Design and analysis of composite skew plate bending

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Abstract. This paper generalizes and extends a numerical formulation within the method to simulate the behaviour of skew composite plate bending. It is of pivotal importance to note that despite the spread use of skew plates, the published literature shows an important lack of scientific works regarding these particular structures. The main goal of the development described in this work is to provide a numerical tool that could be used to obtain approximate solutions for bending problem of skew composite plates subjected to different loading and various support conditions. Both homogeneous and fibre-based materials are investigated. The skew plate behaviour depend on several parameters namely, the skew angle, boundary conditions and constitutive material. Dependence on these parameters is investigated through numerical results.

1 Introduction

The use of laminated composite in many engineering applications has been expanding rapidly in the past four decades due to their higher strength and stiffness to weight ratios when compared to most metallic materials. In particular, laminated composite skew plates are frequently used in various engineering applications and have a spread economical application in different area of science including aerospace, marine, mechanical, and modern highways.

For example, enormous amounts of money and resources are invested every year in design and construction of new roads. The use of composite skew plates in civil engineering, particularly in skew bridges, can lead to the economical solutions and enables the construction of straight, aesthetic and safe highways needed to carry on increasing speeds of present-days traffic. Despite this practical interest, problems of skew plate bending have not yet been correctly addressed in the published literature. Relatively little is to be found in the engineering literature regarding composite skew plates, partially due to the lack of analytical solutions and to the numerical difficulties encountered with regard to modelling composite plate behaviour specially when dealing with a highly skew angle. Therefore, the design of composite skew plates is at present based on numerical data provided by computational models, and influenced by comparisons with rectangular and square plates which can be solved analytically. A high quality numerical approximation of skew plate behaviour is obtained by the use of a numerical approach like BEM, FEM and FDM.[1-5]

2 Problem formulations

The method offers a solution based on the small deflection theory of plate bending and the classical elasticity. This simplified theoretical model was designed for thin lamina of orthotropic material with different properties in different directions

Consider a linear elastic composite material in which is locally orthotropic and globally orthotropic. This means that in each point of the structure the stress-strain relation is given as:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{32}}{E_3} & \frac{1}{E_3} \\ & & & \frac{1}{G_{23}} \\ & & & & \frac{1}{G_{31}} \\ & & & & & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{Bmatrix} \quad (1)$$

The elastic constants involved in the calculations were E_1 and E_2 , the Young's moduli in the longitudinal and transverse directions of a unidirectional composite, G_{12} , the longitudinal-transverse shear modulus, and ν_{12} the effective Poisson's ratio.

The strength and stiffness of a composite build-up depends on the orientation sequence of the plies. For an orthotropic thin plate under plan stress, the bending moments are expressed in terms of transversal displacement as:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = - \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 \omega}{\partial x^2} \\ \frac{\partial^2 \omega}{\partial y^2} \\ \frac{\partial^2 \omega}{\partial x \partial y} \end{Bmatrix} \quad (2)$$

Where ω is the local deflection and D_{ij} are the bending rigidities.

$$D_{11} = \frac{E_1 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{22} = \frac{E_2 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad (3)$$

$$D_{12} = \frac{\nu_{12} E_2 h^3}{12(1 - \nu_{12} \nu_{21})} = \frac{\nu_{21} E_1 h^3}{12(1 - \nu_{12} \nu_{21})} \quad \text{and} \quad D_{66} = \frac{G_{12} h^3}{12}$$

The problem requires the solution of the homogeneous differential equation of bending given by:

$$D_{11} \frac{\partial^4 \omega}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 \omega}{\partial y^4} = p(x, y) \quad (4)$$

Where $p(x, y)$ is the transversal loading.

3 Numerical implementation

In the following section, we describe an efficient approach for the analysis of skew orthotropic plates. Obviously, it turns out that an adequate meshing is essential for such problems. Figure 1 shows a triangular mesh that is aligned with the skewed geometry of the plate. As it is frequently done, an appropriate coordinate system adapted to the constraints of the problem is chosen. Therefore, triangular coordinate system (x_1, x_2, x_3) is used instead of ordinary cartesian rectangular coordinates as shown in figure 1.

Fig. 1. Triangular meshing for skew plate

Hence, Laplace's harmonic operator is expressed in triangular co-ordinates by the following equation:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = (1 + \cot \alpha \cot \beta) \frac{\partial^2}{\partial x_1^2} - \frac{\cos \beta}{\sin \alpha \sin(\beta - \alpha)} \frac{\partial^2}{\partial x_2^2} + \frac{\cos \alpha}{\sin \beta \sin(\beta - \alpha)} \frac{\partial^2}{\partial x_3^2} \quad (5)$$

Using central finite difference for partial derivatives, particular finite difference schemes are obtained for different operators in figure 2 and 3.

Fig. 2. Finite difference schemes for second order operator differential equations

Fig. 3. Finite difference schemes for fourth order operator differential equations

Where coefficients C_1 , C_2 and C_3 are expressed by:

Table 1. Expressions of C_1 , C_2 and C_3 coefficients

	$\frac{\partial^2}{\partial x^2}, \frac{\partial^4}{\partial x^4}$	$\frac{\partial^2}{\partial y^2}, \frac{\partial^4}{\partial y^4}$	$\nabla^2 \text{ et } \nabla^4$	$\frac{\partial^2}{\partial x \partial y}, \frac{\partial^4}{\partial x^2 \partial y^2}$
C_1	$\frac{1}{\Delta X_1^2}$	$\frac{\cot \alpha \cot \beta}{\Delta X_1^2}$	$\frac{1 + \cot \alpha \cot \beta}{\Delta X_1^2}$	$\frac{-\sin(\alpha + \beta)}{2\Delta X_1^2 \sin \alpha \sin(\beta)}$
C_2	0	$\frac{-\cos \beta}{\Delta X_2^2 \sin \alpha \sin(\beta - \alpha)}$	$\frac{-\cos \beta}{\Delta X_2^2 \sin \alpha \sin(\beta - \alpha)}$	$\frac{\sin(\beta)}{2\Delta X_2^2 \sin(\alpha) \sin(\beta - \alpha)}$
C_3	0	$\frac{\cos \alpha}{\Delta X_3^2 \sin \beta \sin(\beta - \alpha)}$	$\frac{\cos \alpha}{\Delta X_3^2 \sin \beta \sin(\beta - \alpha)}$	$\frac{-\sin(\alpha)}{2\Delta X_3^2 \sin(\beta) \sin(\beta - \alpha)}$

Table 2. Central deflexions of simply supported isotropic skew plate under uniform loading

α (degree)	Central deflection w_c (cm)								
	Mesh(4x4)		Mesh(6x6)		Mesh(12x12)		Mesh(16x16)		Analytical (Morley)
	Present	FEAP	Present	FEAP	Present	FEAP	Present	FEAP	
90°	0,683	0,691	0,686	0,698	0,688	0,696	0,688	0,694	0,688
85°	0,691	0,681	0,685	0,690	0,682	0,687	0,681	0,686	0,680
80°	0,680	0,653	0,667	0,664	0,658	0,662	0,657	0,661	0,656
60°	0,480	0,403	0,453	0,426	0,438	0,435	0,436	0,436	0,434
50°	0,331	0,252	0,307	0,274	0,295	0,288	0,293	0,290	0,292
40°	0,191	0,129	0,174	0,145	0,165	0,157	0,164	0,160	0,162
30°	0,084	0,050	0,075	0,057	0,070	0,065	0,070	0,067	0,069

4 Numerical examples

4.1 Simply supported isotropic skew plate

As a first example, an isotropic thin skew plate is first investigated since we can compare with existing solutions from literatures. The edges of the plate are all simply supported.

Fig. 4. Geometry of skew plate

The plate has length a in x and y direction and thickness h in z directions. The data assumed for geometric parameters and material properties of the skew plate are: side length $a=254$ cm, thickness $h=2,54$ cm; Young's modulus $E=210$ GPa, Poisson's ratio, $\nu=0,3$. The plate is subjected to uniformly distributed transversal pressure of intensity, $q=1,95$ kN/m².

The maximal deflexion is expected to be at the center of the plate. In table 2, numerical values of central deflexions of investigated skew plates is represented for different meshes and various skew angles (α varying from 30° to 90°). Other numerical analyses are also carried out by using FEAP program version 7.5 [8]. Four node quadrilateral plate elements with tree degrees of freedom on each node are used for calculation. Obtained numerical results are shown in table 2 and compared with an analytical solution given by Morley[6].

A very good agreement is achieved between finite element data and our numerical results. From results given in Table 2, we can observe that the proposed numerical method yields a high level of accuracy using regular meshes

A much more refined meshing of (12x12) and (16x16) grids is required to obtain closer values to Morley's analytical solutions.

4.2 Simply supported orthotropic skew plate

In this section, we are interested to analyse orthotropic skew plate for two different materials E-Glass/Epoxy and Graphite/Epoxy with simply supported boundary conditions.

The table 3 and 4 below shows the data for geometric parameters and the specifications of mechanical properties of E-Glass/Epoxy and Graphite/ Epoxy.

Table 3. Geometric parameters of the plate

Parameter	Value
Side length a	254cm
Thickness h	2,54 cm
Uniform load q	6,89 kN/m ²

Table 4. Mechanical properties of E-Glass/Epoxy WR/5920 and Graphite/Epoxy T300/5208

	E-Glass/Epoxy WR/5920	Graphite/Epoxy T300/5208
E_1 (GPa)	23,72	128
E_2 (GPa)	21,92	13
G_{12} (GPa)	16,54	6,4
ν	0,13	0,3

Using the numerical model proposed in this paper, a computer program package capable of simulating the mechanical behaviour of orthotropic skew plates is developed using Compaq / Digital Visual Fortran. Finite-difference schemes on grids consisting of regular triangles are used for modelling skew plate as described in the previous section.

Results on tables 6 and 7 show numerical solutions obtained respectively for E-Glass/Epoxy WR/5920 and Graphite/Epoxy T300/5208 composite plates. The central deflection has been reported for simply supported rhombic plates for different skew angles (α angle) and different grids.

Numerical results given by our model are checked against the only existing analytical solutions of uniformly loaded thin square plate. Under the same conditions of the problem, the analytical solutions are shown in table 5.

Table 5. Analytical solution for central deflexion of thin square plate

Material type	Central deflection w_c (cm)
E-Glass/Epoxy	2,881
Graphite/Epoxy	1,914

Table 6. Central deflexions of simply supported E-Glass/Epoxy composite skew plate

E-Glass/ Epoxy	Central deflection w_c (cm)				
α (degree)	Mesh (4x4)	Mesh (6x6)	Mesh (8x8)	Mesh (12x12)	Mesh (16x16)
90°	2,858	2,871	2,875	2,878	2,879
80°	2,955	2,876	2,838	2,794	2,762
70°	2,874	2,702	2,615	2,505	2,424
60°	2,593	2,351	2,226	2,067	1,955
50°	2,086	1,824	1,688	1,519	1,404
40°	1,364	1,158	1,050	0,919	0,834
30°	0,631	0,530	0,478	0,416	0,376

Table 7. Central deflexions of simply supported Graphite/Epoxy composite skew plate

Graphite/ Epoxy	Central deflection w_c (cm)				
α (degree)	Mesh (4x4)	Mesh (6x6)	Mesh (8x8)	Mesh (12x12)	Mesh (16x16)
90°	0,747	0,751	0,752	0,753	0,753
80°	0,776	0,751	0,742	0,735	0,731
70°	0,732	0,687	0,670	0,652	0,640
60°	0,606	0,544	0,519	0,492	0,475
50°	0,400	0,336	0,309	0,282	0,267
40°	0,189	0,142	0,123	0,105	0,097
30°	0,053	0,036	0,029	0,024	0,023

From these tables, it is clear that the central displacement of square plate ($\alpha=90$) compare well with analytical solutions given in table 5. Furthermore the numerical values of central deflexions are much closer to the analytical solutions with refined meshes of (12x12) and (16x16).

5 Conclusions

In this brief note we have presented a numerical study on the bending problems of skew composite plates. Typical cases of skew plates are analysed with various skew angles. The validity of the proposed procedure is confirmed by numerical examples. Two quite different examples are selected for the investigation. Hence, central deflexions obtained for the two examples are compared to available analytical solutions and other finite element solutions. A very close agreement is observed in all cases.

References

1. J. N. Reddy, *A review of refined theories of laminated composite plates*, Shock Vib. Dig. **22**, 3-7 (1990)
2. J. N. Reddy, *A simple higher-order theory for laminated composite plates*, J. Appl. Mech., ASME **51**, 745-752 (1984)
3. N. J. Pagano, S. J. Hatfield, *Elastic behavior of multi-layered bi-directional composites*, AIAA J. **10** (7) 931-933 (1972)
4. S. Timoshenko, S. Woinowsky-Krieger, *Plates and shells*. McGraw-Hill, New York, (1959)
5. O. C. Zienkiewicz, Y. K. Ceung, *The finite element method for analysis of elastic isotropic and orthotropic slabs*. Proc. Inst. Civ. Eng. **28**, 471-488, (1964).
6. C. T. Morley, *Skew plates and structures*, MacMillan Company, New York (1963).
7. S. G. Lekhnitskii, *Anisotropic plates*, Gordon and Breach, Science Publishers (1968)
8. R. L. Taylor, FEAP - A Finite Element Analysis Program. Version 7.5 Dept. of Civil and Environmental Engineering, User Manual, Berkeley, California 94720-1710, January (2003).