

Vibration Based Methods For Damage Detection In Structures

E. Manoach¹, J. Warminski², L. Kloda², A. Teter²

¹Institute of Mechanics, Bulgarian Academy of Sciences, Bulgaria

²Department of Applied Mechanics, Lublin University of Technology, Lublin, Poland

Abstract. Vibration based damage detection methods are among the most popular and promising approaches for health monitoring of structures. In this work a critical review of different methods for damage detection methods of structures is presented. The theoretical bases of the most popular methods based on the changes in the modal properties of the structures are deduced. The review includes the modal displacements, the mode shape slopes, the modal curvatures and the strain energy methods. The efficiency of all these methods is compared by using a finite element analysis of intact and damaged beams. The methods are tested experimentally by using a scanning laser vibrometer to measure the modal properties of specially prepared composite beams with defects. All this methods are compared with the damage detection method based on the analysis of the Poincaré maps of the motion of the structures). Conclusions concerning the advantages and the applicability of the considered methods are deduced.

1 Introduction

Vibration based methods are among the popular and very important methods for structural health monitoring (SHM) and damage detection (DD). In this work only a small part of the vibrational based damage detection methods are considered. The easiest and the most popular from the vibration based methods are the methods based on the modal analysis of the structure. The earliest technique for DD was based on analyzing the eigen-frequencies of the system. The natural frequencies are easily measurable with high accuracy and are usually less contaminated by experimental noise. It is well known, however, that the natural frequencies are not very sensitive to damages. It was shown by many authors that significant damage may cause very small changes in natural frequencies, particularly for larger structures, and these changes may go undetected due to measurement or processing errors. [1-3]. The natural frequencies are global parameters of the systems and often they are weakly affected by local faults

Developing of the experimental techniques – contactless method with sensors or specially Scanning Laser Doppler Vibrometers (SLDV) makes possible to measure the modal displacements in large numbers of nodes. This fact extended a lot the possibility to use extensively the modal based damage detection methods.

In this work some basic modal based methods for damage detection and localization will be presented briefly. Time domain method based on modal

measurements will be also deduced. Then, all listed methods will be tested numerically and experimentally and conclusions about their applicability will be invoked.

2 Modal based methods

In this work are reviewed some well know vibration based methods. The detailed analysis of the most of them can be found in the excellent review articles [4-7]

2.1 Modal displacements

Mode shapes are found to be quite sensitive to damage and are able to directly provide damage location information [8]. Moreover, the mode shapes are less sensitive to environmental effects, such as temperature, than natural frequencies [9, 10].

The simplest method based on the mode shapes is the so-called modal displacements method (MD) [11].

It seeks for a maximal difference in the measured displacements of modes of damaged and healthy structures:

$$\Delta W_i = \sum_{j=1}^N |w_{ij}^* - w_{ij}| \quad (1)$$

In Eqn (1) w_{ij} is the displacements of i -th mode at j -th node of the intact beam and w_{ij}^* is the same for damaged beam. Often, it is accepted $N=1$, i.e. only first mode is used for DD purpose.

2.2. Modal slopes square

Another simple method is the based on the modal slopes. It estimates the absolute differences between the squared modal slopes (the first derivative of modal shapes) of the damaged and healthy structures

$$\Delta W_i'^2 = \sum_{j=1}^N |w_{ij}'^{*2} - w_{ij}'^2| \quad (2)$$

This method is also more often used considering only the first mode , i.e. $N=1$.

2.3. Modal curvatures and modal curvatures square

The modal curvature method (MC) is widely used because it shows a good sensitivity to damage [12]. The methods have a few drawbacks such as the high noise sensitivity due to curvature calculations and no explicit relationship between damage and damage features [13]. By using SLDV, however, it is expected that the efficiency of the method to be very good.

The method can be expressed as:

$$\Delta W_i'' = \sum_{j=1}^N |w_{ij}''^{*} - w_{ij}''| \quad (3)$$

In [10] it is shown that the MC based methods are much more promising than the MD based methods, especially when curvatures are directly measured from the strain mode shape.

Some authors [13] mention that together with the peak of MC at the damage location, some smaller peaks at different undamaged locations can appear, especially for the higher modes. This can cause confusion in a practical application in which one does not known in advance the location of faults. A variation of modal curvature method is the modal curvatures square method which some authors [11] have used in order to enforce the differences in the curvatures of the intact and damaged structure.

2.4. Modal strain energy methods

The modal strain energy method (**MSEM**) was developed by Stubbs and Kim [14] and [15] to locate damage in structures. It was developed for Bernoulli -Euler beams. Later it has been extended by Cornwell et. al. in [16] for plate-lake structures. The method will be presented briefly here for the case of Timoshenko beam where the strain energy consists of two parts – due to bending and to shear. For location j on the beam this change in the i^{th} mode strain energy healthy and damaged beams can be presented as:

$$U_{ij} = \frac{1}{2} \int_{a_j}^{a_{j+1}} \left\{ (EI)_j \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 + k \frac{(Ebh)_j}{2(1+\nu)} \left(\frac{\partial w_i}{\partial x} - \psi_i \right)^2 \right\} dx; \quad (4a,b)$$

$$U_{ij}^* = \frac{1}{2} \int_{a_j}^{a_{j+1}} \left\{ (EI)_j^* \left(\frac{\partial^2 w_i^*}{\partial x^2} \right)^2 + k \frac{(Ebh)_j^*}{2(1+\nu)} \left(\frac{\partial w_i^*}{\partial x} - \psi_i^* \right)^2 \right\} dx,$$

where i is the mode number, j is the element number U_{ij}^* is the strain energy of i^{th} mode for the j^{th} element . With w_i and ψ_i are denoted the variables for the damaged beam and ψ_i

and ψ_i^* are the rotational angles of the cross-section of i^{th} elements for healthy and damaged beams. It can be shown that the following relation can be obtained:

$$\frac{(EI)_j^*}{(EI)_j} = \frac{J_{ij}^*/U^*}{J_{ij}^*/U} = \frac{\gamma_j^*}{\gamma_j} = f_j; \gamma_j^* = J_{ij}^* U^*, \gamma_j = J_{ij}^* U \quad (5)$$

where

$$J_{ij}^* = \int_{a_j}^{a_{j+1}} \left\{ \left(\frac{\partial^2 w_i^*}{\partial x^2} \right)^2 + k \frac{6}{h^2(1+\nu)} \left(\frac{\partial w_i^*}{\partial x} - \psi_i^* \right)^2 \right\} dx$$

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The strain energy ratio f_j potentially provides indications for damage location as well as its extent. It is expected to be equal to one over the undamaged regions and different than one over a damaged region. It is well known that damage affects the strain energy when it is located close to the regions of maximum strain energy. Therefore, the damage index f_j is affected by the frequency of excitation. For this reason it is reasonable to use the cumulative strain energy ratio:

$$f_m = \frac{1}{N} \sum_{j=1}^N f_{j_m} \quad (6)$$

This approach including strain energy due to bending and shear could be considered as a small extension of the method developed in [14], if the angular rotation of ψ_i could be measured experimentally. This, however, is not an easy task. Therefore, in this work we have checked the influence of the second term in the strain energy (due to shear strains) only numerically.

3. Forced response methods

A big group of SHM methods are based on the analysis of the measured forced response of the structure. This is actually the most popular alternative for in-operation machinery monitoring. Most of these methods are trying to extract some features from the nonlinear forced response of the structures which are sensitive to the damage.

3.1. Poincare map based method

In [17] and [18] a method for damage detection and location on the base of the analysis of the Poincaré map of the forced response of the structure was developed. It was tested numerically and experimentally for plates and beams. The intention of the authors of this study is to check the method effectiveness and to extent it if only the modal properties of the damaged and healthy structures are known.

According to this method a damage index can be constructed in the following way:

$$I_i^d = C \left(S_i^u - S_i^d \right), \quad (7)$$

where

$$S_i^u = \sum_{j=1}^{N_p-1} \sqrt{(w_{i,j+1}^u - w_{i,j}^u)^2 + (\dot{w}_{i,j+1}^u - \dot{w}_{i,j}^u)^2} \quad (8 \text{ a-b})$$

$$S_i^d = \sum_{j=1}^{N_p-1} \sqrt{(w_{i,j+1}^d - w_{i,j}^d)^2 + (\dot{w}_{i,j+1}^d - \dot{w}_{i,j}^d)^2}$$

and C is a constant. This constant could be connected with the number of the nodes in the Poincaré map, for example $C=1/N_p$ where, $i=1,2\dots N_{nodes}$, N_{node} is the number of nodes, N_p is the number of points on the Poincaré map and $(w_{ij}^u, \dot{w}_{ij}^u)$ and $(w_{ij}^d, \dot{w}_{ij}^d)$ denotes the j^{th} point on the Poincaré maps of the undamaged and the damaged states of i^{th} node, respectively.

This damage index represents the relative difference between the lengths of two curves formed by connecting the dots on the Poincaré maps in i^{th} node for the non-damaged and the damaged structure. This difference is accepted as a measure for the global change (during of the total period of vibration) in the dynamic behaviour of the damaged structure in comparison with the undamaged one. The assumption is that if the function $I^D(x)$ has maximum and it is strongly concave in the vicinity of the maximum, then the structure has damage and the nodes close to the maximal value of the function will represent the damaged area.

3.2. Obtaining the time series from the modal data

In this way, if we know from the experimental measurements the natural frequencies and modes of vibrations ω_n and w_n , following a simple numerical procedure we obtain time series describing the forced response of the beam subjected to artificially applied harmonic loading $p(x,t)=p_0 \sin(\omega_e t)$, where p_0 is the amplitude of the artificial excitation and ω_e is the excitation frequency (see, for example [3]). In order to check these formulas for DD and to check the applicability of a few of the popular and easy modal methods for damage detections, numerical and experimental testing were performed.

4. Numerical results

A beam with length $l=0.44$ m, thickness $h=0.0033$ m and width $b=0.0181$ m was considered.

The material properties of the beam are $E=4.6^{10}$ Pa, $\rho=2032\text{kg/m}^3$ and $\nu=0.4$. They correspond of the effective properties of composite cross ply laminated beam made of 10 layers glass-epoxy composite. The beam was modeled as a Timoshenko beam by a finite element program. The length of the beam was divided to 80 linear beam elements (81 nodes) with equal length.

Three cases of reductions of beam thickness (cuts) were considered:

- a) Reduction with 0.125 mm (Cut1)
- b) Reduction with 0.25 mm (Cut 2)
- c) Reduction with 0.33 mm (Cut 3)

All reductions were made in elements 49 and 50 (between nodes 48-50).

For the intact beam and the beams with 3 different depth of the cuts were calculated the first seven eigen frequencies and normal modes. The introduced minimal damages lead to very small changes in the natural frequencies of the beam. The decrease of the natural frequencies in the damaged beams does not exceeds 1 % (not shown here).

The computed damage indexes (DI) based on modal displacements (Eq. (1)) for the case a) – cut with depth 0.125 mm are shown in figure 1. Calculations were performed for $N=1$. Despite of the fact that the thickness in damaged elements is reduced by only 3.78 %, the damage and its location is very well predicted in all cases ($N=1,2,3$). The inclusion of more modes in the computations enlarge the value of the DI (figure 2). The next criterion, mode shape slope squared, is demonstrated in figure 3. For this criterion the results from the computations of beam with Cut 2 are shown. As can be seen the location of the fault is well predicted, but there are two peaks in the damage location. These two peaks appeared in several cases which have been tested. This fact can provoke confusion in the potential experiment. The inclusion of more modes in the criteria leads to increasing the value of DI but the figures remain the same.

In the next figure (figure 4) the computed modal curvatures for each node of the beam are shown. It is clearly seen that the fault is predicted very well. The increased number of modes in the formula (3) leads to increasing the value of the peak of the curve. Modal curvatures square method gives even more sharp demonstration of the damage location (not shown here).

The application of strain energy method for damage detection is shown in figure 5. The FE program allows obtaining of modes of vibrations w_n and angular rotations ψ_n which allows checking the method's performance for the cases of the total energy and for the case of the bending energy only. Three deductions could be made from this figure: 1)The damage is extremely well predicted for both cases; 2) The influence of the shear deformation is obviously very strong for this kind of damage and therefore the consideration of the total energy gives much better results for damage identification. It must be noted that for different damages (delamination) which we have tested, the inclusion of the term due to shear stresses in strain energy is not essential. This calculations are not shown here.; 3) The method could be used to estimate the severity of the damage because the undamaged case corresponds to value $DI_{\text{energy}} \approx 1$.

Then a forced response of the beam was simulated. The beams were subjected to harmonic force with amplitude $P = 20$ Pa and excitation frequency $\omega_e = 523.5$ rad/s. The forced responses of the intact and damaged beams were computed, Poincaré maps were constructed and damage index (Eq. (8)) was computed. The results for the beams with three different damages (Cut1, Cut2 and Cut3) are plotted in figure 6. In all three cases the damage and its location are predicted very well, in spite of the fact that the picks are not very sharp. These low grade slopes of the Poincaré map DI are typical for this method and do not decrease its ability to predict damage.

5. Experimental setup

The natural frequencies and mode shapes of free vibrations were obtained by using scanned laser vibrometer Polytec PSV-500 and mini SmartShaker K2007E01. The beam was manually discretized by 41 grid points. The structure of fiber glass generates internal scattering which leads to high levels of noise. That is why the reflective labels were stuck on each of measured points. The first and last points have been set at the clamped ends. 41 markers were located along the beam length at equal distance from each other. (Fig. 7a).

The specimens are composite beams prepared from 10-layer glass-epoxy composite. The measurements were made for the intact beam. After that measurement of the intact beam 3 different cuts were made consecutively. The cut was made on the entire width of the beam (Fig. 8).

After the introduction of each cut modes shape and frequencies were measured by SLDV. This technique allow us to avoid the influence of non-perfect boundary conditions.

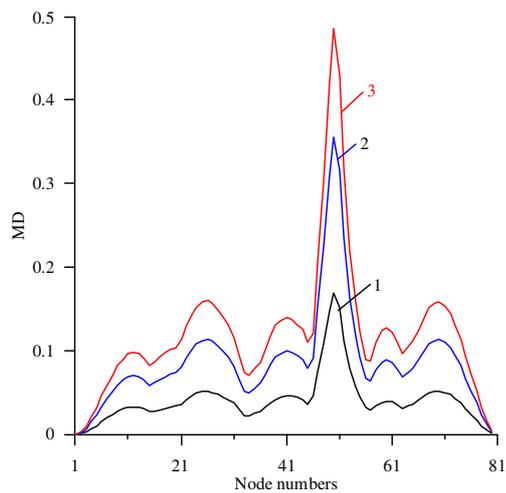


Fig. 1. Modal displacements based damage indexes for three different depth of the cuts. 1 – Cut1; 2 - Cut2; 3 – Cut3

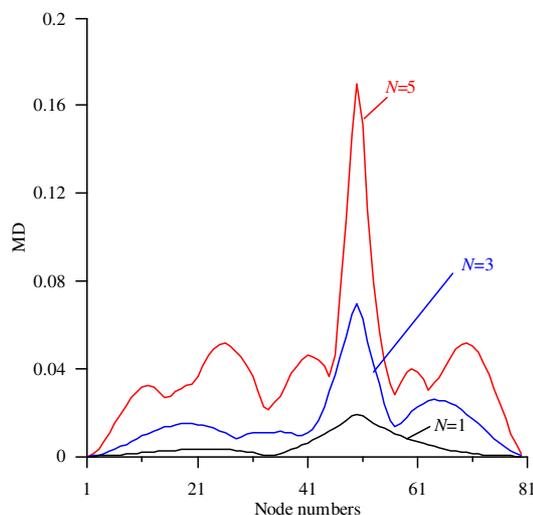


Fig. 2. Influence of the modes numbers taken to compute MD DI.

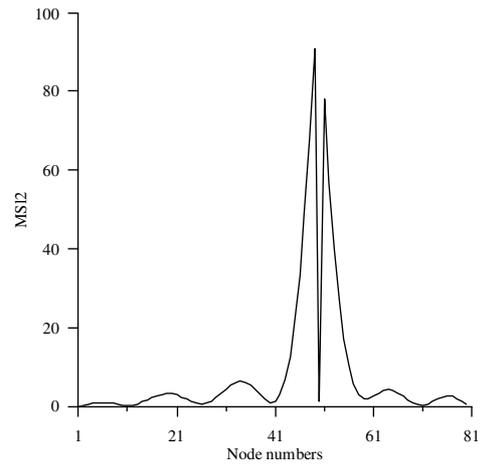


Fig.3 Mode shape slope criterion for the beam with CUT2. $N=1$.

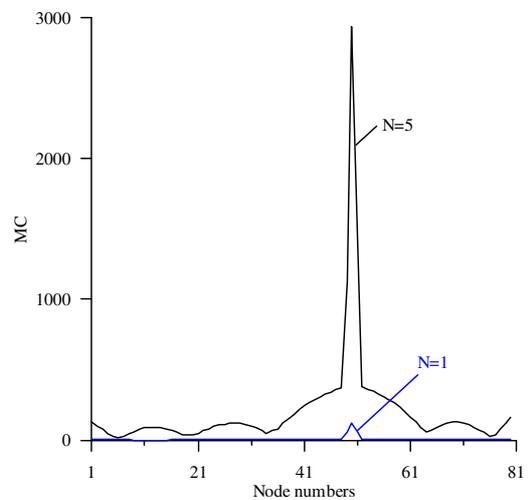


Fig. 4. Modal curvature criterion.

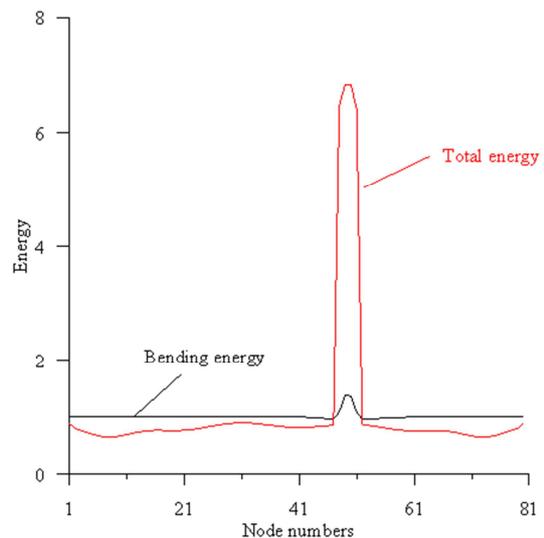


Fig.5. Modal strain energy criterion.

6. Experimental results

First step in the experimental study was to compare the natural frequency measured experimentally and numerically. The difference between the first 7 frequencies did not exceeded 5 %. This fact confirmed

that the experimental measurements are performed correctly.

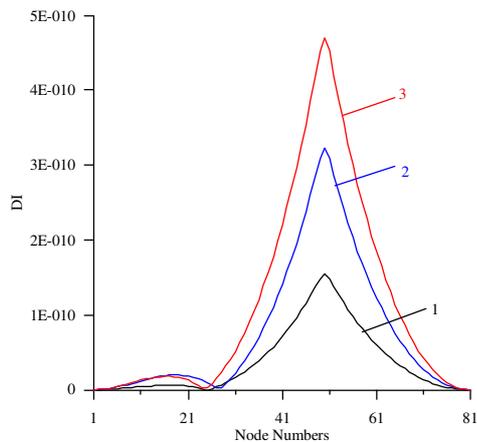


Fig. 6. Poincaré map based DI.

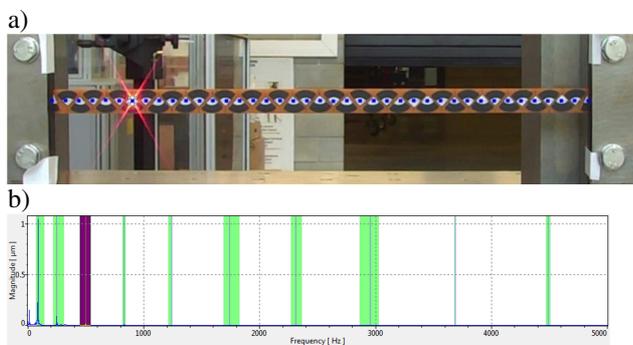


Fig. 7 Clamped-clamped beam with markers -(a) and amplitude-frequency response of the healthy beam- (b)



Fig. 8 Beam samples with a cut.

The DD methods listed above were checked on the base of the experimentally obtained data. The results obtained by the modal slope method did not predict the damage and its location and that is why they are not shown here. The modal displacement method was checked for $N=1,3,5$ and the obtained results were similar. The slope of the curve of MD index is not sharp but the method indicates exactly the damage location (figure 9). The similar are the results obtained by the Poincaré map based method (see figure 11). The beam was artificially considered to be subjected to harmonic loading with $p= 10^3$ MPa at three nodes at the beam

center with frequency equal to the first natural frequency ($\omega_c=540$ rad/s). The modal curvature method gives also good results, but smaller peaks appear along the beam length which could make a confusion in the case of multiple delaminations. The best results are obtained by the strain energy method (the bending energy is considered only) – figure 11. For the case of $N=1$, however this method give wrong results, i.e. the method requires more modes to be included in the analysis.

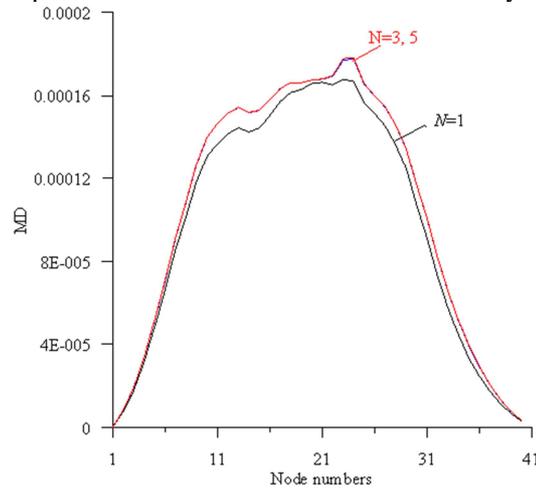


Fig.9. Modal displacements difference for beam with cut2.

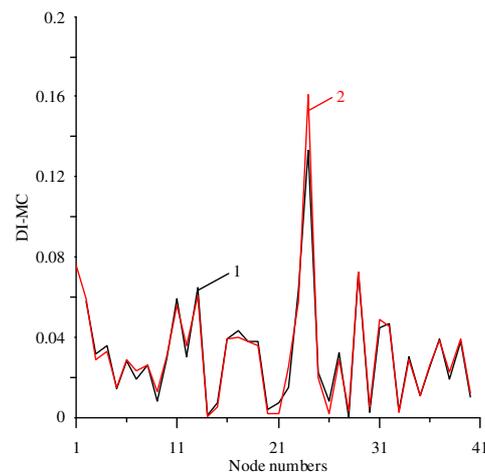


Fig. 10. Modal curvature difference for beam with Cut 2. 1- (black color) – $N=1$; 2 (-red color) $N=7$.

7. Conclusions

The numerical and experimental study shows that the modal analysis obtained by using SLDV can be used effectively for DD. Among the most popular modal based methods the one based on the modal strain energy gives the best results for experimentally obtained data (real data with noise). It requires, however, more modes to be considered. The modal properties of a structure could be successfully used for Poincaré map based forced response method and therefore for on line SHM. The improved strain energy method which includes the shear deformation could be much more effective when the shear stresses are essential.

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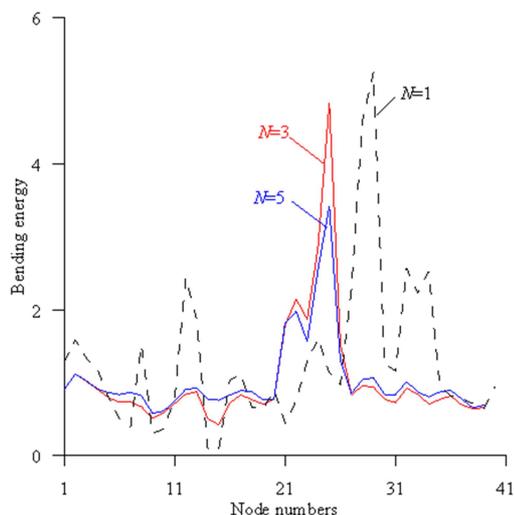


Figure 11 Strain energy based DI for beam with Cut3.

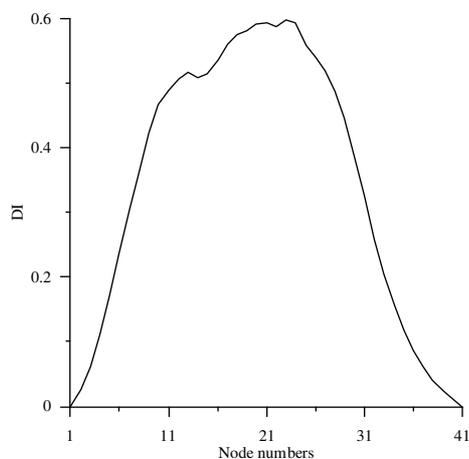


Fig. 12. Poincaré map based index for beam with Cut3.

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