Metamodel for nonlinear dynamic response analysis of damaged laminated composites

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Abstract. Damage affects negatively the safety of the structure and can lead to failure. Thus, it is recommended to use structural health monitoring techniques in order to detect, localize and quantify damage. The main aim of the current work is the development of a numerical metamodel to investigate the dynamic behavior of damaged composite structures. Hence, a metamodelling for damage prediction and dynamic behavior analysis of laminate composite structures is proposed, wherein the stress state in the structure is used as indicative parameters and artificial neural networks as a learning tool.

1 Introduction

Damage in composite structures can be classified in three forms, matrix cracking, fibre fracture and delamination. These forms of damage may not be visible from surface, but they may have a significant effect on the structure stiffness, its strength and its life-time. Hence, the damage assessment is the subject of many studies in the literature, especially with regard to techniques of detection, quantification and localization of the damage. Kachanov [1] proposed a first meso-modelling of damage which is introduced as the degree of reduction in the structure resistance and the loss of rigidity. For laminated composite structures, fibre rupture generally occurs in the final phase of layer rupture whereas matrix cracking is considered as the first damage mechanism and presents a common defect in laminated composites.

Over the last decades, several methods of damage detection in composite materials were developed to analyse the damage mechanism and as a result to ensure the reliability and safety of service structures. Most of these methods are costly and they may need to take the structure out of service. Indeed, in some cases, the identification of damage can be considered as an inverse problem where the damage is determined with respect to its effects and the solution become not obvious to get. Thus, for reasons of simplicity, advanced identification techniques based on artificial intelligence methods such as artificial neural networks are increasingly used due to their excellent pattern recognition capability.

Artificial neural networks (ANNs) are defined as computational models whose design is schematically inspired from basic operating concept of biological neurons [2]. The key strength of ANNs is generalisation. Once the relationship between the input and output variables is established via the training process, the ANNs model can be able to accurately deal with inputs which were not present in the training set. Indeed, they are generally considered as a tool for pattern recognition or classification, modelling and prediction problem. And, they represent a promising metamodelling technique, especially for data sets having non-linear relationships as the case of damage detection and quantification problem. Several researchers used ANNs to detect, localize, and quantify damage in homogeneous and composite structures. Sahin et al. [3] proposed a damage detection algorithm using a combination of changes in eigenfrequencies and curvature mode shapes as inputs to ANNs for damage detection and prediction of its severity. Zhang et al. [4] used numerical data from a finite element model of a delaminated composite beam and eigenfrequencies from modal testing as inputs for ANNs in order to predict the interface, lengthwise location and size of delamination. Xu et al. [5] proposed an adaptive multiplayer perceptron (MLP) for cracks detection in anisotropic laminated plates where the only displacement response on the plate surface is provided to the MLP as input. A literature reviews in which the ANNs are used in the damage detection is provided in [6,7].

In this paper, the dynamic behaviour is expressed through elasticity coupled with damage using a phenomenological macro-model for cracked composites structures made of polymer reinforced with long glass fibres. The damage is fully described by a single scalar variable whose evolution law is expressed through the maximum dissipation principle [8,9]. Then, taking into account the nonlinearity induced by damage and using the classical linear Kirchhoff-Love theory of plates, the resulting nonlinear problem is solved in time domain. Several numerical simulations have been performed to generate a dataset consisting with stress and damage states for various combinations of layer orientation and applied load. These data have been used to train a feed-forward neural network till the network learns to an acceptable level of accuracy. The trained ANN has been tested to predict the damage from the input stress state. The established ANN can learn effectively about the damage location and severity present in the composite structure and can predict reasonably well when tested with unknown data set. This approach provides a quick response for damage level prediction in online applications reducing significantly the computational costs.

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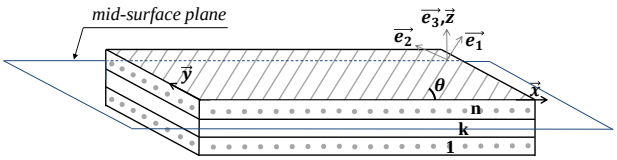

Fig. 1. Coordinates system of a multilayered composite structure

2 Damaged dynamic behaviour

2.1 Finite element model

The thin composite structure, whose thickness is constant h, is composed of n layers having different fibre orientations θ. The global coordinate system $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ is chosen such as the plan $(\mathbf{x}, \mathbf{y})$ is located in the mid-plane of the structure and the $\mathbf{z}$ is oriented in the thickness direction as shown in figure 1. Assuming that there is no stress in the $\mathbf{z}$ direction and according to the First-order Shear Deformation Theory, the displacement field can be written as follows:

$$\left\{\underline{u}(x,y,z,t)\right\} = \left\{\begin{matrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{matrix}\right\} + z\left\{\begin{matrix} \varphi_y(x,y,t) \\ -\varphi_x(x,y,t) \\ 0 \end{matrix}\right\} \quad (1)$$

The problem is discretized by finite elements method where the quadrangular element Serendip $Q8$ with eight nodes is used to mesh the 2D model. Then, using the variational Hamilton principle, the elementary stiffness and mass matrices are carried out for each ply. The global mass $[M]$ and stiffness $[K]$ matrices are then obtained by assembling the elementary contributions all over the mesh elements. The damping is assumed proportional such as $[B] = \alpha_1[M] + \alpha_2[K]$. When the damage occurs, the stiffness will depend on the the damage state. This allows to rewrite it as $[K(D)]$ and the dynamic equation of motion is expressed as follow:

$$[M]\ddot{u}(t) + [B]\dot{u}(t) + [K(D)]u(t) = F(t) \quad (2)$$

where $u(t)$ is the dynamic displacement vector and $F(t)$ is the external load vector. The last equation depicts a nonlinear behaviour of the composite structure due to the material nonlinearity induced by the nonlinear evolution of the stiffness during the load.

2.2 Bilateral damage model

Generally, the matrix cracking is considered as the first damage mechanism and results in a common defect in composites. It evolves and grows leading mainly to the degradation of the structure stiffness through the decrease of the transverse elastic Young's modulus. The investigated structures in this paper are made of a thin layer of polymer reinforced with long glass fibres oriented with respect to $\mathbf{e_1}$ direction as depicted in figure 2(a) where the micro-cracks are parallel to the fibre direction. Boubakar et al [8] deduced, through experimental analysis, that the damage in the polymer matrix depends on the micro-cracks opening modes $(M1)$ and $(M2)$ as shown in figure 2(b).
Three parameters D_I, D_{II} and D_{III} are introduced to describe the damage effects such as:

$$D_I = \frac{\Delta E_2}{E_2} \quad ; \quad D_{II} = \frac{\Delta G_{12}}{G_{12}} \quad ; \quad D_{III} = \frac{\Delta G_{23}}{G_{23}} \quad (3)$$

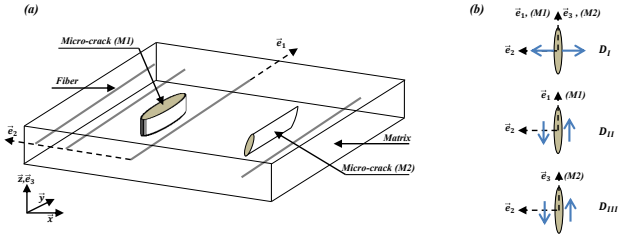

Fig. 2. Thin layer of polymer reinforced with long glass fibres: (a) orientation of the micro-cracks in the matrix, (b) micro-cracks opening modes [8]

A damage matrix $[H(D)]$ is fully expressed by a single scalar variable D representing the relative reduction of the transverse Young's modulus [9]:

$$\begin{aligned} H_{22} &= \frac{D_I}{1-D_I}S_{22} = \frac{D}{1-D}S_{22} \\ H_{44} &= \frac{D_{II}}{1-D_{II}}S_{44} = \frac{D}{\sqrt{1-D}}\sqrt{S_{11}S_{22}} \\ H_{66} &= \frac{D_{III}}{1-D_{III}}S_{66} = \frac{D}{\sqrt{1-D}}S_{22} \\ Others \quad & H_{ij} = 0 \end{aligned} \quad (4)$$

where S_{ii} depict the flexibility components of $[S]$ of the undamaged material. The $[H(D)]$ is added to the flexibility matrix to describe the damaged elastic behavior of the composite structure.

$$\varepsilon = [S + H(D)] : \sigma \quad (5)$$

where σ is the stress tensor. A threshold function $\bar{Y}$ is defined in order to record the damage level and adapt the threshold of damage activation to the damage level. Thereby, the following charge function is established:

$$f_d = Y - \bar{Y} = \frac{1}{2}\sigma^T\left(\frac{\partial H}{\partial D}\right)\sigma - (Y_c + qD^p) \quad (6)$$

where the constants Y_c, q and p are damage parameters. $Y = \frac{1}{2}\sigma^T\left(\frac{\partial H}{\partial D}\right)\sigma$ is the thermodynamic force associated to the damage and it is obtained using the thermodynamics of irreversible processes.

3 Metamodel approach

The damage prediction procedure is fully numerical and is divided into two main steps: the first one consists in the offline simulations where several numerical simulations using the finite element model are carried out. The second one is the application of the ANNs to estimate the damage level and its location.

3.1 Damage simulation

The implicit time integration scheme of Newmark for nonlinear dynamic problems is improved and adapted in order to solve the equation (2). For each time step, the displacement field is known and the effective and predicted stress tensors are calculated at each integration Gauss point

with Newton-Raphson method. Then the damage level is obtained through the charge function in equation (6). If $f_d \leq 0$, the last value of damage is kept as the damage level. But, if f_d tends to be positive, damage occurs and the damage increment will be obtained using the Newton-Raphson method. After getting the damage level, the stiffness matrix in equation (2) is updated to take into account the change in material properties.

3.2 Artificial neural networks metamodel

ANNs are composed of a numerous neurons arranged in layers and connected together by weights. Each neuron communicates with each other to process probabilistic or deterministic information. The full model for damage computing described in the section 3.1 is used in offline simulations in order to get enough data able to obtain a high accuracy in severity and location prediction of the damage. Since the damage mechanism depends on the orthotropic stresses $(\sigma_{22}, \sigma_{12}, \sigma_{23})$, these latter are chosen as inputs of the ANNs and the corresponding damage level D is the ANNs target. Different damage scenarios are simulated using a three layered clamped-free composite beam oriented as $(90/0/90)$, $(67/0/-67)$ and $(45/0/-45)$. Each structure is subjected to a sufficient load able to get fully damaged zones. After carrying out these simulations, some Gauss points are selected where the three stresses $(\sigma_{22}, \sigma_{12}, \sigma_{23})$ and corresponding damage level are extrated and thereby a database is made. Then the obtained samples of the database are normalized in the interval $[-1, 1]$ to make the training process faster and to ensure the uniform distribution of the samples between the network inputs and the targets. Once the input and output sets are organised, the ANNs is now ready to be trained. The organised database is randomly divided into training, validation and test ratios. During the training process, the predicted ANNs output (*i.e: using the ANNs model*) is compared to the given target (*i.e: full model*) and the weights are adjusted until achieving an optimal set of weights that minimizes the mean square error of the entire training data set. The validation set is used to control the over fitting and finally, the performance of the fully trained ANNs is evaluated with the test set. To summarize, the offline simulations are used to obtain the relationship between the state of stress and the damage level. This relationship is introduced in online simulations instead of the Newton-Raphson method to reduce the computational costs. The solving procedure is more detailed in [10]

4 Numerical Results

Several numerical simulations have been performed in order to highlight how the stress state can be used as a damage indicator in terms of location and severity. The mechanical properties of the considered laminated structure are given in table 1. The investigated structures are clamped at $x = 0$ and free at $x = L$ and they have the same geometric data as $(0.3, 0.03, 0.001)\,m$ in $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ directions, respectively. A distributed impact load is applied along the free side in $\mathbf{x}$ direction with F as magnitude and $\tau = 1\,ms$ as a duration. Firstly, to validate the proposed ANNs metamodel, the results provided by the ANNs are compared to those given by the full damage model. Thus, the ANNs method is applied to the structures used in the offline simulations. The decrease ratio of eigenfrequencies obtained via the two methods is used to ensure the performance of ANNs in terms of damage severity. This ratio is defined as $R_d = 100\|\frac{f_0 - f_D}{f_0}\|$. As shown in figure 3, the ANNs metamodel predict the damage severity with a good agreement comparing to the full model. Then, comparing the figure 5 to figure 4 and the figure 7 to figure 6 where (a), (b) and (c) are the upper, middle and the lower layers respectively, the ANNs demonstrate their ability to perfectly estimate the spatial damage levels in the layered beams with high accuracy.

Now, the designed ANNs is applied to a new layered beam. This latter has the same dimensions of the beams used previously but the layer orientations $(60/0/60)$ are not present is the the trainig data set. An impact distributed load is applied to the free side of this beam where $F = 1.52\,10^5 N/m$. As shown in figure 8, the used ANNs permit to obtain the eigenfrequencies shift with a good agreement with those given by the full damage model. Then, the damage states

Table 1. Mechanical properties of the laminated structure

Elastic modulus E_1 ($\mathbf{e}_1$) (MPa)	45680
Elastic modulus E_2 ($\mathbf{e}_2$) (MPa)	16470
Shear modulus G_{12} (MPa)	6760
Poisson ratio ν_{12}	0.34
Poisson ratio ν_{23}	0.34
Y_c (MPa)	0.0027
q (MPa)	1.246
p	0.816

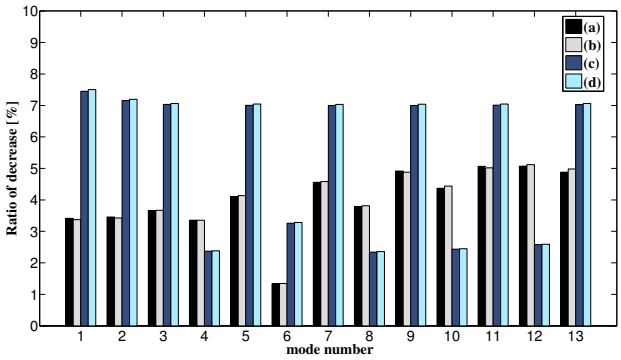

Fig. 3. Damage severity performance: (a) and (b) are the decrease ratios obtained via the full damage model and the ANNs metamodel, of the $(45/0/-45)$ laminated beam, (c) and (d) are the same corresponding ratios of the $(90/0/90)$ laminated beam

Fig. 4. Damage state, obtained via the full model, per ply of the $(45/0/-45)$ beam where $F = 2.28\,10^5 N/m$

Fig. 5. ANNs estimed damage state per ply of the $(45/0/-45)$ beam where $F = 2.28\ 10^5 N/m$

Fig. 6. Damage state, obtained via the full model, per ply of the $(90/0/90)$ beam where $F = 1.2\ 10^5 N/m$

Fig. 7. ANNs estimed damage state per ply of the $(90/0/90)$ beam where $F = 1.2\ 10^5 N/m$

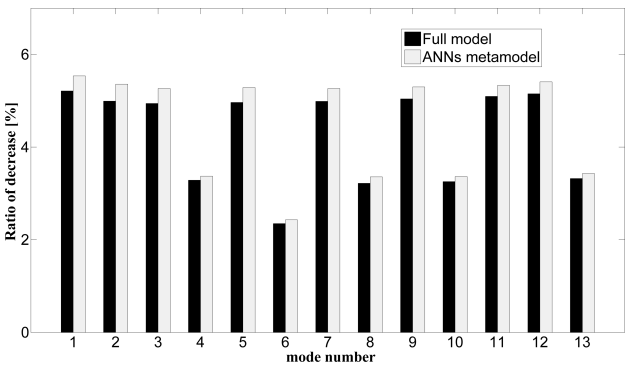

Fig. 8. Damage severity performance in terms of the decrease ratios obtained via the full damage model and the ANNs metamodel of the $(60/0/-60)$ layered beam

Fig. 9. Damage state, obtained via the full model, per ply of the $(60/0/-60)$ layered beam

Fig. 10. ANNs estimed damage state per ply of the $(60/0/-60)$ layered beam

obtained through the full model and the ANNs metamodel are used to verify the damage dispersion in each layer. As shown in figures 9 and 10, ANNs localize the damage and estimate its level in each layer with high accuracy. In addition, ANNs metamodel reduces significantly the computational cost in the online simulations. Hence, once the relationship between the stresses and the corresponding damage level is obtained using the offline simulations, it is introduced in online simulations instead of the Newton-Raphson method to compute the damage. The damage estimation becomes faster reducing about 66% of the computational cost as shown in table 2.

Table 2. Computational costs (CPU time [*hour*]) of the numerical simulations using the full model and the ANNs metamodel

	$(45/0/-45)$	$(60/0/-60)$	$(90/0/90)$
Full model	77.5	72.4	67.8
ANNs (online)	29	27.6	22.8
Reduction ratio [%]	**62.6**	**62**	**66.4**

5 Conclusion

This paper proposes a named stress-based method which describes the damage evolution in the laminate structures made of polymer reinforced with long glass fibers. The only considered damage is the matrix micro-cracking and is fully described by a single scalar variable. The damage depends on the stress state which can be used as a damage indicator in terms of location and severity. Hence, an ANNs metamodel for severity and location prediction of damage in composite structures is proposed and investigated using features extracted from stress-based analy-

sis data as inputs and the corresponding damage level as target. Numerical simulations validate the proposed metamodel and have shown a high accuracy in terms of damage evolution, severity and localization. Moreover, the ANNs metamodel permits to reduce significantly the computational costs. The proposed method is flexible in use since only the stress state is required to predict the damage level. Thereby, this method can be used in life-time estimations and monitoring strategies of composite structures.

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