MATEC Web of Conferences **82**, 01007 (2016) DOI: 10.1051/matecconf/20168201007
D2ME 2016

Vibration Analysis of Suspension Cable with Attached Masses by Non-linear Spline Function Method

Jian Qin[a], Liang Qiao, Jun Chen, Jiancheng Wan and Yongjun Xia

China Electric Power Research Institute, 100055, Beijing, China

Abstract. The nonlinear strain and stress expressions of suspension cable are established from the basic condition of suspension structure on the Lagrange coordinates and the equilibrium equation of the suspension structure is obtained. The dynamics equations of motion of the suspended cable with attached masses are proposed according to the virtual work principle. Using the spline function as interpolation functions of displacement and spatial position, the spline function method of dynamics equation of suspension cable is formed in which the stiffness matrix is expressed by spline function, and the solution method of stiffness matrix, matrix assembly method based on spline integral, is put forwards which can save cost time efficiency. The vibration frequency of the suspension cable is calculated with different attached masses, which provides theoretical basis for valuing of safety coefficient of the bearing cable of the cableway.

1 Preface

Cable structures play an important role in many engineering fields, such as civil, bridges, forestry and electrical engineering, and their application is more and more wide with different structural variety[1,2]. Therefore, it is of interest to investigate the dynamics of suspended cables and cable structures. In recent years, numerous journal articles have addressed the nonlinear dynamics of suspended cables. In addition, some studies have focused on the vibration analysis of suspended cables with attached masses, which make the singularities in the deformed cable profile[3].

On the other hand, the moving mass problem of distributed parameter systems is a very important topic in structural dynamics, and the response of structures subjected to a moving mass is of considerable practical importance in different engineering fields[4-6].

The selection of the displacement function is key for calculation method[7-8]. For the one dimensional cable structure, the spline function can be used as efficient interpolation function which has strong continuity and high accuracy[9-11].

In this study, the dynamic model of the suspended cable with attached masses is developed based on the virtual work principle. The stiffness matrix composed by spline function for the cable structure is obtained.

2 Continuum model of suspension cable

[a] Corresponding author: qinjian@epri.sgcc.com.cn

2.1 The strain of cable

There is a suspension element under fixed coordinate system, the starting point is X, the endpoint is $X+dX$, the length of the element is ds. Here $ds^2=(X+dX-X)^2=dX^2$. Under the action of external force, the infinitesimal element moves to the new position, the starting point becomes $x=X+dU$, the endpoint becomes $x+dX+dU$, and the length of element becomes ds^*, $ds^{*2}=(x+dX+dU-x)^2=(dX+dU)^2$. The process is shown as Figure 1.

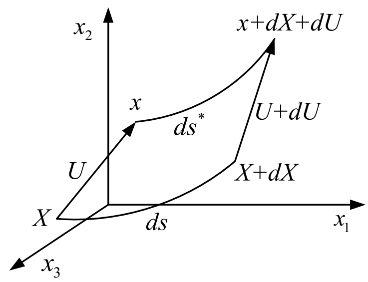

Figure 1. The Infinitesimal Element of Cable.

The transformation from ds to dx can be expressed as

$$dx=\frac{dx}{ds}ds=\frac{d(X+U)}{ds}ds=(\frac{dX}{ds}+\frac{dU}{ds})ds \tag{1}$$

The length changes of infinitesimal element is expressed using the initial coordinates s, i.e., Lagrange coordinates as

$$ds^{*2}-ds^2=(dX+dU)^2-dX^2=2(dXdU+\frac{1}{2}dUdU)=2(\frac{dX}{ds}\frac{dU}{ds}+\frac{1}{2}\frac{dU}{ds}\frac{dU}{ds})dsds \tag{2}$$

So the Green strain of the suspension cable is

$$\varepsilon=\frac{1}{2}\frac{ds^{*2}-ds^2}{ds^2}=\frac{dX}{ds}\frac{dU}{ds}+\frac{1}{2}\frac{dU}{ds}\frac{dU}{ds} \tag{3}$$

2.2 Kirchhoff stress of suspension cable

The tension T^* in the current configuration (ds^*) is the true force, which can be seen as the Euler stress tensor (also known as the Cauchy stress tensor).

The Green strain is in the initial configuration (ds), so the stress tensor should be consistent. We move T^* to the endpoint of initial configuration, and transform the direction of T^* according to the coordinates of the endpoint. Then the Kirchhoff stress T in the initial configuration is obtained.

$$T=(\frac{dX}{ds}+\frac{dU}{ds})T^* \tag{4}$$

2.3 The equilibrium equation of suspension cable

The equilibrium equation of the suspension structure is established in the initial configuration.

$$\frac{dT}{ds}+\mathbf{f}=\frac{d}{ds}\left[(\frac{dX}{ds}+\frac{dU}{ds})T^*\right]+\mathbf{f}=0 \tag{5}$$

Here **f** is volume force.

3 Nonlinear dynamic analysis of suspension cable

3.1 The dynamic equations

For dynamic issue of the suspension cable, its equilibrium equation is to add inertial force and damping force on basis of the static equation, so the dynamic equilibrium equation of the suspension cable in the initial configuration is

$$\frac{d}{ds}\left[\left(\frac{dX}{ds}+\frac{dU}{ds}\right)T^{*}\right]+\mathbf{f}=\rho\ddot{U}+\mu\dot{U} \tag{6}$$

And the boundary condition of displacement is

$$U\big|_{S_u}=U_u \tag{7}$$

Initial conditions are

$$U(X,0)=U(X)\,,\ \dot{U}(X,0)=\dot{U}(X) \tag{8}$$

In which, **f** is volume force, ρ is line density of the suspension cable, μ is damping coefficient. $\dot{U}$ and $\ddot{U}$ are first order derivative and second order derivative of U to t, i.e., represent speed and acceleration respectively. Inertial force and damping force in the balance equation are one of the basic characteristics for difference between the elastic dynamic and static force.

The displacement U, strain ε and stress σ are all functions of time. Therefore fixed solution condition of the dynamic issue also includes initial conditions.

It must note the attached masses on the suspension cable are also move synchronously with vibration of the suspension cable. Therefore for dynamic issue, iteration force generated by the attached masses shall be considered in vibration analysis of the suspension cable.

When the attached mass is m_p, action force between the suspension cable and the attached masses is

$$\mathbf{f}=\begin{bmatrix}0\\ m_p g\end{bmatrix}-m_p\ddot{U}_p=m_pG-m_p\ddot{U}_p \tag{9}$$

In which, U_p is displacement vector of the suspension cable at location of the attached masses, G is gravity acceleration vector.

The former formula is placed in the dynamic equation of the suspension cable, and then

$$\frac{d}{ds}\left[\left(\frac{dX}{ds}+\frac{dU}{ds}\right)T^{*}\right]+m_pG=\rho\ddot{U}+m_p\ddot{U}_p+\mu\dot{U} \tag{10}$$

3.2 Spline function method

The cubic B spline function is a class of piecewise continuous function, and the function space is constructed by the B spline functions at difference nodes such as ϕ_{-1}, ϕ_0, …, ϕ_{n+1} (shown in Figure 2).

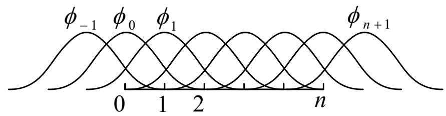

Figure 2. The Base of Function Space.

So the displacement function in the interval [0, s_0] can be interpolated by space base and expressed as the product of spline function and coefficient.

$$U(s,t)=\begin{bmatrix}u_1(s,t)\\u_2(s,t)\\u_3(s,t)\end{bmatrix}=\begin{bmatrix}\Phi(s)B_1(t)\\\Phi(s)B_2(t)\\\Phi(s)B_3(t)\end{bmatrix}=\begin{bmatrix}\Phi & & \\ & \Phi & \\ & & \Phi\end{bmatrix}\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}=N(s)\cdot B(t) \tag{11}$$

In the equation, the spline function is $\Phi=[\phi_{-1}(\xi),\phi_0(\xi),\cdots,\phi_n(\xi),\phi_{n+1}(\xi)]$, A is the position coefficient vector of every point of the initial suspension cable, and B is the displacement vector.

Let $C=\left[\dfrac{dN^T}{ds}\cdot\dfrac{dN}{ds}\right]$, and then

$$\varepsilon(s,t)=A^TCB+\frac{1}{2}B^TCB=(A+\frac{1}{2}B)^TCB \tag{12}$$

Considering the suspension cable is elastic and follow Hooke’s law, tangential force of the suspension cable can be expressed as

$$T=A_0\sigma=EA_0\varepsilon=EA_0(A+\frac{1}{2}B)^TCB \tag{13}$$

Here E and A_0 denote the modulus of elasticity and the cross-sectional area of the cable, respectively.

3.3 Vibration analysis of suspension cable

If displacement U of the elastic cable meets with the geometrical equation and the displacement boundary condition, the integral equation is then obtained according to the virtual work principle in the initial configuration s_0

$$\int_{s_0}(\delta\varepsilon\cdot T^*+\rho\ddot{U}\delta U+m_p\ddot{U}_p\delta U+\mu\dot{U}\delta U-m_pG\delta U)ds=0 \tag{14}$$

The δU is arbitrary, so the motion equation is shown as following:

$$M\ddot{B}(t)+C_t\dot{B}(t)+KB(t)=Q(t) \tag{15}$$

In which M, C_t, K and Q are mass matrix, damp matrix, rigidity matrix and load vector of the system respectively.

$$M=\rho\int_{s_0}N^TNds_0+\sum_k m_kN^T(s_k)N(s_k) \tag{16}$$

$$C_t=\mu\int_{s_0}N^TNds_0 \tag{17}$$

$$K=EA_0\int_{s_0}[C\left(A+B\right)(A+\frac{1}{2}B)^TC]ds_0 \tag{18}$$

$$Q = \mathbf{q}\int_{s_0} N^T ds_0 + \sum_k m_k g N^T(s_k) \tag{19}$$

Where **q** is the gravity vector of unit length.

Considering vibration occurs on foundation of the static force balance configuration, increment equation of static force balance shall be analysed when vibration issue of the system is analysed.

So the variation of equation become

$$M\delta\ddot{B}(t) + C_t\delta\dot{B}(t) + (K + K_\sigma)\cdot\delta B(t) = 0 \tag{20}$$

Here

$$K_\sigma = EA_0\int_{s_0}[(A+\frac{1}{2}B)^T CB\cdot C + \frac{1}{2}C\left(A+B\right)B^T C]ds_0 \tag{21}$$

Therefore the free vibration equation of the suspension cable without damping is

$$M\delta\ddot{B}(t) + (K + K_\sigma)\cdot\delta B(t) = 0 \tag{22}$$

Set solution of the vibration equation as $B(t) = \phi\sin\omega t$, and substitute it into the above equation, the generalized eigenvalue problem is obtained:

$$[(K + K_\sigma) - \omega^2 M]\phi = 0 \tag{23}$$

Solve the above equation and obtain eigenvalues and eigenvectors of the suspension cable with attached masses, and then obtain the inherent vibration model.

4 Example

The span of one suspension cable is 400m, height difference between the starting point and the end point is -50m. Initial length of the suspension cable is 412m, unit weight is 30N/m, elastic module *E*=200e9Pa, radius of cable is 12mm. The suspension cable is evenly divided into 40 units.

The nonlinear spline element method is applied to calculate inherent vibration model of the suspension cable, and calculation result is shown as Figure 3. The two kinds of mass conditions are 1000kg and (1000 kg, 1000kg).

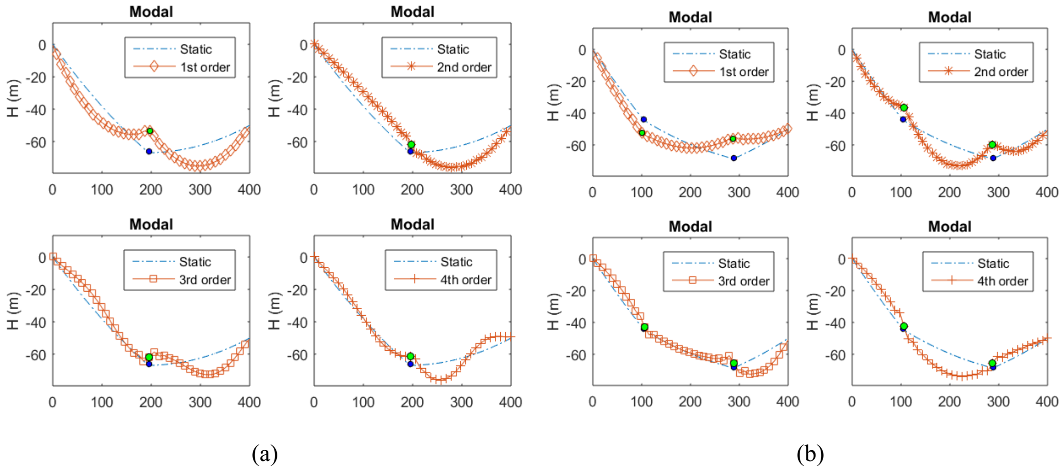

Figure 3. The 1-4th Order Modes with Different Attached Masses

5 Summary

In this paper, the spline function is applied to the vibration analysis of the suspension cable. Based on the principle of virtual work, the formula of the dynamic calculation of the suspension cable with attached masses is proposed.

It can be seen from the calculation results that the attached masses has a great influence on the mode. In the positions of the attached masses, the suspension cable has obvious turning points, and the mass at the lowest end has the largest amplitude and the maximum frequency change. Through the example, the effect of the attached masses to the cable can be used to guide the transportation of cable structures in engineering.

References

1. Okada A. Advanced Structural Design for Cable Structure[J]. Journal of Architecture & Building Science, 2014, **129**
2. Li S, Zhou M, Yang Y. Broken wires diagnosis method numerical simulation based on smart cable structure[J]. *Photonic Sensors*, 2014, **4(4)**:366-372
3. Gavrilov, S.. Non-stationary problems in dynamics of a string on an elastic foundation subjected to a moving load[J]. Journal of Sound and Vibration, 1999, **222(1)**, 345–361
4. Al-Qassab, M., Nair, S.. Wavelet-Galerkin method for the free vibrations of an elastic cable carrying an attached mass[J]. Journal of Sound and Vibration, 2004, **270 (1–2)**, 191–206
5. Cheng, S.P., Perkins, N.C.. Theoretical and experimental analysis of the forced response of sagged cable/mass suspensions[J]. Journal of Applied Mechanics, 1994, **61**, 944–948
6. Di Tomasso, F., Di Egidio, A., Luongo, A., 2003. Modelling and dynamic analysis of strings carrying a single mass. In: Proc. of 16th AIMETA Congress, CD-Rom, Ferrara, Italy (in Italian).
7. Zhang Le-le, Tan Nan-lin, Liu Cai. Simulation and Application of Spline Finite Strip Method in Process of Cold-roll Forming[J]. Journal of System Simulation, 2008, **20(7)**:1703-1706
8. G. Eccher, K.J.R. Rasmussena, R. Zandonini. Geometric nonlinear isoparametric spline finite strip analysis of perforated thin-walled structures[J]. Thin-Walled Structures, 2009, **47**: 219– 232
9. Shen Pengcheng, He Peixiang. The Developments of Spline Finite Element Method in Computational Mechanics[J]. *ADVANCES IN MECHANICS*, 2000, **30(2)**:191-199
10. Qin Rong, Spline function method and program for structural mechanics[M]. Beijing: Science Press, 2015
11. QIN Jian, HUANG Ke-fu. B-Spline Finite Element Method for the Plane Problem of Arbitrary Region[J]. *Engineering Mechanics*, 2010(6):29-34