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Catenary Analysis and Calculation Method of Track Rope of Cargo Cableway with Multiple Loads

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Abstract. According to actual working condition, the catenary equations of elastic track tope for setting-up is proposed based on the cableway erection requirement, such as the erection angle of end, the initial cable length and midpoint position of cable. The mechanics equilibrium equations, the loads span equations and the consistent equations are presented by analysis of track rope stress state under loads. The nonlinear equations are constructed for elastic track tope with multiple loads and the initial values of newton iteration method are obtained to solve the nonlinear equations. The results of this method are compared with the testing results and numerical results in other literatures and the contrast verifies the reliability of this method. The method is more concise and has smaller amount of calculations with a unified form. It can provide effective means to design the cargo cableway and to check the engineering safety during the erection stage and running stage of cableway.

1 Preface

The overhead cargo cableway has many characteristics, easy line selection, high transportation capability and strong adaptability, etc. The problems of the cableway about design, security assessment, and component selection are more and more important.

Calculation methods of the track rope of the cableway are mainly the analytic method and the finite element method. The analytic method mainly consists of the catenary method [1-3] and the parabola method[4-5]. The catenary method can really reflect the cable shape of the actual suspension rope, the results obtained through theoretical calculation of the catenary method are considered as true solution. Dead weight of the rope structure is assumed to distribute evenly in the parabola theory, it is an approximate calculation method which takes the former two items in the catenary theory. The finite element method mainly includes the two node rod element [6-8] and the multiple nodes curve element [9,10], based on the complete Lagrange description or Euler description, calculation method approximately consists of the increment method and the superposition method. Because it is necessary to carry out simplification or approximate during establishing the finite element format of the rope structure, the obtained calculation models are also different.

For the cargo freight cableway, it is a common acknowledge that track rope without elastic elongation is the catenary. Many references have given out the theoretical reduction process and the relevant calculation equation of the balance equation when the rope structure only bears dead weight[11,12]. For the cargo cableway with the great span, the track rope will generate elastic

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elongation under dead weight and centralized load, the corresponding tension in the rope and the drawing force of the support point will also change. When the track rope has several concentrated loads, these centralized loads are converted to the evenly distributed load in some approximate method, in which tension of track rope is calculated as rope without load. But there is big error with accurate result.

In this paper studies the calculation method of track rope under multiple centralized loads is established to provide practicable theory instruction on calculation and selection of the cargo cableway track rope based on catenary equation of the elastic cable.

2 Catenary equation of elastic cable without centralized load

The following assumptions are introduced in basic theory for design and calculation of the cableway track rope:

(1) The track rope is absolutely flexible, i.e., any section only bears drawing force, and it can't bear bending moment;

(2) The material of the track rope material follows Hooke's law, i.e., the stress and strain relationship of the track rope material is linear elasticity;

(3) The dead load of the track rope is distributed evenly along the arc.

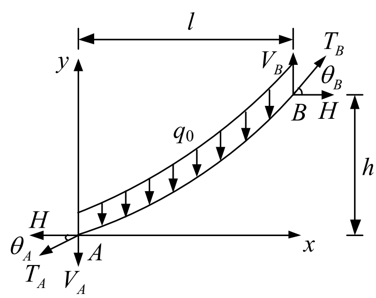

Figure 1. Forcing Schematic Figure of Track Rope without Elasticity

The track rope section shown as figure 1 only bears dead weight load of the rope without elasticity elongation. In which T_A, T_B, V_A, V_B, θ_A and θ_B are tangential tension at end point A and end point B, vertical components of tangential tension, angles between tangential tension and horizontal direction respectively, H is horizontal component of tangential tension, l and h are horizontal span and height different between the end points respectively, q_0 is dead weight intensity of the track rope which is distributed along the arc.

Assume the initial section area of the track rope is A_0, the section area is changed to A with elasticity elongation, intensity of the dead weight distributed along the arc becomes q. Shown as figure 1, the micro-section length of the elastic track rope is ds, the initial arc length is ds_0. Set E as elastic module of the track rope material, the following equation is obtained from Hooke's law: $\mathrm{d}s=[1+T/(EA_0)]\mathrm{d}s_0$. According to the mass conservation of the track rope micro-section after and before deformation: $q_0\mathrm{d}s_0=q\mathrm{d}s$. And it is obtained:

$$q=q_0/[1+T/(EA_0)] \tag{1}$$

Considering forcing condition of the track rope micro-body, the force balance equations at x direction and y direction are obtained:

$$\sum X=0\ :\ H+\mathrm{d}H-H=0 \tag{2}$$

$$\sum Y=0: H\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{d}(H\frac{\mathrm{d}y}{\mathrm{d}x})-H\frac{\mathrm{d}y}{\mathrm{d}x}-q\,\mathrm{d}s=0 \tag{3}$$

From equation (2), H is constant which means horizontal component of tension of the track rope is equal at any location. According to equation (3), there is

$$\frac{\mathrm{d}^2 y}{\mathrm{d} x^2} = \frac{q}{H}\frac{\mathrm{d} s}{\mathrm{d} x} \tag{4}$$

Substituting equation (1) into equation (4), the basic catenary equation of the elastic track rope without centralized load is obtained by $\mathrm{d}s/\mathrm{d}x = \sqrt{1+(\mathrm{d}y/\mathrm{d}x)^2}$ and $T = H\sqrt{1+(\mathrm{d}y/\mathrm{d}x)^2}$ according to deduction process in reference [11]:

$$x = t(a-b) + Kt^2[\sinh(a) - \sinh(b)] \tag{5}$$

$$y = t[\cosh(a) - \cosh(b)] + \frac{Kt^2}{2}[\cosh^2(a) - \cosh^2(b)] \tag{6}$$

$$s_0 = t[\sinh(a) - \sinh(b)] \tag{7}$$

$$s = t[\sinh(a) - \sinh(b)] + \frac{Kt^2}{4}[\sinh(2a) - \sinh(2a)] + \frac{Kt^2}{2}(a-b) \tag{8}$$

In which, $K = q_0/(EA_0)$ (constant), $t = H/q_0$, $a = \sinh^{-1}(V_A/H)$, $b = \sinh^{-1}(V/H)$. V is vertical component of tangential tension at any point. s_0 is initial rope length, s is length of the rope after elastic elongation.

3 Balance equation of elastic track rope under multiple centralized loads

After the setting-up of cableway, the height difference h, the horizontal span l and the initial length s_0 of the whole track rope are determined. The track rope bearing multiple centralized loads is shown as figure 2.

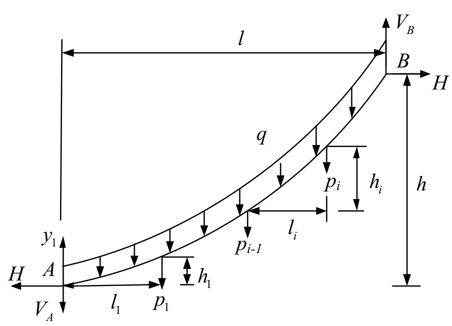

Figure 2. The Track Rope under Multiple Centralized Loads

The rope section splited by centralized load only bears the dead weight of the rope, so the whole rope can be considered as combination of the catenaries. Assuming the track rope has n-1 centralized loads, which are p_1, p_2, …, p_{n-1} respectively, the track rope is divided into n sections, the end points of every section are A_i and B_i, initial arc length is s_i, horizontal span is l_i and height difference is h_i(i=1, 2, …, n).

Established the local coordination system for every section of the track rope, the catenary equation for the every rope section without centralized load is obtained, shown as equations (5)-(8). The nonlinear equations of the whole track rope under multiple centralized loads are established in the following.

3.1 Centralized load

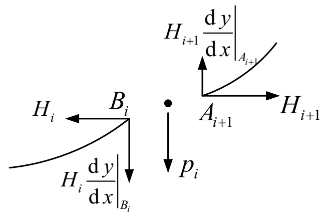

Figure 3. Forcing Analysis at Action Point of Centralized Load

Shown as figure 3, from the force on the action point between every rope section, the force balance conditions at every load point is obtained:

$$H_i - H_{i+1} = 0 \tag{9}$$

$$H_i \frac{dy}{dx}\Big|_{B_i} - H_{i+1}\frac{dy}{dx}\Big|_{A_{i+1}} = p_i \quad (i=1, 2, \ldots, n-1) \tag{10}$$

It is known from the balance conditions that the horizontal force of every rope section H_i is same, which is marked as H. And equation (10) can be written as

$$t[\sinh(b_i) - \sinh(a_{i+1})] = \frac{p_i}{q_0} \quad (i=1, 2, \ldots, n-1) \tag{11}$$

Here $t=H/q_0$, $a_i=\sinh^{-1}(V_{Ai}/H)$, $b_i=\sinh^{-1}(V_{Bi}/H)$ $(i=1, 2, \ldots, n)$.

3.2 Length of suspension rope

During operation of the cableway, the rope length of the track rope between the centralized load (heavy objects) is fixed and unchanged, initial length s_i of every rope section appears in the equation as the known quantity, the horizontal span l_i and height difference h_i of the rope section will change following different loads and different positions.

Therefore, when initial rope length of every rope section $s_i(i=1, 2, \ldots, n)$ is known, the rope section length equation is obtained according to equation (7):

$$t[\sinh(a_i) - \sinh(b_i)] = s_i \qquad (i=1, 2, \ldots, n) \tag{12}$$

3.3 Compatibility condition

1) Every section of the track rope shall meet with the compatibility equation of the whole horizontal span $\sum_{i=1}^{n} l_i = l$, according to equation (5), it can be obtained:

$$t\sum_{i=1}^{n}(a_i - b_i) + Kt^2\sum_{i=1}^{n}[\sinh(a_i) - \sinh(b_i)] = l \tag{13}$$

2) According to deformed compatibility equation in height $\sum_{i=1}^{n} h_i = h$ and equation (6)

$$t\sum_{i=1}^{n}[\cosh(a_i)-\cosh(b_i)]+\frac{Kt^2}{2}\sum_{i=1}^{n}[\cosh^2(a_i)-\cosh^2(b_i)]=h \tag{14}$$

Here $K=q_0/(EA_0)$.

3.4 Nonlinear equation set

Summarizing the above equations, the nonlinear equation set of the elastic track rope under multiple centralized loads is obtained:

$$\begin{cases} t[\sinh(b_i)-\sinh(a_{i+1})]-\dfrac{p_i}{q_0}=0 & (i=1,2,\cdots,n-1) \\ t[\sinh(a_i)-\sinh(b_i)]-s_i=0 & (i=1,2,\cdots,n) \\ t\sum_{i=1}^{n}(a_i-b_i)+Kt^2\sum_{i=1}^{n}[\sinh(a_i)-\sinh(b_i)]-l=0 \\ t\sum_{i=1}^{n}[\cosh(a_i)-\cosh(b_i)]+\dfrac{Kt^2}{2}\sum_{i=1}^{n}[\cosh^2(a_i)-\cosh^2(b_i)]-h=0 \end{cases} \tag{15}$$

The equation set includes 2n+1 nonlinear equations, 2n+1 unknown number ai, bi(i=1, 2 ,…, n) and t, therefore the nonlinear equation set is closed, which can be expressed as:

$$F(X)=0 \tag{16}$$

In which $X=[a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, t]^T$.

After X is calculated, the vertical component V_x and tangential tension T_x at any rope length s_x point of the track rope are obtained respectively:

$$V_x = tq_0\sinh(a_m)+q_0\left(s_x-\sum_{i=1}^{m}s_i\right) \tag{17}$$

$$T_x=\sqrt{(tq_0)^2+V_x^{\,2}} \tag{18}$$

In which, m is number of centralized load in the $[0,s_x]$.

4 Examples

In order to validate calculation accuracy of the cargo cableway track rope, testing data in the reference document [1] and calculation data in reference document [8] are applied for comparison.

Load weight in the reference[1] is P=6130N. The cross area of the track rope is A_0=289.95mm^2, weight of unit length is q_0=27.0N/m, elastic module E=90GPa. The cableway set up for testing is two span with single track rope. Testing data are complete in the first span of the cableway. The horizontal span of first span is l=70.9m, height difference h=15.2m. This paper selects the testing data with deflection coefficient $S_f=f/l$=0.05 for comparison and validation.

Table 1. Testing Comparison and Validation

Comparison items	Test value	Calcula-tion value	Error %

No-load	Drawing force of lower support point /N		5000	4911	-1.780
	Torsion at 0.3*l* location /m		3.077	3.060	-0.552
	Torsion at 0.5*l* location /m		3.475	3.544	1.986
	Torsion at 0.8*l* location /m		2.105	2.292	8.884
On load	Load at 0.3*l* location	Drawing force of lower support point /N	23500	27002	14.902
		Torsion of loading point /m	4.097	3.881	-5.272
	Load at 0.5*l* location	Drawing force of lower support point /N	25300	28812	13.881
		Torsion of loading point /m	4.555	4.292	-5.774
	Load at 0.8*l* location	Drawing force of lower support point /N	20500	22625	10.366
		Torsion of loading point /m	3.570	3.380	-5.322

Seen from comparison and validation data in Table 1, calculated drawing force of the lower support point of the track rope and deflection value at different position are almost same as testing value. When there is centralized load, drawing force of the lower support point of the track rope obtained through calculation is greater than testing value, deflection value at different position is smaller than testing value. Friction force generated between the saddle and the track rope at no-load belongs to static friction force, no displacement occurs in the track rope. When the cableway is loaded, friction force generated between the saddle and the track rope is insufficient to overcome tension difference between two sides of the saddle. The track rope moves, the sag increases, and tension reduces, corresponding deflection at different position increases.

The reference [8] calculates the large sag single span cableway with the nonlinear finite element method in the two node ropes unit with Euler description. The horizontal span of the cableway l=305m, height difference is 0, and the middle deflection coefficient S_f=0.1. The elastic module of the rope is E=130GPa, the section area is A_0=550mm^2, centralized load acting on C point 122m away from the end point is P=35.6kN.

Table 2. Value Calculation Comparison and Validation

Comparison item		Value in reference document	Value in text	Error %
No load	Horizontal tension /kN	17.728	17.955	1.280
	Sag at *C* point /m	29.923	29.296	-2.095
On load	Displacement of *C* point after load acts /m	5.64	5.56	1.418

Seen from Table 2, the calculation value in this paper after and before acting of centralized load are basically same as the results in the reference documents, it means the calculation method in this paper has extensive adaptability.

5 Conclusions

The track rope is an important weight bearing member in the cargo cableway, its shape and tension calculation relationship is directly related to speciation and model selection of the track rope, etc. This paper establishes the balance equations of the elastic track rope under the multiple centralized loads based on the catenary equation of the elastic track rope. The formation of equations is brief and the structure is clear. The Newton iteration method is applied to calculate the equations, in which calculation quantity is small, and accuracy of result is high. Through comparison with result data in other reference documents, reliability of the calculation method in this paper is validated, which can meet with requirement of design and engineering safety in the cargo cableway.

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