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Hysteretic damper based on the Ishlinsky-Prandtl model

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Abstract. In this paper we consider the mathematical model of hysteretic damper based on the Ishlinsky-Prandtl model. The numerical results for the observable characteristics such as force transmission function and “force-displacement” transmission function are obtained and analyzed. The comparison of the efficiency of non-linear viscous damper and hysteretic damper is also presented and discussed.

1 Introduction

The damper is a device used for damping the mechanical, electrical and other modes of vibration arising in the machines during its operation. Damping is an important task that has a wide range of applications. In general the *damping* is a process whereby energy is taken from the vibrating system and is being absorbed by the surroundings. The examples of damping include:

- internal forces of a spring;
- viscous force in a fluid;
- electromagnetic damping in galvanometers;
- shock absorber in a car;
- anti seismic damping devices in buildings etc.

Also the damping devices are widely used in modern avionics (damper of aero-elastic vibrations, which is the electronic system for automatic cancelation of short-aircraft vibrations that inevitably arise when the flight modes change).

Let us also list the types (relative to the physical nature of damping process) of damping devices:

- Structural;
- Coulomb Friction;
- Elastomer;
- Active Drivers;
- Passive Hydraulic;
- Semi-Active Hydraulic;
- Adaptive Hydraulic etc.

In the case of oscillations of mechanical systems we consider the linear viscous damping which is based on the energy dissipation due to viscous friction. However, this type of damping has a significant drawback, namely the low efficiency outside the region of resonance of the system. One way to solve this problem is to use a nonlinear viscous damper [1–5] or damper with hysteretic properties. It should also be pointed out the exciting and interesting (generally, from the fundamental point of view) model of viscous damping which is based on the technique of fractional derivatives and can be called as a fractional damping [6,7].

Main purpose of this work is to study the dynamics of a mechanical system under external affection (forced oscillations) in the case of a damper with hysteretic nonlinearity. As a mathematical model of hysteretic damper we consider the Ishlinsky-Prandtl inverter which is a type of continual hysterons and can be presented as a system of parallel connected nonlinear links such as “stops” [8].

2 Hysteretic material

The materials that are used for hysteretic dampers in general are polymers (synthetic rubber). The composition of such materials is chosen appropriately to provide the high damping properties in a specific range of frequencies and temperatures. When the material is damped the energy dissipation occurs within the material itself. Such an effect is caused by friction between the inner layers that are “flow” when the damping occurs. In the case when such a structure is under damping vibrations the hysteresis loop appears in the “stress-strain”. The area of the loop determines the energy loss (per unit volume of the body) in one cycle due to damping.

Let us briefly consider a mathematical model of hysteresis (based on the operator technique). The carrier of hysteretic nonlinearities is a converter W with scalar input $u(t)$ and outputs $x(t)$. The state of this converter is a pair $\{u, x\}$, i.e., an input-output pair. Let the set of possible states of the converter W is a strip $\Omega = \Omega(W)$ which is placed between the two horizontal lines Φ_l and Φ_r, as is shown in Fig. 1.

If the input $u(t)\,(t \geqslant t_0)$ is continuous and monotone then the output can be defined as:

$$x(t) = W[t_0, x_0]u(t)\,(t \geqslant t0) \qquad (1)$$

so that the variable state $\{u(t), x(t)\}$ is a point in the broken line as is shown in Fig. 1 by thick line; this broken line (passing through the initial state $M_0 = \{u(t_0), x_0\}$ and the

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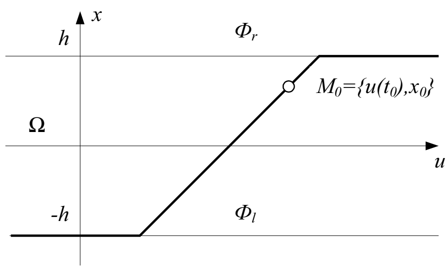

Fig. 1. Action of "stop"-nonlinearity

ends on the lines Φ_l and Φ_r) consists of segment with slope 1 and two horizontal half-lines. In other words, when the input is monotonic the output is determined by the equation:

$$x(t) = \begin{cases} \min\{h, u(t) - u(t_0) + x(t_0)\}, u(t) \text{ nondecreasing}, \\ \max\{-h, u(t) - u(t_0) + x(t_0)\}, u(t) \text{ nonincreasing}. \end{cases} \quad (2)$$

To determine the output (1) for piecewise monotonic continuous inputs we can use the semigroup identity

$$W[t_0, x_0]u(t) = W[t_1, W[t_0, x_0]u(t_1)]u(t)\ (t_0 \leqslant t_1 \leqslant t). \quad (3)$$

The described converter is called as a "stop" [8].

In the most common models of elastic-plastic fibers their states are completely determined by the variables of deformation u and stress x. The parameter h in this case is called as the yield strength of the material. Such fibers can be considered as converters with an input in the form of deformation and an output in the form of strain. In the Prandtl model the strain is determined by the deformation in the same manner as in "stop"-nonlinearity, but the trajectories of possible states between the boundary horizontal lines have the slope E which differs from 1 (for small deformations the fiber can be considered as an elastic material with the elastic modulus E).

Let us consider the inverter W which can be presented in the form of a simple flow diagram (without feedback) based on the finite number of hysterons $W^1, \dots, W^N$ and elementary functional units (as is shown in Fig. 2). Usually

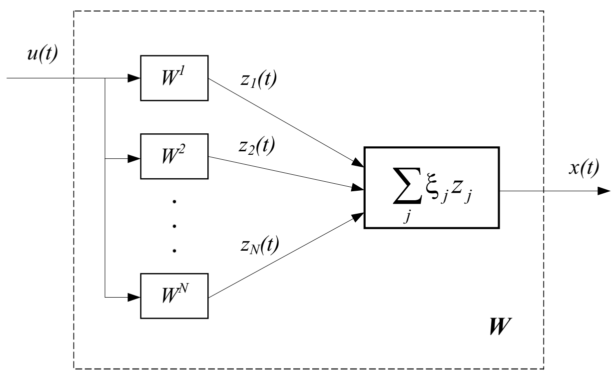

Fig. 2. Parallel connection of hysterons

such converters W are nondeterministic. Their condition can be described (instead of the input-output description) by the set $\{u, z_1, \dots, z_N\} \in R^{N-1}$, where u is the converter's input and z_j is an output of hysteron W^j in the flow diagram.

Suppose that hysterons $W^1, \dots, W^N$ with areas of permissible states $\Omega(W^1), \dots, \Omega(W^N)$ are determined by the input-output relations

$$z_j(t) = W^j[t_0, z_j(t_0)]u(t)\ (j = 1, \dots, N). \quad (4)$$

Let

$$\Omega(W) = \{\{u, z_1, \dots, z_N\} : \{u, z_j\} \in \Omega(W^j),\ u \in R^1\}. \quad (5)$$

Parallel connection of hysterons W^j with weights ξ_j we call the converter W with an area of permissible states (5) such that for every initial state

$$q(t_0) = \{u_0, z_0\} = \{u(t_0), z_1(t_0), \dots, z_N(t_0)\} \in \Omega(W) \subset R^{N+1} \quad (6)$$

all the continuous scalar inputs $u(t)\ (t \geqslant t_0)$ that satisfy the condition $u(t_0) = u_0$ are allowed. The output is determined by the relation

$$x(t) = W[t_0, z_0]u(t) = \sum_{j=1}^{N} \xi_j W^j[t_0, z_j(t_0)]u(t),\ (t \geqslant t_0). \quad (7)$$

The considered converter W is called as Ishlinsky-Prandtl material [6]. Of course the "classical" Ishlinsky-Prandtl converter is determined as a continuos system (sum should be replaced by an integral). However in our numerical simulations it is more appropriate to use the discrete analog of the Ishlinsky-Prandtl converter (7). In this way in the following consideration we will call the converter (7) as an Ishlinsky-Prandtl converter.

As an example we consider the reaction of the Ishlinsky-Prandtl material on sinusoidal impact. We use the converter W with the parameters $E = 1$, $W^j[t_0, z_j(t_0)] = 0$, $\xi_j = 1$, $W^j : h = \{-j, j\}$, $j = 1, \dots, 10$ and the input in the form $u(t) = 12\sin(t)$. "Stress-strain" diagram of such a converter is shown in Fig. 3.

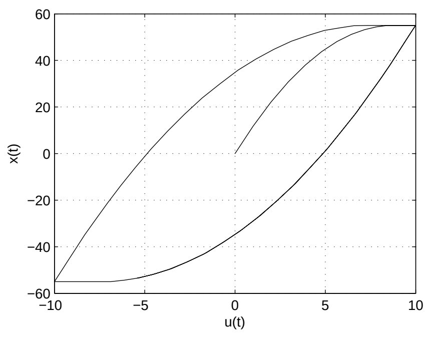

Fig. 3. "Stress-strain" diagram for Ishlinsky-Prandtl material

3 Physical model

Let us consider a mechanical system under external affection and in the presence of the damping part $f_d(t)$ as is shown in Fig. 4. The system is presented as a cylinder with mass M under external affection $f(t)$ of harmonic nature.

In the cylinder there is a car of mass m (moving without friction in the horizontal plane) connected to the border by a spring with stiffness k. For simplicity we assume that the system is one-dimensional.

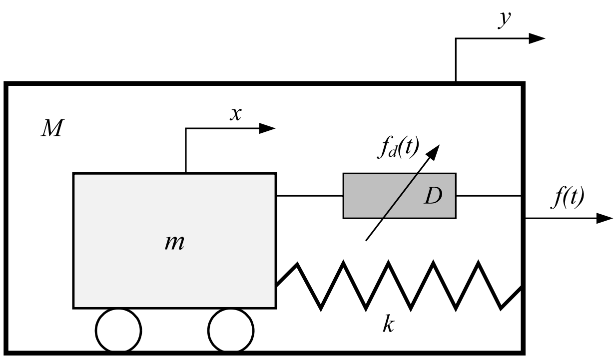

Fig. 4. The considered physical system

Suppose that the affection force $f(t)$ is described by the following relation:

$$f(t) = Y\omega^2 \sin(\omega t), \tag{8}$$

where $Y\omega^2$ is an amplitude and ω is a frequency of the affection force.

The equation of motion for considered system is

$$\begin{cases} M\ddot{y} + kz + f_d(t) = f(t), \\ m\ddot{x} - kz - f_d(t) = 0, \\ f(t) = Y\omega^2 \sin(\omega t),\ z(t) = y(t) - x(t). \end{cases} \tag{9}$$

Here overdot determines the time derivative and $z(t)$ is a relative displacement.

3.1 Viscous damping

Let us consider the case of viscous damping. In general case the viscous friction can be described as follows:

$$f_d(t) = c(1 + z(t))^n \dot{z},\ n \geqslant 0, \tag{10}$$

where c is a damping coefficient. In the case $n = 0$ we have a linear viscous friction. For $n > 0$ the nonlinear viscous fricition takes place.

The equation of motion for relative displacement $z(t)$ becomes

$$\ddot{z}(t) + \frac{M+m}{Mm}\left[c(1+z(t))^n \dot{z}(t) + kz(t)\right] = \frac{Y}{M}\omega^2 \sin(\omega t). \tag{11}$$

It is more suitable to introduce the dimensionless variables

$$\Omega = \frac{\omega}{\omega_0},\ \mu = \frac{mM}{m+M},\ A = \frac{Y}{M},\ \tau = \omega_0 t,$$

$$\omega_0 = \sqrt{\frac{k}{\mu}},\ \zeta = \frac{c}{2\omega_0\mu},\ u = z.$$

After such notations the final equation of motion has the form:

$$\frac{d^2u}{d\tau^2} + 2\zeta(1+u)^n \frac{du}{d\tau} + u = A\Omega^2 \sin(\Omega\tau). \tag{12}$$

3.2 Hysteretic damping

Now we consider the hysteretic damper. In this case (using the notations introduced above) the damping force can be presented as:

$$f_d(\tau) = W[\tau, z_j(\tau)]u,\ j = \overline{1, N}, \tag{13}$$

where W is an Ishlinsky-Prandtl operator which is determined by the relation (7).

In this case the equation of motion for the considered system becomes:

$$\frac{d^2u}{d\tau^2} + \alpha W[\tau, z_j(\tau)]u + u = A\Omega^2 \sin(\Omega\tau),\ j = \overline{1, N}, \tag{14}$$

where $\alpha = S/k$ and S is an area of damping material's section, k is a spring stiffness.

3.3 Main characteristics

Let us consider the main characteristics reflecting the efficiency of the damper in the resonance system and beyond. Such characteristics are force transmission function and "force-displacement" transmission function.

The *force transmission function* is determined by the ratio of the force applied to the cylinder M and the force applied to the car m (Fig. 4). This function reflects the efficiency of suppression of the external affection by the force transmission from an external source to the load. This characteristic is expressed as follows:

$$T_{ff} = \frac{1}{Y\omega^2} \max\left| m\omega_0^2 \frac{d^2x}{d\tau^2} \right|. \tag{15}$$

The *"force-displacement" transmission function* is determined by the relation of the motion of car m relative to the cylinder M and the force applied to the cylinder. This quantity reflects the efficiency of vibration absorption by the ability of the damper to reduce the relative motion of the car under influence of external forces. This characteristic is expressed as

$$T_{ff} = \frac{\max|x(\tau)|}{Y\omega^2}. \tag{16}$$

During the following simulations we use these quantities for comparison of the linear viscous, nonlinear viscous and hysteretic dampers.

4 Numerical simulation

4.1 Difference scheme

The dynamics of the considered system can be simulated using the explicit difference scheme applied to equations (12) and (14). Finite-difference equations for viscous damper is:

$$\begin{aligned} &\frac{u_{i+2} - 2u_{i+1} + u_i}{h_\tau^2} + \\ &+2\zeta(1+u_{i+1})^n \tfrac{u_{i+1}-u_i}{h_\tau} + u_{i+1} = A\Omega^2 \sin(\Omega i h_\tau), \end{aligned} \tag{17}$$

and for hysteretic damper is:

$$\frac{u_{i+2} - 2u_{i+1} + u_i}{h_\tau^2} + \\ +\alpha W[ih_\tau, z_j(ih_\tau)]u_{i+1} + u_{i+1} = A\Omega^2 \sin(\Omega ih_\tau). \tag{18}$$

Here $i = \overline{0, T}$, $j = \overline{1, N}$, h_τ is a step of a grid by τ axis, N is a number of the Ishlinsky-Prandtl hysterons, T is a time period.

The explicit solutions for the viscous damper and for hysteretic damper are:

$$u_{i+2} = A\Omega^2 h_\tau^2 \sin(\Omega ih_\tau) - \\ -2\zeta h_\tau (1 + u_{i+1})^n (u_{i+1} - u_i) - h_\tau^2 u_{i+1} + 2u_{i+1} - u_i, \tag{19}$$

$$u_{i+2} = A\Omega^2 h_\tau^2 \sin(\Omega ih_\tau) - \\ -\alpha h_\tau^2 W[ih_\tau, z_j(ih_\tau)]u_{i+1} - h_\tau^2 u_{i+1} + 2u_{i+1} - u_i, \tag{20}$$

respectively.

4.2 Results

We make the numerical simulations using the explicit results (19) and (20). In order to compare the viscous damper and hysteretic damper we present the numerical results for two characteristics of the system the force transmission function and "force-displacement" transmission function. For the nonlinear viscous damper we use the following set $n = \{0, 2, 4\}$.

For the hysteretic damper we use the Ishlinsky-Prandtl material (which corresponds to rubber) with the parameters $E = 10000$, $W^j[t_0, z_j(t_0)] = 0$, $\xi_j = 1$, W^j: $h = \{-j, j\}$, $j = 1, \ldots, 50$ (with a step 0.1), $\alpha = 0,0001$.

The characteristics of the mechanical system (per dimensionless units): $M = 1$, $m = 1$, $\zeta = 0.8$, $\omega_0 = 10$; the external affection with parameters $A = 1$, $\omega = 0, \ldots, 30$ (with a step 0.2); parameters of the difference scheme: $h_\tau = 0.0167$, $T = 10000$.

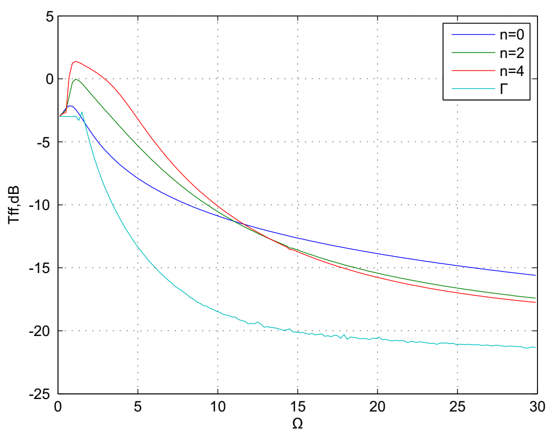

Fig. 5. Force transmission function

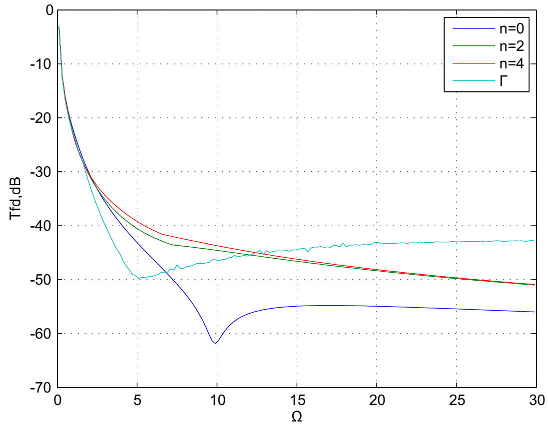

Fig. 6. "Force-displacement" transmission function

The simulation results are shown in Fig. 5 and Fig. 6. As it can be seen from these figures the linear viscous damper (dark blue curve $n = 0$) has a high efficiency in the resonance region of the system, however outside the resonance region the damping properties sharply decrease. At the same time the nonlinear viscous damper (green $n = 2$ and red $n = 4$ curves) has a wide range of effective use, but loses in efficiency to linear damper in the resonance region of the system.

The hysteretic damper (light blue curve) based on Ishlinsky-Prandtl material has a high efficiency both in the resonance region and beyond. The disadvantage of the hysteretic damper is in decreasing of the ability to reduce the relative movement of car under external forces outside the resonance region of the system.

5 Conclusions

In the presented paper we have considered the hysteretic damper based on the Ishlinsky-Prandtl model. We presented the numerical results for the observable characteristics of the system under consideration such as force transmission function and "force-displacement" transmission function. These results allow to compare the various types of viscous dampers (linear and nonlinear) and hysteretic damper. Based on the obtained numerical results we can formulate the following concluding notes:

- **Linear viscous damper** has a high efficiency in the resonance region of the system, however outside the resonance region damping properties sharply decrease.
- **Nonlinear viscous** damping has a wide range of effective use, but loses in efficiency to linear damper in the resonance region of the system.
- **Hysteretic damper** based on Ishlinsky-Prandtl material **has a high efficiency both in the resonance region and beyond**. The disadvantage of the hysteretic damper is in decreasing of the ability to reduce the relative movement of car under external forces outside the resonance region of the system.

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