, so that equation (6) takes the form

$$N = \sigma(-K + 1 - \cos\theta_0) r_0 \delta\varphi \tag{7}$$

Here $K = \int_{(L)} \frac{\cos^2\psi}{r(\psi)} R(\psi) d\psi$

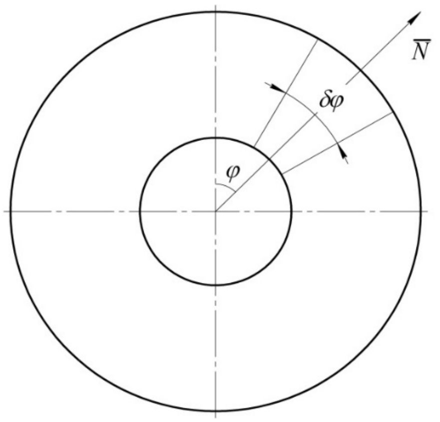

Figure 1. Schematic diagram of the liquid flow.

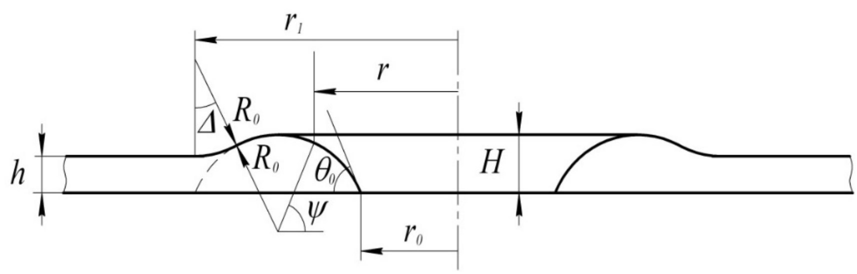

Figure 2. The cross-section of the rim surrounding the dry spot.

3 Condition of the critical state of a dry spot in the film

Under the forces acting on the rim elements, the rim begins to extend symmetrically, absorbing the liquid film, or conversely shrink. We neglect the liquid flow within the rim assuming that each element of the rim moves along the corresponding radius of the rim integrally. Then the behavior of a small dry spot in the film is determined by the sign of the total force acting on the rim element towards the radius of the dry spot. The spot disappears if this force is directed toward the center of the rim, and grows in size if the force is directed from the centre.

Hence, the equality

$$P = N \tag{8}$$

is the condition for the equilibrium state of the rim. Substituting the expressions for summands in this equality, we obtain the equation for determining the critical parameters of dry spot in the liquid film

$$\frac{1}{2}\rho g h^2 r_1 - \sigma(1 - \cos\theta_0 - K) r_0 = 0 \tag{9}$$

or in a dimensionless form

$$\frac{r_1}{2 r_0 Bo} - (1 - \cos\theta_0 - K) = 0 \tag{10}$$

where $Bo = \sigma/\rho g h^2$ is the Bond number.

In the overwhelming number of works dealing with the modeling of dry spots and rivulets flowing along the interfaces of dry spots, the interfaces of the cross section of the rim or rivulet are approximated by circular arcs. Let us assume that the curved interface of the rim cross section is composed of two arcs of a circle with a radius R_0 (Fig. 2).

Thus, we believe that the cross-section of the free interface of the film in the area of a dry spot has the shape of an arc of a circle with a certain radius R_0, which is adjacent to the plate substrate at an angle θ_0. The most visible characteristic of the initial perturbation, obviously, is the rim height H, which is associated with the radius R_0 by the relationship

$$H = R_0(1 - \cos\theta_0) \tag{11}$$

Let assume the following scenario for the initial dry spot formation. A gas (steam) bubble appears in the liquid film directly on the wall. It is separated from the atmosphere by a thin film membrane. At some point, the membrane collapses and a round dry spot without a rim appears in the film.

In this case $H = h$ and the cross section of the rim is a half segment of a circle with a radius R_0 that is adjacent to the substrate at an angle equal to θ_0. Then

$$R_0 = \frac{h}{(1-\cos\theta_0)}\ , \qquad r_1 = r_0 + R_0 \sin\theta_0 \tag{12}$$

After the cross-sectional shape of the rim is known, the K integral in the expression for N (7) takes a definite value

$$K = K_0\left(\frac{\pi}{2} - \theta_0,\ \frac{\pi}{2},\ \frac{r_0}{h}(1 - cos\theta_0) + sin\theta_0\right), \tag{13}$$

where $K_0(\psi_1, \psi_2, C) = \int_{\psi_1}^{\psi_2} \frac{\cos^2\psi}{C-\cos\psi}$.

Substituting all the necessary values in equation (10), we obtain the equation for determining the critical parameters of an initial perturbation in the following form

$$\frac{1}{2Bo}\left(1 + \frac{h}{r_0}\frac{sin\theta_0}{1-cos\theta_0}\right) - (1 - cos\theta_0 - K) = 0 \tag{14}$$

Solving equation (14) with respect to r_0/h, we see that for a given geometry of the free interface of the rim, critical value of the dry spot radius is determined by the liquid film thickness, Bond number and contact angle. Figure 3 shows the dependences of r_0/h versus (θ_0) for three values of Bond number. For clarity, Table 1 shows film thicknesses for water at various Bond numbers selected for the calculations.

Table 1.

g=9.8 m/s, ρ=1000 kg/m³, σ=0.073 N/m			
Bo	3	10	100
h, mm	1.56	0.86	0.27

4 Conclusions

Critical ratios of the dry spot radius to the liquid film thickness are determined by Bond number and contact angle. In this case, at the contact angles greater than θ_0 = 70–80 degrees, the effect of Bond number on the ratio r_0/h is quite small, and the values of r_0/h are also small. However, at moderate and small values of θ_0, the role of the Bond number is extremely important, and the values of r_0/h become substantially greater than unity, i.e. for small values of θ_0 even large dry spots tend to disappear.

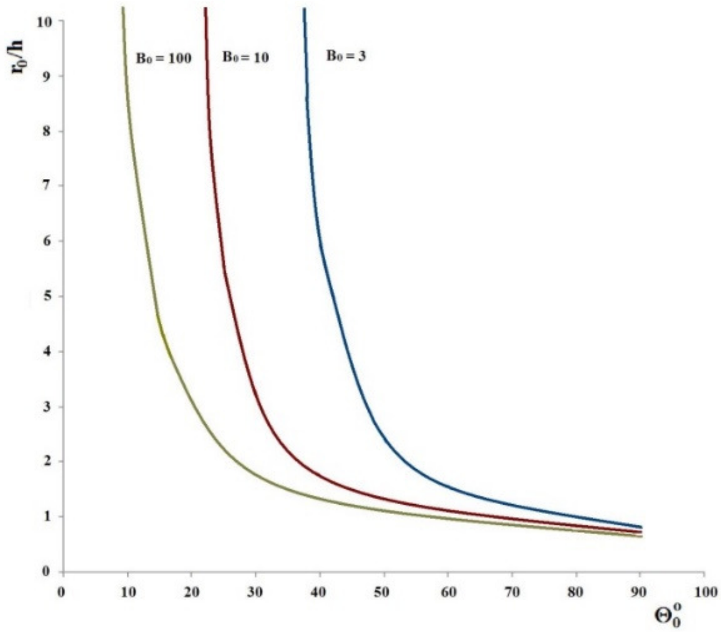

Figure 3. The calculation results of critical size of dry spot in the liquid film depending on the contact angle and Bond number.

We gratefully acknowledge the support from the Ministry of Education and Science of the Russian Federation (Agreement No. 14.613.21.0011, project identifier RFMEFI61314X0011)

References

1. Ajaev V.S., Interfacial Phenomena and Heat Transfer, **1**(1), 81–92 (2013).
2. Ajaev V.S., Gatapova, E.Ya., Kabov O.A., Advances in Colloid and Interface Science, **228**, 92-104 (2016).
3. Zaitsev D.V., Kabov O.A., Cheverda V.V., and Bufetov N.S., High temperature, **42**, 450 (2004).
4. Zaitsev D.V., Kabov, O.A., Microgravity science and technology, **19**, 174 (2007).
5. Zaitsev D.V., Rodionov D.A., Kabov O.A., Microgravity science and technology, **19**, 100 (2007).
6. Zaitsev D.V., Kirichenko D.P., Kabov O.A., Technical Physics Letters, **41**, 551 (2015).
7. El-Genk M.S., Saber H.H., International Journal of Heat and Mass Transfer, **44**, 2809 (2001).
8. Kadoura M., Chandra S., Experiments in Fluids, **54**, 1465 (2013).
9. El-Genk M.S., Saber H.H., J. Heat Transfer, **124**, 39 (2002).
10. Potash M. and Wayner Jr., Int. J. Heat and Mass Transfer, **15**, 1851 (1972).
11. Stephan P.C. and Busse C.A., Int. J. Heat Mass Transfer, **35**, 383-391 (1992).
12. Karchevsky A.L., Marchuk I.V. and Kabov O.A., Applied Mathematical Modeling, **40**, 1029–1037, (2016).
13. Lyulin Yu. V., Spesivtsev S. E., Marchuk I. V., and Kabov O. A., Technical Physics Letters, **41**, 1034–1037 (2015).
14. Korn G., Korn T. *Handbook of Mathematics* (Nauka, Moscow, 720 p., 1968).