

Dynamics of structures of clusters of tethered satellites

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Abstract. In this paper, we study the nonlinear dynamics of clusters of tethered satellites via numerical step-by-step time integration of the differential equations of motion. The basic algorithm is Central Difference explicit method. The masses of the space vehicles are considered lumped and the connection cables are considered massless and of linear elastic material. The considered nonlinearities are due to large displacements leading to changes in masses coordinates. The proposed algorithm does not involve assemblage of stiffness matrices. Instead, the elastic restoring forces are directly computed at each time step. A new time step optimization procedure is implemented. As an example, a cluster of four satellites in a tetrahedral disposition is considered. Free damped vibrations due to large initial conditions are computed.

1 Introduction

In this paper, we study the nonlinear dynamics of clusters of tethered satellites via numerical step-by-step time integration of the differential equations of motion. The basic algorithm is the Central Finite Difference explicit method, (Brasil [2][3]). The masses of the space vehicles are considered lumped and the connection cables are considered massless and of linear elastic material. The considered nonlinearities are due to large displacements leading to changes in masses coordinates. The proposed algorithm does not involve assemblage of stiffness matrices. Instead, the elastic restoring forces are directly computed at each time step. A new time step optimization procedure is implemented. As an example, a cluster of four satellites in a tetrahedral disposition is considered. Free damped vibrations due to large initial conditions are considered.

Cable structures have been widely used in Civil Engineering applications, as described by Pauletti [12] and Irvine [9]. Recent spacecraft applications, specifically for tethered satellites, are reported by Cosmo and Lorenzini, [5], Ellis and Hall [7], Kumar [10], Mankala and Agrawal [11], Stevens and Wiesel [18]. A problem to be overcome is the high strength and low weight necessary for cable spacecraft applications. Suggestions have been made to future use of carbon nanotubes, as commented by Baughman et al. [1].

2 Theoretical development

We write down the dynamic equilibrium equation of the masses of our tethered satellites system via Newton's Second Law, supposing that we have n generalized coordinates, the three orthogonal free displacements of each mass. The resulting system of second order Ordinary Differential Equations (Clough and Penzien, [4], Paz [17], Thomson [19]) is

$$\mathbf{f}_{inertia} = \mathbf{p}(t) - \mathbf{f}_{dissipation} - \mathbf{f}_{elastic} \quad (1)$$

$\mathbf{f}_{inertia}$ is the $n \times 1$ vector of inertia forces that we supposed to be linearly proportional to the accelerations vector as $\mathbf{f}_{inertia} = \mathbf{M}\ddot{\mathbf{u}}$, \mathbf{M} being the $n \times n$ mass matrix. This is a diagonal matrix in our formulation.

$\mathbf{p}(t)$ is the $n \times 1$ vector of external applied forces, time dependent in general.

$\mathbf{f}_{dissipation}$ is the $n \times 1$ vector of dissipation forces that we suppose to be linearly proportional to the velocities vector as $\mathbf{f}_{dissipation} = \mathbf{C}\dot{\mathbf{u}}$, \mathbf{C} being the $n \times n$ damping matrix. This is a diagonal matrix in our formulation.

$\mathbf{f}_{elastic}$ is the $n \times 1$ vector of elastic restoring forces, non linear, in general, due to the considered large displacements of the masses and the presence of previous tension of the cables and to the fact that the cables will not work on compression.

The Eq. (1) may be rewritten as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{f}_{elastic} = \mathbf{p}(t) \quad (2)$$

This equation will be numerically integrated using a Central Finite Differences approach (Brasil [2][3]) as follows. Let us write down the expressions of Taylor expansions forward and backwards of the vector function $\mathbf{u} = \mathbf{u}(t)$ for the displacements of the masses of our tethered satellites system, involving time steps t_{i-1} , t_i , t_{i+1} , separated by a time interval h (in seconds).

$$\mathbf{u}_{i+1} = \mathbf{u}_i + h\dot{\mathbf{u}}_i + \frac{1}{2}h^2\ddot{\mathbf{u}}_i + \dots \quad (3)$$

$$\mathbf{u}_{i-1} = \mathbf{u}_i - h\dot{\mathbf{u}}_i + \frac{1}{2}h^2\ddot{\mathbf{u}}_i - \dots \quad (4)$$

Subtracting Eq. (4) of Eq. (3) we obtain a Central Finite Differences approximation of the velocity vector

$$\dot{\mathbf{u}}_i = \frac{\mathbf{u}_{i+1} - \mathbf{u}_{i-1}}{2h} \quad (5)$$

Adding Eq. (3) to Eq. (4) we obtain a Central Finite Differences approximation of the acceleration vector

$$\ddot{\mathbf{u}}_i = \frac{\mathbf{u}_{i+1} - 2\mathbf{u}_i + \mathbf{u}_{i-1}}{h^2} \quad (6)$$

Substituting approximations (5) and (6) in the Eq. (2) we get an algebraic linear system of equations

$$\hat{\mathbf{K}}\mathbf{u}_{i+1} = \hat{\mathbf{p}}_i \quad (7)$$

where

$$\hat{\mathbf{K}} = \frac{1}{h^2}\mathbf{M} + \frac{1}{2h}\mathbf{C} \quad (8)$$

and

$$\hat{\mathbf{p}}_i = \mathbf{p}_i + \frac{2}{h^2}\mathbf{M}\mathbf{u}_i - \left(\frac{1}{h^2}\mathbf{M} - \frac{1}{2h}\mathbf{C}\right)\mathbf{u}_{i-1} - \mathbf{f}_{\text{elastic},i} \quad (9)$$

is the pseudo static loading vector at this time step. No system solution procedure is actually needed in Eq. 7, as matrix of Eq. 8 is diagonal. The computation is carried out at vector components level.

The value of the elastic forces $\mathbf{f}_{\text{elastic},i}$ at any time step t_i is computed as follows. Let us consider one certain cable of the structure. For this particular cable, we get the 6 x 1 vector of the displacements of the two end nodes of Eq. (10), displayed in figure 1, considering the present coordinates of these nodes and the present inclination of the cable at this time step,

$$\mathbf{q} = [q_1 \ q_2 \ \dots \ q_6]^T \quad (10)$$

the change of length of this cable will be

$$\Delta L = (q_4 - q_1)cx + (q_5 - q_2)cy + (q_6 - q_3)cz \quad (11)$$

where, cx , cy and cz are the direction cosines of the cable.

If N_0 is the initial traction of this cable, L_0 its initial length, E the modulus of elasticity of the material and A the section area, the axial force at this time step will be

$$N = N_0 + \frac{EA}{L_0}\Delta L \quad (12)$$

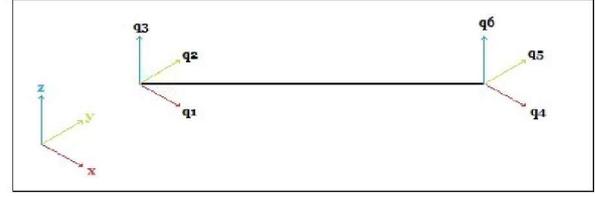


Fig. 1. Cable end nodes displacements

provided the value of Eq. (12) is positive. Otherwise it will be set to zero. Next, this axial force will be decomposed in the horizontal and vertical directions at each end node of the cable to be added to the elastic forces vector. The same routine will be repeated for all other cables.

One should note from Eq. (9) that \mathbf{u}_{i-1} is needed at each time step. This poses a problem at the initial time t_0 , when initial conditions u_0 and \dot{u}_0 are known, but not before. A simple scheme to solve it is to suppose a Uniformly Accelerated Motion in the first time interval, that is, $\ddot{u}_1 = \ddot{u}_0$, which allows for the computation of u_1 by writing the dynamical equilibrium Eq. (2) at time t_1 .

3 Numerical simulations

We next analyse a sample model of four masses connected by cables initially forming a tetrahedral configuration, as displayed in figure 2.

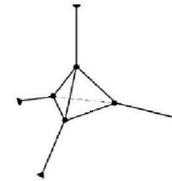


Fig. 2. The model

Damped free vibrations due to non-trivial large initial conditions lead to time histories of Fig. 3. Displacements in the three directions of one of the tetrahedron corner masses are shown. Severe nonlinearities may be observed due to large displacements and the possibility of cables becoming slack.

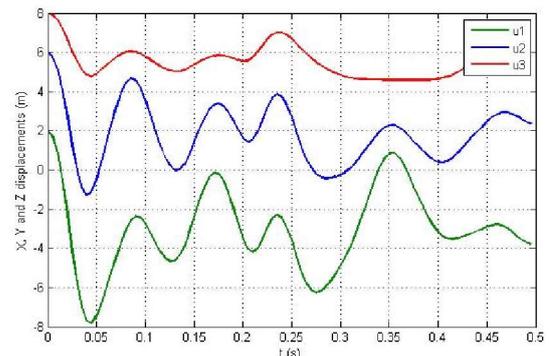


Fig. 3. Displacements time histories

4 Time step optimization

It can be shown that the central difference method is only conditionally stable, and, for linear multiple degrees of freedom systems, stability is guaranteed bounding the time-step by the Courant, Friedrichs and Lewys [6] condition

$$\Delta t \leq \frac{2}{\omega_{\max}^0}, \quad (13)$$

where ω_{\max}^0 is the largest natural frequency of the system from the solution of the eigenvalue problem

$$\det(\mathbf{K}_0 - \omega^2 \mathbf{M}) = 0, \quad (14)$$

where \mathbf{K}_0 and \mathbf{M} are the system's stiffness and mass matrices. There is no guarantee that criterion (13) is on the safe side for non-linear systems.

The problem can be circumvented by repeatedly updating the stiffness matrix \mathbf{K}_k , evaluated at time t_k . Considering that the main attractiveness of the central difference method is the possibility of dealing exclusively with force vectors, this is in contradiction with the spirit of the method.

Irons and Treharne [8] have shown that an upper-limit estimative of the maximum frequency of a discrete system is given by the maximum of its maximum elements' natural frequencies, i.e.,

$$\omega_{\max}^k = \overbrace{\max_{e=1}^{n_e}} \{ \omega_{e,\max}^k \} \quad (15)$$

where $\omega_{e,\max}^k$ is the maximum natural frequency of element e , evaluated at time t_k , by means of a series of eigenvalue problems

$$\det(\mathbf{K}^e - \omega^2 \mathbf{M}^e) = 0, \text{ for all elements} \quad (16)$$

Taking into account that the central difference method only is fully vectorised when masses are lumped into the system's degrees of freed, Pauletti et al. [15][16] show that the maximum element frequency is given by

$$\omega_{e,\max}^k = \sqrt{\frac{n_n^e \lambda_{e,\max}}{m_e}}, \quad (17)$$

where m_e and n_n^e are respectively the total mass and the number of nodes of the element and $\lambda_{e,\max}$ is maximum eigenvalue of its stiffness matrix.

Let us consider a column-wise partition of the element stiffness matrix, evaluated at a given configuration \mathbf{u}_k^e according to $\mathbf{K}_k^e = \mathbf{K}^e(\mathbf{u}_k^e) = [\mathbf{k}_1^k \ \mathbf{k}_2^k \ \dots \ \mathbf{k}_{n_{\text{dof}}}^k]$, where n_{dof}^e is the number of degrees of freedom of the element. Then, an upper bound estimative of $\lambda_{e,\max}$ can be obtained according to

$$|\lambda_{\max}^e| \leq \overbrace{\max_{j=1}^{n_{\text{dof}}^e}} \left(\|\mathbf{k}_j^e\| \right) \quad (19)$$

where is the L1-norm of each column of the stiffness matrix, that is,

$$\|\mathbf{k}_j^e\|_1 = \sum_{i=1}^{n_{\text{dof}}^e} |(\mathbf{k}_j^e)_i|. \quad (20)$$

Besides, Pauletti and Almeida-Neto [13][14] have shown that columns can be conveniently evaluated by finite-difference approximations, according to

$$\mathbf{K}_j^k \cong \frac{1}{2h^e} [\mathbf{f}^e(\mathbf{u}_k^e + h^e \boldsymbol{\delta}_j) - \mathbf{f}^e(\mathbf{u}_k^e - h^e \boldsymbol{\delta}_j)], \quad (21)$$

$j = 1, \dots, n_{\text{dof}}^e$

where \mathbf{f}^e is the internal load vector for element e , h is a scalar parameter and $\pm h \boldsymbol{\delta}_j$ are backward and forward Kronecker-delta perturbations of the j^{th} element degree of freedom, such that $\boldsymbol{\delta}_j^e = [\delta_{ij}]$, $i = 1, \dots, n_{\text{dof}}^e$ and where δ_{ij} is the Kronecker delta. This finite-difference approximation provides excellent estimates for the stiffness matrices, as long as h is small enough.

In our present algorithm, no stiffness matrix is actually employed, but simply the internal force vectors are backward and forward perturbed, providing an upper-bound limit for λ_{\max}^e , according to Eqs. (21) and (19), leading to an upper-bound estimative of the system maximum frequency ω_{\max}^k , according to Eq. (15), and thus a lower-bound estimative for the instantaneous critical time-step Δt_k given by Eq. (13).

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