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Mathematical Modelling of the Evaporating Liquid Films on the Basis of the Generalized Interface Conditions

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Abstract. The two-dimensional films, flowing down an inclined, non-uniformly heated substrate are studied. The results contain the new mathematical models developed with the help of the long-wave approximation of the Navier-Stokes and heat transfer equations or Oberbeck-Boussinesq equations in the case, when the generalized conditions are formulated at thermocapillary interface. The evolution equations for the film thickness include the effects of gravity, viscosity, capillarity, thermocapillarity, additional stress effects and evaporation.

1 Introduction

Modelling of the convective processes caused by impact of various forces on the fluid media is important nowadays [1-4]. The tangential stresses due to flow in the gas phase on a thermocapillary gas-liquid interface and the evaporation effects should be taken into consideration in the case when the fluid flows are accompanied by the adjacent gas flows. One of the most critical points in the mathematical modelling of the above mentioned processes is the statement of the boundary conditions on the interface. The conditions at the interface between two interacting gas-liquid phases have been presented in [6-8]. The conditions at the interface are the result of the conservation laws of mass, momentum and energy. The additional relations concerning a continuity of some flow characteristics, laws of the heat and mass fluxes, of the dynamic, energy and phenomenological properties etc. are usually assumed to be fulfilled in order to obtain the closed problem statements. The generalized interface conditions allow to model the evaporative processes in the liquid and gas-vapor phases with interface in the full problem statement and in the long-wave approximation of the governing equations.

The effects of evaporation on the nonlinear stability of uniformly heated films have been investigated experimentally, numerically and analytically in many papers [1-5, 9-11]. In this paper we present new mathematical models of flows of the thin evaporating liquid layers on the basis of the long-wave approach (see also [9-11]). These models are constructed on the basis of the Navier-Stokes equations or their Boussinesq approximation and generalized kinematic, dynamic and energetic interface conditions. One-sided two-dimensional models of the evaporating falling film are considered here in the case of moderate Reynolds numbers. The evolution equations of the film thickness take into account the effects of gravity, viscosity, capillarity, thermocapillarity, evaporation and of the

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additional tangential stresses at interface. Numerical results demonstrate the differences in the flow characteristics obtained with the help of alternative mathematical models.

2 Mathematical models of the problem

Let the impermeable substrate be inclined at an angle α to the horizon. The coordinate system is chosen so that the direction of the x-axis coincides with the direction of fluid flow (see Fig. 1a), $\mathbf{g}=(g_1, g_2)=(g \sin \alpha, -g \cos \alpha)$ is the gravitation vector, $g = |\mathbf{g}|$. The position of the interface is determined by the equation $z=h(x,t)$, $z=0$ is the equation of solid plate. If a longitudinal characteristic length (length of the surface waves) l is much larger than a transverse characteristic length (the mean film thickness) d then $\varepsilon = d/l$ is a small parameter. The one-sided mathematical model based on the Navier-Stokes and heat transfer equations or the Oberbeck-Boussinesq equations in dimensionless form can be written as follows [9-11]:

$$Re\varepsilon^2\left(u_t + uu_x + wu_z\right) - \varepsilon^2 u_{xx} = u_{zz} - p_x + f_u \sin\alpha, \tag{1}$$

$$Re\varepsilon^4\left(w_t + uw_x + ww_z\right) - \varepsilon^4 w_{xx} - \varepsilon^2 w_{zz} = -p_z + f_w \cos\alpha, \tag{2}$$

$$u_x + w_z = 0,$$

$$RePr\varepsilon^2\left(T_t + uT_x + Tw_z\right) - \varepsilon^2 T_{xx} = T_{zz}.$$

Here $\mathbf{v}=(u, w)$ is the velocity vector; p is the pressure or the modified pressure in the case of the Oberbeck-Boussinesq equations; T is the temperature; ρ is some relative value of liquid density; $Re=u_*l/\nu$ is the Reynolds number; $Pr=\nu/\chi$ is the Prandtl number; $f_u = \gamma_1$, $f_w = -\gamma_2$ in the case of the Navier-Stokes and heat transfer equations (here $\gamma_1 = Ga/(Re\varepsilon)$, $\gamma_2 = Ga/Re$) and $f_u = -\gamma_1 T$, $f_w = \gamma_2 T$ in the case of the Oberbeck-Boussinesq equations (here $\gamma_1 = Gr/(Re\varepsilon)$, $\gamma_2 = Gr/Re$); $Ga=gd^3/\nu^2$ is the Galileo number; $Gr=gT_*\beta d^3/\nu^2$ is the Grashof number; ν, χ are the kinematic viscosity and thermal diffusivity coefficients; β is the thermal expansion coefficient; u_* is the characteristic velocity, T_* is the characteristic temperature drop.

We restrict ourselves to the case of moderate Reynolds numbers ($Re=O(1)$). The generalized kinematic, dynamic and energetic conditions are satisfied at the thermocapillary interface [6-8] and take into account evaporation:

$$-\varepsilon\left(h_t + h_x - w\right) = J_{ev}E,$$

$$\varepsilon^2 h_x\left(w_z - u_x\right) + 0.5\left(u_z + \varepsilon^2 w_x\right) = \alpha_\tau\left[\varepsilon h_x\left(w_z^g - u_x^g\right) + 0.5\left(w_x^g + u_z^g\right)\right] - 0.5\alpha_{Ma}(T_x + h_x T_z),$$

$$-p + 2\varepsilon^2\left(w_z - h_x u_z\right) = -p^g + \alpha_E J_{ev}^2 + \alpha_{Ca} h_{xx}\left(1 - \alpha_\sigma T\right), \tag{3}$$

$$-T_z + \varepsilon^2 h_x T_x + \beta_2\left\{T div_\Gamma \mathbf{v}\right\} = \beta_3 J_{ev} + \beta_4 J_{ev}\left[-p + 2\varepsilon^2\left(w_z - h_x u_z\right)\right] + 0.5\beta_5 J_{ev}^3 + \beta_6 \varepsilon h_{xx} J_{ev}\left(1 - \alpha_\sigma T\right).$$

The conditions are presented in the form, where the terms of order higher that ε^2, are neglected. The local mass flux is determined with the help of the Hertz-Knudsen equation [4]: $J_{ev} = \alpha_J T \,|_{z=h(x,t)}$.

Here J_{ev} is the local mass flux at the thermocapillary interface, $E=\kappa T_*/(\lambda_U \rho\nu)$ is the evaporation coefficient, κ is the heat conductivity coefficient, λ_U is the latent heat of vaporization, $\alpha_\sigma=MaCa/(RePr)$, $Ma=\sigma_T T_* l/(\rho\nu\chi)$ is the Marangoni number, $Ca=u_*\rho\nu/\sigma_0$ is the capillary number, σ_T is the temperature coefficient of surface tension, σ_0 is the value of the coefficient of surface tension σ at the same temperature ($\sigma=\sigma_0-\sigma_T(T-T_0)$), $\alpha_{Ma}=Ma\varepsilon/Pr$, $\alpha_{Ca}=\varepsilon/Ca$, $\alpha_\tau = \bar{\rho}\bar{\nu}\bar{\upsilon}\varepsilon / \bar{h}$, $\bar{\rho}, \bar{\nu}$ are the ratio of the densities and coefficients of kinematic viscosity of the gas and liquid, $\bar{\upsilon}$ is the ratio of the characteristic gas velocity to u_*, $\bar{h}$ is the ratio of the characteristic scale of gas layer to l, $\alpha_J = \alpha\rho_s\lambda_U T_* / J_*\left(M / \left(2\pi R_g T_s^3\right)\right)^{1/2}$, α is the accommodation coefficient, ρ_s is the vapor density, J_* is the characteristic value of the vapor mass flux, M is the molecular weight, R_g is the universal gas constant, T_s is the saturated vapor temperature, $\beta_2 = Ma\varepsilon/(RePrE\bar{U})$, $\bar{U} = \lambda_U / u_*^2$, $\beta_3 = 1$, $\beta_4 = (1/\bar{\rho}-1)/(Re\bar{U}\varepsilon^2)$, $\beta_5 = (1-1/\bar{\rho})^2 E^2/(Re^2\bar{U}\varepsilon^2)$, $\beta_6 = (1-1/\bar{\rho})(Re\bar{U}Ca)$, $div_\Gamma \mathbf{v}$ is the surface divergence of velocity.

The no-slip conditions on the non-uniformly heated solid substrate are fulfilled. Statement of the problem must be added by the initial interface position and the initial values of the functions. The results of the parametric analysis of the problem are demonstrated in Table 1, where we present the values of the dimensionless parameters in (3).

3 The evolution equation for the thin layer

In the case when the mathematical model is based on the equations of Navier-Stokes, we have the evolution equation of the film thickness (lubrication theory approach):

$$h_t + h_x\left\{\frac{h^2}{2}\left((C_0)_x - \gamma_1 \sin\alpha\right) + hC_1\right\} + \left\{\frac{h_3}{6}(C_0)_{xx} + \frac{h^2}{2}\left(C_1\right)_x\right\} + \frac{E\alpha_J}{\varepsilon}\{Ah + \Theta_0\} = 0. \quad (4)$$

Here the functions A and Θ_0 determine the heat flow regime. The function Θ_0 is given, the function A satisfied a certain differential equation. Functions C_0, C_1 can be found as follows:

$$C_0(x,t) = p^g - \alpha_{Ca}h_{xx}\left(1-\alpha_\sigma\Theta^0\right) + \gamma_2 h\cos\alpha,\ \ C_1(x,t) = \alpha_\tau \tau(x,t) - \alpha_{Ma}\tilde{\Theta} - (C_0)_x h + \gamma_1 h\sin\alpha.$$

In the case when mathematical modelling is based on the Oberbeck-Boussinesq equations, the evolution equation of the film thickness has the form:

$$\begin{aligned} &h_t + h_x\left\{\frac{h^2}{2}\left((C_0)_x + \gamma_1\sin\alpha\Theta_0\right) + hC_1 + \frac{h^4}{24}\gamma_2\cos\alpha A_x + \frac{h^3}{6}\left(\gamma_2\cos\alpha(\Theta_0)_x + \gamma_1\sin\alpha A\right)\right\} + \\ &+\left\{\frac{h^3}{6}(C_0)_{xx} + \frac{h^2}{2}\left(C_1\right)_x + \frac{h^5}{120}\gamma_2\cos\alpha A_{xx} + \frac{h^4}{24}\left(\gamma_2\cos\alpha(\Theta_0)_{xx} + \gamma_1\sin\alpha A_x\right) + \right. \\ &\left. + \frac{h^3}{6}(\Theta_0)_x\gamma_1\sin\alpha\right\} + \frac{E\alpha_J}{\varepsilon}\{Ah + \Theta_0\} = 0. \end{aligned} \quad (5)$$

Here the functions C_0, C_1 are found with the help of the relations

$$C_0(x,t) = p^g - \alpha_{Ca}h_{xx}\left(1-\alpha_\sigma\Theta^0\right) - \gamma_2\cos\alpha\left(A\frac{h^2}{2} + \Theta_0 h\right) - \frac{\gamma_1}{Bu}x\sin\alpha + \gamma_2 h\cos\alpha,$$

$$C_1(x,t) = \alpha_\tau\tau(x,t) - \alpha_{Ma}\tilde{\Theta} - (C_0)_x h - \gamma_1 h\sin\alpha\left(A\frac{h^2}{2} + \Theta_0 h\right) - \gamma_2\cos\alpha\left(A_x\frac{h^3}{6} + (\Theta_0)_x\frac{h}{2}\right).$$

Table 1. The values of the parameters α and β

"α"	Ethanol-nitrogen	HFE-7100-nitrogen	FC-72-nitrogen	"β"	Ethanol-nitrogen	HFE-7100-nitrogen	FC-72-nitrogen
α_σ	10^{-2}	10^{-1}	10^{-1}	β_2	ε	ε	ε
α_{Ca}	$\varepsilon^3 10^6$	$\varepsilon^3 10^6$	$\varepsilon^3 10^5$	β_3	*1*	*1*	*1*
α_E	-10^{-3}	-10^{-2}	-10^{-2}	β_4	$\varepsilon^{-2}10^{-3}$	$\varepsilon^{-2}10^{-3}$	$\varepsilon^{-2}10^{-3}$
α_τ	$\varepsilon 10^{-2}$	$\varepsilon 10^{-2}$	$\varepsilon 10^{-2}$	β_5	$\varepsilon^{-2}10^{-6}$	$\varepsilon^{-2}10^{-5}$	$\varepsilon^{-2}10^{-5}$
α_{Ma}	$\varepsilon 10^3$	$\varepsilon 10^4$	$\varepsilon 10^3$	β_6	-10^{-2}	-10^{-2}	-10^{-2}

4 The numerical results

The evolution of the evaporating film has been investigated with use of two mathematical models (see Section 3, eqs. (4) and (5)). The periodic problems of moving of the liquid layers (ethanol and

HFE7100) accompanied by the gas flow (nitrogen) have been solved numerically on the interval [$-L$, L] with the initial disturbance $h_0=1-\delta_1 \cos(kx)$ (δ_1=0.01, $k=\pi/2$) and periodic conditions at $x=-L$, $x=L$. The non-uniform heating is realized with the help of tempearture distribution $\Theta_0 = 1 + \delta_0\cos k_1 x\cdot\cos k_2 t$ (δ_0=0.25, $k_1=\pi/2$, k_2=2). Numerical investigations have been carried out on the basis of the finite-difference scheme of the second order of approximation.

Figure 1b presents the results of numerical study of the liquid layer dynamics in the case of low gravity. We have observed the qualitatively similar results using the alternative mathematial models and found the quantitative differences. The evaporation process is significantly more intensive for "HFE7100-nitrogen" system (see lines 3 and 4 on the Fig. 1.b). A small increase in the intensity of evaporation is observed in the case of mathematical model based on the Oberbeck-Boussinesq equations (see (5) and lines 2 and 4 on the Fig. 1.b).

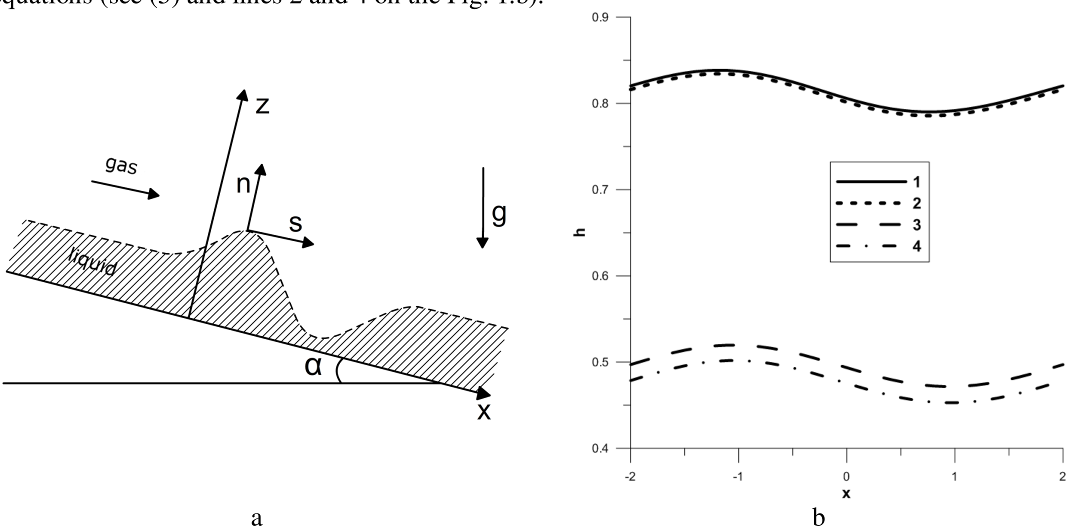

Figure 1. a) Geometry of flow domain; b) The behaviour of the liquid layer thickness, the case of low gravity (g=9.81·10^{-2} m/s^2), $\alpha = \pi/6$, t = 0.1: 1 – the model (4), "ethanol-nitrogen" system; 2 – the model (5), "ethanol-nitrogen" system; 3 – the model (4), "HFE7100-nitrogen" system; 4 – the model (5), "HFE7100-nitrogen" system.

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