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Application of Multivariate Adaptive Regression Splines to Sheet Metal Bending Process for Springback Compensation

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Abstract. An intelligent regression technique is applied for sheet metal bending processes to improve bending performance. This study is a part of another extensive study, automated sheet bending assistance for press brakes. Data related to material properties of sheet metal is collected in an online manner and fed to an intelligent system for determining the most accurate punch displacement without any offline iteration or calibration. The overall system aims to reduce the production time while increasing the performance of press brakes.

1 Introduction

Mechanical systems such as press brakes and punches are widely used production machines in the industry. Especially, press brakes performing sheet metal forming are one of the major production machines providing a variety of basic parts to several industries. Sheet metal forming is a very effective and highly developed process for high output production situations. Though, the process itself has some impediments and these impediments would increase the production time, which in return would increase the cost of the process in today's competitive world of production.

Press brakes, like the several other mechanical systems, are composed of mechanical systems, hydraulic systems and electrical control systems. Furthermore, these subsystems are also composed of primitive components. For proper functioning, every component should be configured and the harmony of these individual components in overall system should be maintained as well. One of the crucial components of these systems is the material that is being processed on them. For press brakes, processed material is the sheet metal being bent. Sheet metal processed on press brakes actually completes the overall mechanical model of the press brake and affects the performance and functioning of the press brake in every manner. Correct evaluation of material behavior is the utmost important aspect to consider for obtaining suitable metal-forming analysis, simulations and results [1]. Moreover, behavior of the material affects the quality and pace of the production drastically. It is due to the fact that, during the production process some iterations or calibrations would be required to be performed for different types of sheet metals (even the different batches of the same material) in order to get the correct bending result and this would increase the production time. In other words, physical properties of even the different batches of the same material would deviate from the nominal values, and for every different batch of material, a production calibration becomes necessary. Therefore, the cost of the production would increase as well as the time spent on the production.

To sum up, the physical properties of the raw material used in press brakes affects the overall system significantly and the automatisation of the whole bending process with compensating the effects of different material properties of the raw material would be the key purpose for increasing the efficiency and speed of bending process.

2 Background

Press brake system can be described as a servo, electro, hydro, mechanical system. The desired target is the final bending angle of a sheet metal processed by the press brake. Therefore, the input of the system is the desired angle. This desired bending reference is taken by the press brake system and a displacement command is generated by the system and this command is executed on sheet metal. The result is the bended material. However, in this system there is no feedback that would be taken and fed to the system. The overall described system is depicted in Figure 1.

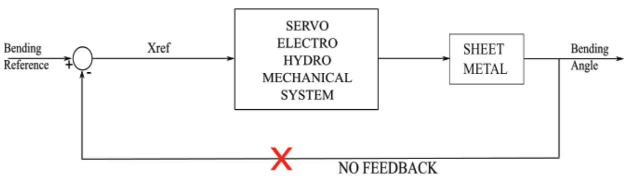

Figure 1. Overview of Sheet Metal Bending in Press Brake

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The adverse effects that would deteriorate the quality and accuracy of bending should be predicted beforehand and a feedforward mechanism or module (see Figure 2) should be added to the system. The adverse effects would stem from the change of material properties, elastic and plastic behaviors of the materials as well as the tooling and the body. Moreover, the changes in environmental conditions would also affect the performance of the press brake. In other words, quality analytical or black box models should be generated for each adverse effect or for their overall effect and the bending reference should be regulated according to these models before feeding to the press brake system.

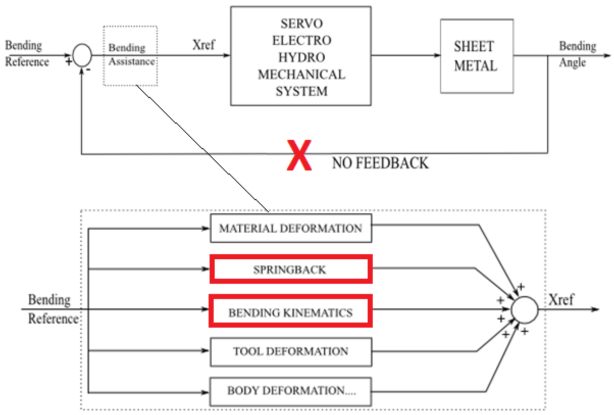

Figure 2. Sheet Metal Bending Assistance

2.1 Adverse Effects on Accurate Bending

There are several factors affecting the bending process. As a result of these effects, the shape of bended sheet metal deviates from the desired final shape. Understanding all of the factors truly has a great importance for this study, since the aim of this study involves reducing these effects so far as possible to enhance the bending performance. Predicting or evaluating the adverse effects, which deteriorates the quality and accuracy of bending, can create an opportunity for us to add a feedforward mechanism (see Figure 2) or module to the system enhancing the performance. The adverse effects can be collected under three titles, namely material properties, springback effect and other adverse effects interfering an accurate bending.

Material properties are intensely interfering the quality and accuracy of the bending. The input parameters for bending has to be based on values provided by the steel manufacturers, or values from standards or material certificates. However, these values are never exact, but reflect ranges that describe between which boundaries a certain parameter can vary. This undoubtedly presents a large uncertainty in the input of the bending model, which also has a profound influence on both parameters like the calculated punch displacement and the calculated sheet length correction [6].

One of the physical properties affecting bending performance is modulus of elasticity. In the initial stage of the bending process, the forces imposed by the punch are too small to cause plastic deformation in the sheet. Consequently, no strain results in the sheet if the punch force is removed. The extent to which the sheet deforms under influence of the small punch force is related to the modulus of elasticity of the sheet material [6]. Obviously, the modulus of elasticity influences the required punch displacement, as the ratio between elastic and plastic deformation is an important indication for this. Evidently this has large influence on the springback of a sheet.
Another important physical property is yield stress. The transition point between elastic and plastic deformation is related to the yield stress of the material. With increasing deformation, certain regions in the sheet located at the surface of the sheet are subject to stresses that exceed the yield stress. In these regions, the plastic deformation occurs first, and the applicable material model changes from elastic to plastic behavior. The value of yield stress is crucial for bending process.

In a piece of material, a number of material properties can alter in value, depending on the direction in which this property is determined. This phenomenon, known as anisotropy, can only be manifested by more-dimensional properties. A one-dimensional property (e.g. the density of a material), being a scalar value, merely assigns a value to a certain point in the material [8]. This does not imply the property to be a constant within the specimen, but only that the property is direction-independent. During the sheet forming operation, the metal crystals are enforced a certain prestraining. Due to this, the material shows an orientation of the crystals called rolling texture. This crystal orientation causes the varying results in the tensile test. Therefore, it is clear that the effects of anisotropy are important for sheet metal bending.

The Ludwik–Nadai relation is used to describe the plastic material behavior and it is also a crucial parameter for bending process. The advantage of this model is the non-linear description of work hardening. Plastic material behavior is important for bending.

Bauschinger effect is a physical property of material that also affects bending process. Not only the bending direction in a sheet is important in tensile experiments, the direction of the force that is applied to the test specimen plays an important role as well. This effect signifies that the behavior of a test specimen under pressure differs from a specimen in tension. The Bauschinger effect is an anisotropic material property and it is affecting the quality of bending [11].

Friction is also an important aspect of the bending process, which cannot be related to sheer material behavior is the friction. During the process, friction forces arise at two locations: the contact between sheet and punch and between sheet and die. From a kinematic point of view, the first friction force is very difficult to prove. The friction between the sheet and the die is often modeled according to the friction force by Coulomb friction and it is affecting the bending process.

Another vital phenomenon affecting the quality of a bending process is springback effect. For predicting the final geometry of produced sheet metal part as well as the determination of value for springback is an important feature. A simple description of springback is that after the punch removal, generated dimensional change is because of material elastic recovery [15].

The amount of springback is related to the material thickness, material type, bending angle, bending direction, etc. as presented in lots of the studies [15]-[21]. Moreover there are several quality springback models for compensation the springback. Some of these are empirical models and some of them are parametric models.

In this study, the compensation for the springback effect would be studied.

2.2 Geometric Interpretation of Three Point Bending

Bending kinematics of air bending (three point bending / V-bending) will be discussed with the assumption of no slippage of the sheet on the press brake V-die. Bending kinematics is a geometric calculation that provides information for the preliminary value of punch displacement regarding the material and punch are rigid and it is independent of the type and physical properties of the material used in the bending process.

First of all, in Figure 3, a sheet metal while being bended by press brake punch is depicted with important features presented. Nomenclature for this figure is listed in Table 1.

Table 1. Nomenclature for Bending Kinematics.

Symbol	Description
V	width of V-opening of the die
R_p	tip radius of the punch
R_d	radius of the die sides
t	thickness of the sheet metal
β	angle that supplements the bending angle
z	needed punch penetration to bend the sheet metal to (180-ß) degrees

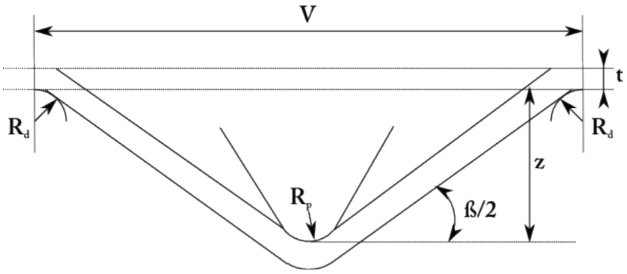

Figure 3. Bending Operation on a Punch-Die Configuration.

In several studies, bending kinematics calculation of V-bending is presented [22]-[25]. In literature, there are several versions for punch displacement equation that generates the same results. The geometrical result is presented in equations (1)-(3). The formulation takes die and punch types, bending angle and sheet metal thickness as inputs and returns the necessary punch displacement without taking material type of the sheet metal into consideration.

$$\mathbf{z} = \frac{V}{2}\tan\left(\frac{\beta}{2}\right) + \left[1 - \cos\left(\frac{\beta}{2}\right) - \tan\left(\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)\right]\left(R_d + R_p + t\right) \quad (1)$$

$$\mathbf{z} = \frac{V}{2}\tan\left(\frac{\beta}{2}\right) + \left[\frac{\cos\left(\frac{\beta}{2}\right) - \cos^2\left(\frac{\beta}{2}\right) - \sin^2\left(\frac{\beta}{2}\right)}{\cos\left(\frac{\beta}{2}\right)}\right]\left(R_d + R_p + t\right) \quad (2)$$

$$\mathbf{z} = \frac{V}{2}\tan\left(\frac{\beta}{2}\right) + \left[1 - \sec\left(\frac{\beta}{2}\right)\right]\left(R_d + R_p + t\right) \quad (3)$$

This formulation is a starting point for searching the correct and accurate punch penetration value. One should take this result as a starting point or initial value for searching the correct punch displacement. Therefore any automation system exploring the true punch displacement for a given bending angle should take this formulation as an input. However, it should be noted that, this formulation does not involve any information on material properties of bended sheet metal.

3 Data Collection and Evaluation

Improvement of bending process on press brake requires a thorough observation of the process on the system, extensive collection of data during the process and good evaluation of the collected data. During the studies on bending process, a hydraulic press brake was used and for several different configurations, bending data had been collected for good understanding of the bending practice. In this section, data collected on the press brake is presented as well as the evaluation and preprocessing of data before feeding the intelligent system.

3.1 Data Collected on Press Brake

During the studies on press brake system, for several configurations, bending data had been collected. This data is categorized in Table 2. Data is collected for 90 different bending angles at 4 different die configurations and for 3 different material thicknesses. This adds up to 90 × 4 × 3 = 1080 data points. For these 1080 different data points, punch displacement after material touch with 1 µm resolution, cylinder chamber pressures of the press brake and the final bent angle of the materials with 0.1 degrees resolution are collected.

Table 2. Data Collected on Press Brake

Bending Angle	Die Config. (Die Width/Die Radius [mm/mm]	Material Thickness [mm]	Collected Data
1 to 90 degrees (with 1 degree res.)	16/2	1	Punch Displacement [mm]
	22/2.5	1.5	
	35/3	3	Cylinder Pressures [bar]
	50/4	-	Bent Angles [deg]

Punch Displacements and the bent angles are used for the construction of a truth table for the intelligent system. The use of this truth table is described in the following sections. Cylinder pressure of the press brake are used for calculating the force transmitted to the material during bending operation. It is easier and cheaper to place a pressure transducer to cylinders when compared to the use of a sensor or strain gauge to be placed within die or press brake body. Therefore the usability of the noisy data of cylinder pressures is tested in this study. A sample data on force transmitted to material during bending phase is depicted on Figure 4. As it can be seen from the figure, force versus displacement data is similar to strain-stress curve which contains lots of information about physical properties of materials. This data is also used for feeding the intelligent system as well as the other available data.

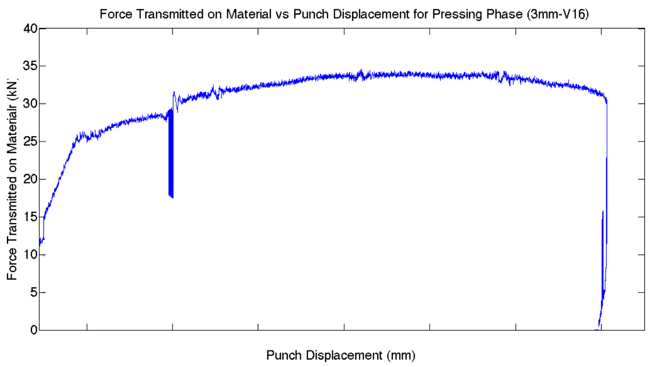

Figure 4. A Sample Force Transmitted on Material vs Punch Displacement Data extracted from Cylinder Pressure Data Collected on Press Brake

3.2 Evaluation of Collected Data

Using the bent angle and punch displacement data collected, a truth table for punch displacement for the correct bending angle had been constructed for 1080 data points. Throughout this paper, mentioned bent angle versus punch displacement table is used as the truth table.

For understanding the behavior of the material during bending, the geometrical interpretation of the punch displacement mentioned in section 2.2 is compared with this truth table. This comparison gave an insight for the rest of the study presented in this paper. Geometrical relation mentioned before lacks material information. However, on the other side, truth table constructed using the data collected has the material information that the former one lacks. The difference between these two relations constructs the punch displacement error results in the deviation between the bending angle and bent angle, in other words, the springback effect. The error between these two relations are given in Figures 5 to 7 for different material thicknesses, for different die configurations and depicted as a function of bending angle.

As the given figures are examined thoroughly, it gives us several good features to be used. These features are given below.

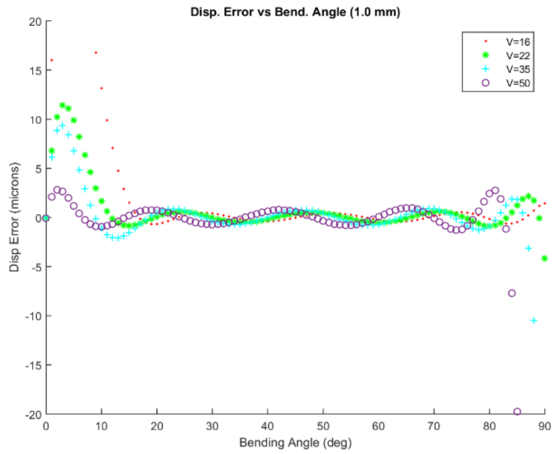

Figure 5. Punch displacement error for material with 1 mm thickness

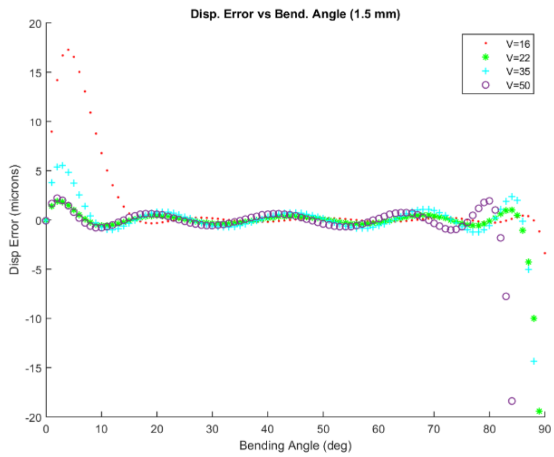

Figure 6. Punch displacement error for material with 1.5 mm thickness

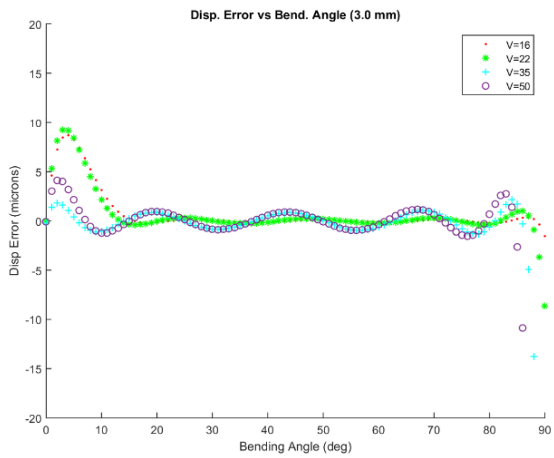

Figure 7. Punch displacement error for material with 3 mm thickness

Displacement error between the geometric relation and the truth table constructed

- increases as the material gets thinner
- increases as the die width gets smaller
- is large for small bending angles as well as large bending angles.

Moreover, as the figures are examined it can be seen that the displacement error resembles a *half sinc* function for both ends of the figures. Using all of this information, it is conceivable that a good function fitting (avoiding overfitting) would be a solution possibility for compensation of springback effect This brings the study to the research of intelligent fitting techniques, and the technique selected is presented at Chapter 4.

3.3 Preprocessing of Collected Data for Feeding the Intelligent System

The data collected on the press brake is preprocessed and constructed with a specific structure before being fed to the intelligent system. Preprocessed input data is composed of 7 different variables constructed from 9 different variables using some normalizations. Preprocessed data is listed in Table 3.

Table 3. Preprocessing of Collected Data

Collected Data	Unit	Normalization →	Processed Input	Unit
θ	deg	θ/90(pi/180)	θn	-
t	mm	t/t_{max}	tn	-
TPD	mm	TPD/TPD_{max}	TPDn	-
V	mm	$V/R/D_{max}$	D	-
R	mm			
F_1	N	$(F_2-F_1)/F_1$	MP_1	-
F_2	N	$(F_3-F_2)/F_2$	MP_2	-
F_3	N	$(F_4-F_3)/F_3$	MP_3[a]	-
F_4	N			

The unprocessed variables are bending angle, material thickness, theoretical punch displacement value, die width, die radius, and four different force values collected on specific four different punch displacement points. All of the inputs are converted into unitless forms in a suitable meaningful way in order to improve convergence ability of the data. The bending angle data is converted into radians. Die widths and die radii are specific to each other for a specific type of die, therefore their ratio will be fed to the system. Material thicknesses normalized to 5mm. For this study, materials with thicker than 5 mm is left beyond the scope. The theoretical punch displacement value is normalized by dividing the value maximum possible theoretical punch displacement value.

For adapting material information force reactions on material is retrieved for 4 points during pressing. These points [P1, P2, P3, P4] are decided after the examination of force versus punch displacement data collected on press brake. Most of the bending results show that the transition between elastic to plastic region of material is reached around 10 degrees of bending. In other words, if data until the end of the elastic region would be covered, this data would represent the elastic features of the material. For that reason, for 4 points until this transition point force data will be collected and will be fed to system. Last of these points are selected as the theoretical punch displacement value for 10 degrees of bending and this punch displacement point is marked as P4, the other 3 points are calculated such that the elastic region is divided into 3 parts equally. Force transmitted to material at these points are taken as input to the intelligent system using the normalization given in the table.

As mentioned before, it all of the preprocessed data has no units and normalized. This feature will allow the model to converge easier to a specific solution since the order of magnitude between variables are removed.

4 Proposed Method

Several neural network and regression techniques are examined for the compensation of the springback effect using priori information. The comparison of these techniques are subject of another paper to be presented. Among several techniques, one of the most successful results acquired by the use of Multivariate Adaptive Regression Splines technique. This technique is suitable for the presented problem, since, as mentioned before, the error of the output of the geometric relation and the truth values are in shape of a common function and easily can be fitted. In this chapter, a brief introduction to the technique and the application of the technique to bending problem is presented.

4.1 Introduction of Multivariate Adaptive Regression Splines

Multivariate Adaptive Regression Splines is a method for nonparametric approximating and inferring general functions of a high-dimensional argument noisy data. The basic assumption of the method is that the function to be estimated is locally smooth [28-31]. Multivariate Adaptive Regression Splines is a faster method in terms of training times compared to that of Feedforward Neural Networks.

Multivariate Adaptive Regression Splines algorithm uses two sided truncated functions of predictors (x) as basis functions and the models built out of these basis functions. Representation of the basis function is given in equation (4). The generalized expression of the method using the mentioned basis function is given in equation (5).

[a] MP : Material Property

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$$(\mathbf{x}-\mathbf{t})_+ = \begin{Bmatrix} x-t & x>t \\ 0 & otherwise \end{Bmatrix} \quad (4)$$

$$y = f(x) = \beta_0 + \sum_{m=1}^{M} \beta_m H_{km}(x_{v(k,m)}) \quad (5)$$

In Equation (5) it can be seen that the summation is over the M terms (number of basis functions) in the model, and β_o, β_m and t (knot values in equation (4)) are all parameters of the model. These parameters are estimated using the training data. Function H is represented as in Equation (6).

$$H_{km}(x_{v(k,m)}) = \prod_{k=1}^{K} h_{km} \quad (6)$$

In Equation (6), $x_v(k,m)$ is the predictor in the k^{th} of the m^{th} product. If K is selected as 1, the model becomes additive, if K is selected as 2, the model becomes pairwise interative.

There are two main phases during the application of the methods. These are forward stepwise procedure and backward pruning procedure. During forward stepwise procedure, several basis functions are added to the model according to a pre-determined number. The forward procedure is a brute one and at the end of the procedure, the model would overfit to data. To avoid this, backward pruning procedure is executed.

A backward pruning procedure is applied in which the model is pruned by removing those basis functions that are associated with the smallest increase in the goodness-of-fit. For measuring the goodness of fit, relation given in the equation (7), namely Generalized Cross Validation, is used. The given relation measures the goodness of fit as well as the complexity of the model.

$$GCV = \frac{\sum_{i=1}^{N}(y_i - f(x_i))}{\left(1-\frac{C}{N}\right)^2}, \quad C = 1 + cd \quad (7)$$

N is the number of cases in the data set (number of training samples) and d is the number of the number of independent basis functions, namely effective degrees of freedom. c is the penalty for adding a basis function. Experiments have shown that the best value for C can be found somewhere in the range 2 < d < 3.

4.2 Application of Multivariate Adaptive Regression Splines to Bending

Using the above basic equations, and "*earth-py*" [32] python library Multivariate Adaptive Regression Splines Method is applied to bending problem.

Regression is performed on data input presented in Table 3 and data output corresponding truth value constructed. Total number of data points available is 1080 as mentioned before. Several training data set sizes and validation data set sizes are used for the application of the method. The number of data points in the different training data sets can be seen in Table 4. Validation data set sizes for these training sets are 80 for the first set and 100 for the rest of the sets.

To validate the performance of the model, for each training session, training set and validation set are selected randomly among 1080 data points for 1000 times, model is trained 1000 times for these selected data set couple and validated. The results are presented in the next chapter.

5 Results

Multivariate Adaptive Regression Splines Techniques has been applied for 12 different training sessions. 12 different training and validation data set sizes are tried during these sessions. For each session, training and data evaluation sets are selected randomly for 1000 times and training is repeated for 1000 times for each session. A sample training validation result for 200 training data points and 100 validation data points is presented in Figure 8. In this figure, 1000 different colors represent 1000 different training model and as it can be seen from the figure, the maximum displacement error of the trained model is around 70 nanometers.

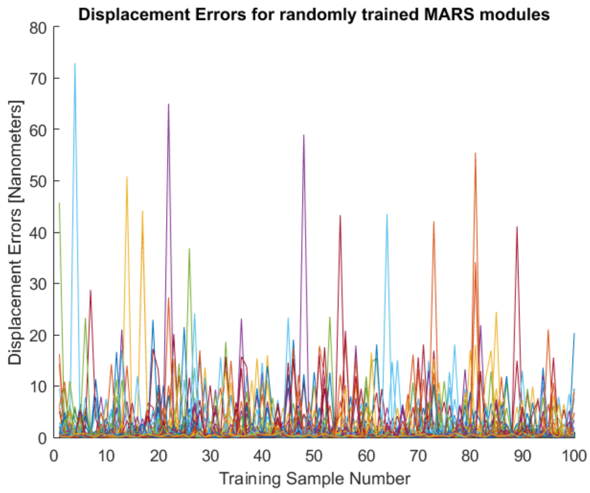

Figure 8. Displacement Errors for Randomly Trained MARS Modules (200 training samples)

For the comparison of the training performances, for each training sessions, mean square error (MSE) and maximum error of the trained models have been calculated. Among 1000 trained models, the maximum value of these two parameters are selected as the performance index of that training session.

As the performance index table is examined, it can be seen that the mean squared error of the models trained decreased gradually as the number of training data size increases. It should also be noted that, maximum error decreases for training sets with size greater than 500 data points; however it seems stable for training sets with size smaller than 500 data points.

It should also be noted that, the punch displacement errors depicted in results table in units of nanometers. Most of the press brakes can provide position control accuracy around 1 micrometer minimum, therefore the output of all of the trained models are acceptable.

It can be concluded that the selected identifying data inputs are selected so that it well covers the data space and using Multivariate Adaptive Regression Splines technique the bending procedure can be defined with success.

Table 4. Mean Squared Error and Maximum Error values for different Training Sample (TS) Numbers.

# of TS	MSE (nm^2)	Max Error (nm)	# of TS	MSE (nm^2)	Max Error (nm)
1000	10.58	27.11	180	112.70	73.70
800	26.85	43.22	160	93.28	74.01
500	60.99	73.04	140	94.90	73.84
300	77.35	72.76	120	87.77	73.88
280	99.49	73.43	100	91.59	74.87
260	96.89	73.14	50	118.43	74.04
240	85.57	73.23	30	164.10	74.63
220	78.01	72.99	20	183.04	74.23
200	85.13	73.35	10	251.55	74.28

6 Conclusion

In this study, using the data collected on a real press brake and by the help of an intelligent regression technique, namely Multivariate Adaptive Regression Splines, a bending assistance model for one type of material of different thicknesses has been generated. Model is generated using different sizes of data and for each case, it has presented satisfactory results.

7 Future Work

In this study, improvement of bending procedure for compensation of springback is achieved for one type of material with different thicknesses using an intelligent regression technique. In the future, the model developed in this study is planned to be extended into a model working for more than one type of material.

Moreover, several studies has also been carried out for compensation of adverse effects like springback in sheet metal bending. In another study, an extrapolating model is being developed, in which the material information is extracted at the early phase of bending and estimates the required punch displacement without a priori information.

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