Multi-cracks identification based on the nonlinear vibration response of beams subjected to moving harmonic load

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Abstract. The aim of this work is to investigate the nonlinear forced vibration of beams containing an arbitrary number of cracks and to perform a multi-crack identification procedure based on the obtained signals. Cracks are assumed to be open and modelled through rotational springs linking two adjacent sub-beams. Forced vibration analysis is performed by a developed time differential quadrature method. The obtained nonlinear vibration responses are analyzed by Huang Hilbert Transform. The instantaneous frequency is used as damage index tool for cracks detection.

1 Introduction

Vibration based techniques of damage identification aim to combine mathematical models with signal processing techniques. Relevant work has been published in this regard [1], but they almost assume that structures and damages behave linearly while in reality signals are nonlinear. Thus, the structural health monitoring researchers appeal to mathematical models of nonlinear dynamics [2, 3]. The problem of vibration of multi-cracked beams subjected to moving loads has also attracted many researchers [4] where free and forced vibrations of the beam are investigated. However, the linear case is usually considered and the forced response is computed following classical schemes of integration mainly the Runge-Kutta method [5, 6]. On the other hand, cracks identification is concerned by analysing the vibration signals using adapted techniques. The focus will be here on time frequency methods known for their local properties in both time and frequency domains [7].

In parallel with our previous work [8, 9], this paper focuses on the nonlinear behaviour of multi-cracked beams subjected to moving harmonic load. For the free and forced responses, a numerical method based on the differential quadrature method has been developed. Crack identification procedures are elaborated based on the numerically computed nonlinear responses.

2 Mathematical formulation

Consider a multi-cracked Euler-Bernoulli beam with length L , cross-section A , mass density ρ , moment of inertia I , and modulus of elasticity E that is subjected to a

moving harmonic load of magnitude F_0 , speed v and excitation frequency Ω , and R is the number of existing cracks as shown in figure 1. It is assumed that the cross-sectional area of the beam is rectangular and its material is homogenous. The ' R ' cracks are assumed to be open and modelled through rotational springs which flexibilities are given by fracture mechanics [8]. The whole beam is sub-divided into $(R+1)$ sub-beams.

Fig. 1. Multi-cracked beam under a moving harmonic load

The equation of motion for the i^{th} mode of vibration is given by:

$$m_{ii}\ddot{q}_i^{(r)} + c_{ii}\dot{q}_i^{(r)} + k_{ii}q_i^{(r)} + \beta_{ii}q_i^{3(r)} = F_i^{(r)}(t) \quad \frac{x_{r-1}}{v} \leq t \leq \frac{x_r}{v} \quad (1)$$

where: m_{ii} , c_{ii} , k_{ii} , β_{ii} are modal parameters defined as follows:

$$m_{ii} = \rho A \sum_{r=1}^{R+1} \int_{x_{r-1}}^{x_r} w_i(x) w_i(x) dx \quad (2)$$

$$c_{ii} = \eta \sum_{r=1}^{R+1} \int_{x_{r-1}}^{x_r} w_i(x) w_i(x) dx \quad (3)$$

$$k_{ii} = EI \sum_{r=1}^{R+1} \int_{x_{r-1}}^{x_r} w_i''''(x) w_i(x) dx \quad (4)$$

$$\beta_{ii} = -\frac{EA}{2L} \sum_{r=1}^{R+1} \int_{x_{r-1}}^{x_r} w_i''(x) \left(\int_{x_{r-1}}^{x_r} (w_i'(x))^2 dx \right) w_i(x) dx \quad (5)$$

$$F_i^{(r)}(t) = \rho A \sum_{r=1}^{R+1} \int_{x_{r-1}}^{x_r} F_0 \sin(\Omega t) \delta(x - vt) w_i(x) dx \quad (6)$$

x_0 and x_{R+1} correspond respectively to the left and right boundaries of the beam ($x=0$ and $x=L$).

Note that the eignemode of a multi-cracked beam is written as [8]:

$$w_i(x) = w_i(0) + \sum_{r=1}^n w_{ri}(x)(H(x-x_{r-1}) - H(x-x_r)) + w_{R+1}(L) \quad (7)$$

The present problem is a set of coupled differential equations which has been solved for the linear case ($\beta=0$) method [8].

Due to the fact that the excitation term depends on piecewise mode, the classical numerical methods cannot be used. In this work, a new numerical approach based differential quadrature method (DQM) has been developed in time domain for nonlinear analyses.

2.1 Differential quadrature method

The Differential quadrature method (DQM) was first introduced by Bellman and developed by many researchers [10]. The DQM requires the discretization of the problem into N points. The derivatives at any point are approximated by a weighted linear summation of all the functional values along the discretized domain, as follows [10]:

$$\begin{cases} \dot{q}(t_i) = \sum_{j=1}^N b_{ij}^{(2)} q(t_j) \\ \dot{q}(t_i) = \sum_{j=1}^N b_{ij}^{(1)} q(t_j) \end{cases} \quad (7)$$

N is the number of distretizing points, $b_{ij}^{(1)}$ and $b_{ij}^{(2)}$ are the first and second order weighting coefficients respectively. The weighting coefficients for the first order derivative to the functional values can be obtained as:

$$b_{kj}^{(1)} = \begin{cases} \frac{L(t)}{(t_k - t_j)L_1(t_j)} & k \neq j \\ - \sum_{j=1, j \neq k}^N b_{kj}^{(1)} & k = j \end{cases} \quad (8)$$

$$L(t_k) = \prod_{i=1}^N (t_i - t_k) \quad (9)$$

The second, third and higher derivatives can be calculated as:

$$b_{kj}^{(m)} = \sum_{j=1, j \neq k}^N b_{kl}^{(1)} b_{lj}^{(m-1)} \quad (10)$$

The N discretizing points are calculated through:

$$t_j = \frac{1}{2} [1 - \cos(\frac{j-1}{N-1} \pi)] \quad j = 1, 2, \dots, N \quad (11)$$

2.2 Nonlinear forced response using the time differential quadrature method

In this work, a new approach for solving nonlinear differential equations by introducing a correction loop in for calculating nonlinear response is presented. Applying the DQ method in each sub-beam Eq.(1) can be discretized as:

$$([M] + [C] + [K] + [B]) \{q\}^r = \{F\}^r \quad (12)$$

where: $[M] = m_{ii} \quad b_{kj}^{(2)}$; $[C] = c_{ii} \quad b_{kj}^{(1)}$; $[K] = k_{ii} \quad \text{Id}(N, N)$;

and $[B] = \beta_{ii} \quad \text{Id}(N, N)$.

We first set $[B] = 0$. The linear solution for the r^{th} time interval is obtained by solving the algebraic problem.

$$\{q\}^r = ([M] + [C] + [K])^{-1} \{F\}^r \quad (13)$$

In order to calculate non linear response, for the sub-beam ' r ', we propose an iterative process written as:

$$\{q\}_{i+1}^r = ([M] + [C] + [K])^{-1} \left(\{F\}^r - [B] \{q\}_{i+1}^r \right) \quad (14)$$

The residue R_{i+1} is calculated by:

$$R_{i+1}^r = [M] \{q\}_{i+1}^r + [C] \{q\}_{i+1}^r + [K] \{q\}_{i+1}^r + [B] \{q\}_{i+1}^r - \{F\}^r \quad (15)$$

The stopping criterion is taken as

$$\frac{R_{i+1}^r}{R_1^r} \leq \varepsilon \quad (16)$$

3 Huang Hilbert transform: an overview

As the identification process, used in this paper, will be based on the Huang Hilbert transform, an overview on the empirical mode decomposition and Hilbert transform is given

3.1. Empirical mode decomposition

The Empirical mode decomposition (EMD) is a technique representing non linear and non-stationary signals as sum of simpler components called Intrinsic Mode Functions (IMFs). An IMF should satisfy the following conditions:

- An IMF may only have one zero between successive extrema
- An IMF must have zero local mean

The decomposition is performed through a repeated sifting procedure. At the end, the time signal $x(t)$ can be expressed in terms of n number of IMFs:

$$s(t) = \sum_{i=1}^n \text{IMF}_i(t) + \text{residue} \quad (17)$$

The Hilbert transform is then applied to each of those components, in order to get instantaneous amplitude and frequency plots.

3.1. Hilbert transform

The Hilbert Transform (HT) of a signal $s(t)$, is an integral transformation, from time domain to time domain, defined by [7] :

$$H(s(t)) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{s(\tau)}{t-\tau} d\tau \quad (18)$$

The HT is the convolution of $s(t)$ with $1/t$ and hence emphasizes the local properties of $s(t)$. The real signal $s(t)$ and its HT $h(t)$, form an analytical complex signal $\tilde{S}(t)$ of the form :

$$\tilde{S}(t) = s(t) + i h(t) = A(t)e^{i\theta(t)} \quad (19)$$

The instantaneous $A(t)$ and phase $\theta(t)$ change with time. The instantaneous amplitude $A(t)$ or envelope, is given by:

$$A(t) = \pm \sqrt{(s(t))^2 + (h(t))^2} \quad (20)$$

where the “ $\pm$ ” signs correspond to the upper positive and the lower negative envelopes

$$\theta(t) = \text{Arc tan}\left(\frac{h(t)}{s(t)}\right) \quad (21)$$

The instantaneous frequency (IF) is defined as the derivative of the phase:

$$\omega(t) = \frac{d\theta(t)}{dt} \quad (22)$$

It measures the rate and direction of a phase in the complex plane. It can be estimated by different algorithms [7].

4 Numerical results and discussion

3.1. Nonlinear forced response

In order to validate the previous developments, an Euler-Bernoulli beam with the following material properties is considered: Young's modulus $E=210$ GPa, material mass density $\rho=7860$ kg/m³ and Poisson ratio $\nu=0.3$. The geometrical parameters of the beam are selected as: depth $h=0.01$ mm, thickness $b=0.01$ mm. First, comparison is made with the reference [11] for the non cracked linear case. The obtained results are plotted in figure 2 and are the same as those presented in [11]

Fig. 2. Dynamic response of a pinned-pinned beam, varying the speed for $\Omega=\omega_{\text{resonance}}$

For the nonlinear case, the numerically obtained forced responses of a multi-cracked beam are depicted for various speeds and different frequencies in figure 3. It is

assumed that the beam contains four equally spaced cracks of equal depth ($a/h=0.1$) located at $x/L=0.1; 0.3; 0.5$ and 0.7 .

Fig. 3. Dynamic response of a pinned-pinned beam, varying the speed for different speeds ($\Omega=\omega_{\text{resonance}}$) and different load frequencies ($v=11.7$ m/s)

3.2. Multi-cracks identification

As the main aim of this work is to investigate crack detection from nonlinear signals, we propose to combine Huang Hilbert transform to moving load properties for a better cracks identification. For that, the nonlinear signal, depicted in figure 4, is decomposed into simpler components (IMFs) using the EMD, then Hilbert spectral analysis is applied to each of those IMFs.

Fig. 4. Analyzed nonlinear signal

We notice that instantaneous frequency of the first IMF identifies positions of all cracks that are localized by sharp transitions in the curve. This is due to the presence of high frequency components in the signal at these locations as shown in figure 5. It should be noted that large peaks are obtained leading to clear crack position detection.

Fig. 5. Instantaneous frequency of the first IMF for $v=11.7$ and $\Omega=\omega_{\text{resonance}}$

Moreover, a better detection is obtained for selected values of the speed v and excitation frequency Ω . In this work we obtain them by trial and found that the best detection is obtained for $\Omega=\omega_{\text{resonance}}$ and $v=11.7$ m/s as shown in figure 5. In spite of that, there are other peaks in the curve which means that there are other higher frequency components in the analyzed signals. This is due to the fact that the nonlinear signal contains various other harmonics and these harmonics may contain other information about cracks positions. For efficient multi-cracks identification based on the nonlinear responses some filters and a deep analysis is required.

5 Conclusion

We developed a numerical algorithm based on the time differential quadrature method in order to get the forced responses of multi-cracked beams under moving harmonic load.

A large number of cracks can be easily considered for the direct problem and identified for the inverse problem. We used for cracks identification Huang Hilbert transform. Higher frequency components are first detected and for the nonlinear case, not only cracks produce sharp transitions in the curve of instantaneous frequency but also some nonlinear signal components. We cannot accurately define those components since we lack of explicit analytic solutions for the multi-cracked beam vibration problems.

The identification is performed for selected values of the speed and excitation frequency. Adjusted values lead to better multi-cracks detection. An optimization procedure can be elaborated to predict the best v and Ω parameters.

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