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Research of Influence of Small Asymmetries Construction on the Dynamics Motion of Space Landing Vehicle with Considering Resonance

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Abstract. This article provides an analytical methodology rapid assessment of the impact of small asymmetries on motion dynamics of space landing vehicle in a resonant motion. The methodology allows to analyze the value of the design parameters and the aerodynamic coefficients on the degree of their influence through the asymmetry of a deviation from the longitudinal axis of the vehicle velocity vector. The methodology is based on the assumption that the rapid development of resonant modes of motion. An example of the calculations for the proposed method, which showed the adequacy of the results according to the calculations for modeling the spatial movement.

1 Introduction

Aspects of the dynamics and sustainability of the movement is not controlled spacecraft, with rotation about the longitudinal axis, the presence of small asymmetries devoted sufficient number of scientific publications [1,2,3,4, etc.]. In [5] and [8], in more detail the methods for calculating the parameters of angular motion of space landing vehicles with an inflatable braking device.

Is shown in these that the presence of small asymmetries may experience resonant mode of motion, in which the longitudinal axis of the landing vehicle is significantly deviates from the velocity vector. Under certain conditions, the landing vehicle may lose stability their movement. In these papers presented the mathematical model of motion of the landing vehicle, taking into account the influence of small asymmetries.

The most complete picture of the spatial movement of the landing vehicle can be obtained by numerical integration of complex nonlinear mathematical models. However, this way of solving the problem is possible only in the final stages of design and gives only a partial (numerical) results.

In this paper we present a methodology for assessing the impact of small asymmetries in the longitudinal axis of the deflection space landing vehicle from the velocity vector in a resonant mode of motion, for use in the early design stages of the landing vehicle.

2 Modelling

Small asymmetries caused additional moments that define the dynamics of change of angular motion of the landing vehicle. Consider the equation of angular motion of the landing vehicle, taking into account the influence of small asymmetries in the projections on the axes related to the coordinate system (OXYZ) presented in [2], [6], [7]:

$$\dot{\omega}_x = \frac{d\omega_x}{dt} = \frac{1}{J_x}\Big[J_{xy}\cdot(\dot{\omega}_y - \omega_x\cdot\omega_z) +$$

$$+ J_{xz}\cdot(\dot{\omega}_z + \omega_x\cdot\omega_y) + J_{yz}\cdot(\omega_y^2 - \omega_z^2) +$$

$$(J_y - J_z)\cdot\omega_y\cdot\omega_z + q\cdot S\cdot\ell\cdot\left(m_{x_0} + m_x^{\bar{\omega}_x}\cdot\frac{\omega_x\cdot\ell}{V} + C_y\cdot\frac{\Delta z}{\ell} - C_z\cdot\frac{\Delta y}{\ell}\right) \Big]$$

$$\dot{\omega}_y = \frac{d\omega_y}{dt} = \frac{1}{J_y}\Big[J_{xy}\cdot(\dot{\omega}_x + \omega_y\cdot\omega_z) + J_{xz}\cdot(\omega_z^2 - \omega_x^2) +$$

$$+ (J_z - J_x)\cdot\omega_x\cdot\omega_z + J_{yz}\cdot(\dot{\omega}_z - \omega_x\cdot\omega_y) +$$

$$+ q\cdot S\cdot\ell\cdot\left(m_{y_0} + m_y^{\beta}\beta + m_y^{\bar{\omega}_y}\cdot\frac{\omega_y\cdot\ell}{V} + C_x\cdot\frac{\Delta z}{\ell}\right)\Big] \quad (1)$$

$$\dot{\omega}_z = \frac{d\omega_z}{dt} = \frac{1}{J_z}\Big[J_{xy}\cdot(\omega_x^2 - \omega_y^2) + J_{xz}\cdot(\dot{\omega}_x - \omega_y\cdot\omega_z) +$$

$$+ J_{yz}\cdot(\dot{\omega}_y + \omega_x\cdot\omega_z) + (J_x - J_y)\cdot\omega_x\cdot\omega_y +$$

$$+ q\cdot S\cdot\ell\cdot\left(m_{z_0} + m_z^{\alpha}\alpha + m_z^{\bar{\omega}_z}\cdot\frac{\omega_z\cdot\ell}{V} - C_x\cdot\frac{\Delta y}{\ell}\right)\Big].$$

where: $\omega_x, \omega_y, \omega_z$ - the projections of the angular velocity of the landing vehicle on the axis of the related coordinate system;

$q = \frac{\rho V^2}{2}$ - dynamic pressure;

V - velocity of landing vehicle;

S - mid-section area;

ℓ - length of landing vehicle;

J_x, J_y, J_z - the principal moments of inertia;

C_x, C_y, C_z - aerodynamic coefficients of axial and transverse forces;

m_{x0}- aerodynamic moment coefficient relative to the longitudinal axis;

m_y^β, m_z^α - derivatives of aerodynamic coefficients on the angle of attack (α) and glide (β);

$m_x^{\varpi_x}, m_y^{\varpi_y}, m_z^{\varpi_z}$ - the derivatives of aerodynamic coefficients on the projections of the angular velocity of the landing vehicle on the axis related coordinate system.

Small asymmetry: $\Delta y, \Delta z$ - the lateral displacement of the real center of mass relative to the longitudinal axis of the of the landing vehicle; J_{zy}, J_{xz}, J_{yz} - the centrifugal moments of inertia; m_{y0}, m_{z0} - the aerodynamic coefficients of transverse moments at zero values of angles of attack and glide angle.

Numerous studies of resonant modes of movement is not operated vehicles [2, 7, 11] have shown that the development of resonant mode (the maximum deviation of the longitudinal axis of the landing vehicle of the velocity vector) is held for a short time of a few seconds.

Based on this, we transform the second and third equations of (1) under the following assumptions: neglect the influence of gravity in the dynamic equations of translational motion; We assumed to be constant all the aerodynamic coefficients and their derivatives; neglected as a second small quantities, works of angular velocity in the dynamic equations of rotational motion. Thus, we can get the next formulas:

$$J_{yz} = 0; J = J_y = J_z; C_Z^\beta = -C_n^\alpha; |m_y^\beta| = |m_z^\alpha|; \; m_y = -|m_z^\alpha| \cdot \beta;$$

$$m_z = -|m_z^\alpha| \cdot \alpha;$$

$$C_x = C_\tau, C_y = C_n^\alpha \cdot \alpha, C_z = -C_n^\alpha \cdot \beta \cdot$$

It further indicated - C_n^α a derivative of the aerodynamic coefficient of the normal force on the angle of attack.

Using an algorithm for transforming equations proposed in [1]. After transformation of the second and third equations in the system (1) according to the introduced estimates we obtain the second order differential equation for the angles of attack and glide angle in the following form:

$$\begin{aligned}
\ddot{\alpha} = &-\left[\frac{S}{m}\left(C_n^\alpha - C_\tau\right) + \frac{Sl^2}{J}\left|m_z^\omega\right|\right]\frac{q}{V}\dot{\alpha} - \\
&-\left[\frac{qSl}{J}\left|m_z^\alpha\right| - \frac{J - J_x}{J}\omega_x^2 + \frac{q^2S^2l^2}{mJV^2}\left(C_n^\alpha - C_\tau\right)\left|m_z^\omega\right|\right]\alpha - \\
&-\frac{2J - J_x}{J}\omega_x\dot{\beta} - \left[\frac{Sl}{J}\left|m_z^\varpi\right| + \frac{J - J_x}{J}\frac{S}{m}\left(C_n^\alpha - C_\tau\right)\right]\frac{q}{V}\omega_x\beta + \\
&+ h_\beta\frac{J - J_x}{J}\omega_x^2 + \frac{qSl}{J}\left|m_z^\alpha\right|\alpha_a - \frac{qSC_x}{J}\Delta y,
\end{aligned}$$

$$\begin{aligned}
\ddot{\beta} = &-\left[\frac{S}{m}\left(C_n^\alpha - C_\tau\right) + \frac{Sl^2}{J}\left|m_z^\omega\right|\right]\frac{q}{V}\dot{\beta} - \\
&-\left[\frac{qSl}{J}\left|m_z^\alpha\right| - \frac{J - J_x}{J}\omega_x^2 + \frac{q^2S^2l^2}{mJV^2}\left(C_n^\alpha - C_\tau\right)\left|m_z^\varpi\right|\right]\beta - \\
&-\frac{2J - J_x}{J}\omega_x\dot{\alpha} - \left[\frac{Sl}{J}\left|m_z^\omega\right| + \frac{J - J_x}{J}\frac{S}{m}\left(C_n^\alpha - C_\tau\right)\right]\frac{q}{V}\omega_x\alpha + \\
&+ h_\alpha\frac{J - J_x}{J}\omega_x^2 + \frac{qSl}{J}\left|m_z^\alpha\right|\beta_a - \frac{qSC_x}{J}\Delta z.
\end{aligned} \tag{2}$$

where additionally indicated:

α_a, β_a- wheel balancers the angles of attack and glide caused a small asymmetry of shape;

$h_\alpha = \frac{J_{xz}}{J - J_x}, h_\beta = \frac{J_{xy}}{J - J_x},$ relative centrifugal moments of inertia.

Transform the equation in the system (2), using complex variables for the angular motion parameters of the landing vehicle and asymmetries:

$$\left.\begin{aligned} \delta = \beta + i \cdot \alpha \quad h = h_\beta + i \cdot h_\alpha \quad \omega = \omega_y + i \cdot \omega_z \\ \delta_0 = \beta_0 + i \cdot \alpha_0 \end{aligned}\right\} \tag{3}$$

where: δ - the spatial angle of attack;

$$\left.\begin{aligned} \alpha_0 = -\alpha_a - \frac{C_\tau}{|m_z^\alpha|} \cdot \frac{\Delta y}{\ell} \;\; ; \;\; \alpha_a = -\frac{m_{z_0}}{|m_z^\alpha|} \;\; ; \\ \beta_0 = -\beta_a + \frac{C_\tau}{|m_z^\alpha|} \cdot \frac{\Delta z}{\ell} \;\; ; \;\; \beta_a = -\frac{m_{y_0}}{|m_z^\alpha|} \;\; ; \end{aligned}\right\} \tag{4}$$

After the transformations obtain the differential equation of angular motion of the landing vehicle, describing the angular fluctuations of landing vehicle in the complex form:

$$\ddot{\delta} + K_1 \cdot \dot{\delta} + K_2 \cdot \delta = K_3 \tag{5}$$

where:

$$K_1 = \frac{1}{V}\left[q \cdot S \cdot \left(\frac{C_n^\alpha - C_\tau}{m} + \frac{|m_z^{\overline{\omega}}|\,\ell^2}{J}\right)\right] + i \cdot \left[\left(2 - \frac{J_x}{J}\right) \cdot \omega_x\right]$$

$$\begin{aligned} K_2 = &\left[\frac{qS\ell}{J} \cdot |m_z^\alpha| - \frac{J - J_x}{J}\omega_x^2 + \frac{q^2S^2\ell^2}{mJV^2} \cdot |m_z^{\overline{\omega}}|\,(C_n^\alpha - C_\tau) + \right] + \\ &+ i \cdot \left[\frac{J - J_x}{J} \cdot \frac{qS}{mV} \cdot (C_n^\alpha - C_\tau) \cdot \omega_x + \frac{qS\ell^2}{JV} \cdot |m_z^{\overline{\omega}}| \cdot \omega_x + \dot{\omega}_x\right] \end{aligned}$$

$$\begin{aligned} K_3 = &\left[\frac{q \cdot S \cdot \ell}{J} \cdot |m_z^\alpha| \cdot \beta_0 + \frac{J - J_x}{J} \cdot \dot{\omega}_x \cdot h + \right] + \\ &+ i \cdot \left[\frac{q \cdot S \cdot \ell}{J} \cdot |m_z^\alpha| \cdot \alpha_0 + \frac{J - J_x}{J} \cdot \omega_x^2 \cdot h\right]. \end{aligned} \tag{6}$$

In Eq. (5) K_1 coefficient determines the damping properties of the of the landing vehicle, K_2 coefficient determines the amplitude of the deviation of the spatial angle of attack, K_3 coefficient characterizes the small asymmetries. We analyze the terms in the equation for the K_2 coefficient.

The calculations for the of the landing vehicle show that the values of the first two terms of about two orders of magnitude greater than the values of all other terms. The maximum value of the spatial angle of attack will be the minimum value of coefficient K_2, which is obtained from the condition that the first two terms.

For steady motion spatial angle of attack b is determined from (5) by the following relationship:

$$\delta_Б = \frac{K_3}{K_2}. \quad (7)$$

In this formula the coefficient K_3 depends on the value of existing small asymmetries, for given asymmetries values: $K_3 = Const$. Thus, the change of the angle is determined by the coefficient value K_2. In order to get $K_2 \to 0$ · $\delta_Б \to \max$. Therefore, the condition $K_2 = \min$ is as follows:

$$\frac{qS\ell}{J} | m_z^\alpha | - \frac{J - J_x}{J} \omega_x^2 = 0 \quad (8)$$

The first term in the equation K_2 coefficient is the square of the frequency of natural transverse fluctuations of the landing vehicle. Formula (8) transform to a new kind:

$$\frac{qS\ell}{J - J_x} | m_z^\alpha | - \omega_x^2 = 0 \quad (9)$$

Denote in the formula (9) the first term as the resonant frequency:

$$\omega_{rez} = \sqrt{\frac{qS\ell |m_z^\alpha|}{J - J_x}} \quad (10)$$

Thus, the necessary condition for maximum spatial angle of attack is the coincidence of the angular velocity of the landing vehicle relative to the longitudinal axis and the resonance frequency:

$$\omega_x = \omega_{rez}. \quad (11)$$

Under the conditions of the balanced movement of the landing vehicle module integrated balancing attack angle b can be determined by the following formula:

$$| \delta_Б | = \sqrt{(\boldsymbol{Re}\delta_Б)^2 + (\boldsymbol{Im}\,\delta_Б)^2} \quad \Rightarrow \quad | \delta_Б | = \sqrt{\beta_Б^2 + \alpha_Б^2} \quad (12)$$

where: $\alpha_Á, \beta_Á$ - angles of attack and glide angle in a balanced movement.

After substituting the equations (2), (6) and carrying out transformations we obtain a formula for determining the maximum value of the angle of attack of the module integrated in a balanced movement in the presence of resonance modes:

$$| \delta_Б |_{max} = K_\delta \cdot \sqrt{\left(K_m m_{y0} + K_\Delta \overline{\Delta z} - K_J h_\alpha\right)^2 + \left(K_m m_{z0} - K_\Delta \overline{\Delta y} + K_J h_\beta\right)^2} \quad (13)$$

where: K_δ - coefficient of influence of small asymmetries in the value of the spatial angle of attack in a resonant mode of movement of the landing vehicle.

Coefficients of the relative asymmetry influence:

$$K_m = 1, K_\Delta = C_\tau, K_J = |m_z^\alpha| \frac{J}{J - J_x}. \quad (14)$$

Dimensionless value of the asymmetry:

$$\overline{\Delta y} = \frac{\Delta y}{\ell}, \overline{\Delta z} = \frac{\Delta z}{\ell}, h_\beta = \frac{J_{xy}}{J}, h_\alpha = \frac{J_{xz}}{J}. \quad (15)$$

Consider the example of estimating the relative impact of a variety of small asymmetries in the maximum value of the spatial angle of attack in a resonant mode for of the landing vehicle with the following characteristics:

$$|m_z^\alpha| = 0{,}343, C_\tau = 0{,}72, J = 8{,}75, J_x = 1{,}5$$

The coefficients of the relative influence of asymmetries, calculated according to the formulas (14) are as follows: $K_m = 1, K_\Delta = 0{,}72, K_J = 0{,}41$

Thus, for the landing vehicle, for equal values of asymmetries greatest impact on the maximum deviation of the spatial angle of attack will have a form of asymmetry external device [13, 14, 15]. The least affected by centrifugal moments of inertia. For equal values of the quantities of small asymmetries $m_{y0} = \overline{\Delta z} = h_\alpha$ ratio of the maximum values of the spatial angle of attack for the lateral deviation and the center of mass of the centrifugal moment of inertia for maximum spatial angle of attack due to the asymmetry of the form will be equal to the coefficients $K_\Delta = 0{,}72, K_J = 0{,}41$.

3 Results

For confirmation results of the calculations on the proposed analytical method of determining the maximum deviation will hold the spatial angle of attack of the landing vehicle by integrating the spatial system of differential equations in the landing vehicle at resonance motion conditions.

At Fig. 1, Fig. 2, Fig. 3 shows graphs of these calculations for the following quantities of small asymmetries $m_{y0} = \overline{\Delta z} = h_\alpha = 4 \cdot 10^{-3}$. In all cases, the start time of the resonance mode is six seconds, and the time to maximum spatial angle of attack equal to fourteen seconds.

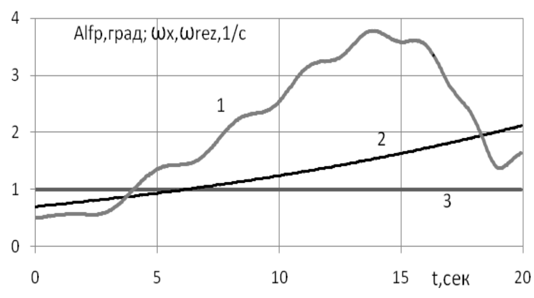

Figure 1. Graphics of spatial angle of attack (1), the resonance frequency (2) and the angular velocity of the descent the landing vehicle about the longitudinal axis (3) to effect only the asymmetry form the landing vehicle ($\alpha_{pvax1} = 3{,}78^0$).

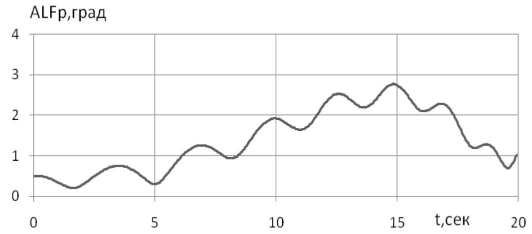

Figure 2. Graphic of spatial angle of attack to affect only the lateral displacement of the center of mass of the landing vehicle ($\alpha_{max2} = 2{,}77^0$).

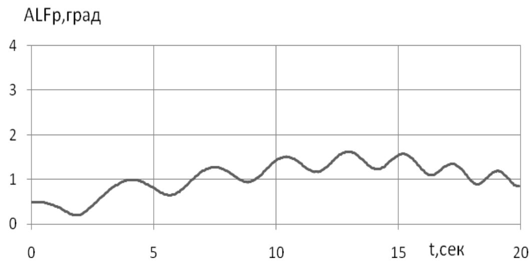

Figure 3. Graphic of spatial angle of attack to affect only the centrifugal moment of inertia ($\alpha_{pvax3} = 1{,}62^0$).

Relationship maximum values of spatial angles of attack for the lateral deviation and the center of mass of the centrifugal moment of inertia for maximum spatial angle of attack due to the asymmetry of shape, will be as follows:

$$\frac{\alpha_{\text{Ï}\ \max 2}}{\alpha_{\text{Ï}\ \max 1}} = 0{,}73, \quad \frac{\alpha_{\text{Ï}\ \max 3}}{\alpha_{\text{Ï}\ \max 1}} = 0{,}42$$

It is evident that they are almost equal coefficients $K_\Delta = 0{,}72, K_J = 0{,}41$.

Thus, confirmed the possibility of using the presented techniques for rapid assessment of the relative impact of a variety of small asymmetries in the value of spatial angle of attack of the landing vehicle in a resonant motion.

4 Conclusions

1. An analytical methodology of rapid assessment of the impact of small asymmetries in the amount of deviation of spatial angle of attack of the landing vehicle in a resonant motion. The methodology allows to analyze the value of the design parameters and the aerodynamic coefficients on the degree of their influence through the asymmetry of a deviation of the longitudinal axis of the landing vehicle of the velocity vector.

2. An example of the influence of small asymmetries in the resonance conditions for of the landing vehicle designed for the descent into the Martian atmosphere. It is shown that the solution of this problem by analytical procedure is almost identical with the results of numerical modeling of spatial movement of the landing vehicle.

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