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The Stochastic Synthesis of the Adaptive Filter for Estimating the Controlled Systems State Based on the Condition of Maximum of the Generalized Power Function

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Abstract. The proposed procedure for the synthesis of the filter of the state estimation is based on a new mathematical model of the dynamic controlled system. It is based on the maximum condition of the function of the generalized power. The optimization boundary problem can be solved using the invariant embedding procedure. The result differs from the known ones as it has lower dimensionality, non-linear structure and provides a more accurate estimation.

1 Introduction

Constructing mathematical models of controlled systems involves the problem to keep balance between simplicity and adequate representation. Greater number of measurements and accuracy can compensate the mismatch between the observed movement and the model. But there are a number of important practical applications where it is impossible or extremely expensive. For example, in radiolocation and navigation the poor mathematical model increases estimation errors. This is due to the fact that the traditional approach to the construction of the model for the perturbation of the estimated process involves the expansion of the state space on the basis of the accepted hypothesis for the movement pattern of the controlled system [1, 2], which hypothesis in many cases is not true.

The difficulties of such processes simulation are often due to nonlinearity of the processes and complexity of their mathematical description. In the case of random factors influence the processes simulation involves the solution of the stochastic differential equations which are even more complex since they lead to the need to solve the multi-dimensional Fokker-Planck-Kolmogorov equation, and in the presence of observations — to the Stratonovich equation. Using the Gaussian approximation can reduce the simulation complexity and let to go to the equations for the extended Kalman filter. In practice further simplification is performed by use of the linearized model for the evolution of the studied process parameters. So there is a known contradiction between the requirement of maximum details of the mathematical model to improve the reliability and accuracy of simulation and the requirement for constructibility that dictated by the researchers' capability. To find a compromise within the framework of the traditional approach is a very difficult problem since the linearized models for the stochastic filters of the state estimation have virtually exhausted reserves for the accuracy increasing.

However, the description of the controlled process being in the quasideterministic approximation can be based on the prime principle of the dynamics, the Hamilton-Ostrogradski variational principle. This method has been successfully applied in the invariance theory for the perturbations. In particular, in the papers of L.I. Rozonoer and V.V. Velichenko the variational approach was used for the synthesis of non-linear perturbation-invariant systems.

The present paper studies the new mathematical model for the controlled process which is developed on the methodology of the combined-maximum principle [3, 4, 5, 6]. The approach developed by the authors assumes that the state change is the result of an unknown generalized force which is determined by the minimum of the Hamilton action [6, 7, 8]. The generalized force structure can be obtained from the condition of maximum of the generalized power function. The new model is the basis for the stochastic synthesis of the estimation equations using the invariant imbedding procedure [9, 10].

The purpose of this research is to develop a mathematical model for the control system based on the combined-maximum principle methodology and perform synthesis of the stochastic estimating filter to effectively solve the problems of filtration under conditions of regular perturbations.

2 Definition of the problem of synthesis of the mathematical model for the controlled system

In a sufficiently general form the dynamics of the continuous stochastic systems can be described by the unstrict Langevin equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u},t) + \mathbf{g}(\mathbf{x},t)\zeta(t), \tag{1}$$

where $\mathbf{x} \in R^r$ is the state vector, $\mathbf{u} \in R^k$ is vector of the controls defining the nonstationarity of the processes, $\mathbf{f}(\mathbf{x},\mathbf{u},t)$ is the continuous vector function of the corresponding arguments and the time t, $\mathbf{g}(\mathbf{x},t)$ is the matrix function of the dimension $r \times l$, $\zeta(t)$ is l-dimensional random process of the white noise kind.
Equation (1) is based on the Newton's second law taking into account the stochastic forces. The structure $\mathbf{f}(\mathbf{x},\mathbf{u},t)$ is determined by the deterministic forces and often has a cumbersome expression. So its approximation is necessary because the stochastic simulation problem may be insoluble in real time. Under the conditions of a priori uncertainty about the nature of the acting forces the structure can be associated with the perturbation dynamic based on the variational principles [8].

Let the dynamics of the controlled object satisfies to the Hamilton-Ostrogradski principle [8]. So we can write the Lagrange equations of the second kind in the phase space of the generalized coordinates and velocities $(\mathbf{q},\dot{\mathbf{q}}) \in R^{2n}$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} = Q_s, \qquad \mathbf{Q} \in \overline{G}_Q,\ s = \overline{1,n}, \tag{2}$$

where $T = \frac{1}{2}\sum_{s=1}^{n} a_{ss}\dot{q}_s^2$ is the kinetic energy of the system, a_{ss} are the matrix elements of the quadratic form of the kinetic energy, n is the number of degrees of freedom, Q_s are the generalized forces, $\overline{G}_Q$ is the admitted region.

From the Hamilton-Ostrogradski principle it follows that eq. (2) is a result of stationarity of the action integral

$$\delta' S = \int_{t_0}^{t_1} (\delta T + \delta' A)dt = 0, \tag{3}$$

where $A = \int_{q_s(t_0)}^{q_s(t_1)} \sum_{s=1}^{n} Q_s(\mathbf{q},\dot{\mathbf{q}})dq_s$ is the work of the generalized forces; t_0, t_1 are the start and end time of observation.

The observation equation is as follows:

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{q},t) + \xi(t), \tag{4}$$

where $\mathbf{h} \in R^n$ is known vector-function; $\xi \in R^n$ is a white noise vector with known level.

The objective functional has the form

$$J = \frac{1}{2}\int_{t_0}^{t_1} [\mathbf{y}(t) - \mathbf{h}(\hat{\mathbf{q}},t)]^{\mathrm{T}} \mathbf{R}_\xi^{-1} [\mathbf{y}(t) - \mathbf{h}(\hat{\mathbf{q}},t)]dt, \tag{5}$$

where $\mathbf{R}_\xi \in R^{n\times n}$ is the diagonal weight matrix characterising the noise level in the observation channel, and the symbol ^ denotes estimating.

The construction of the mathematical model can be presented as the optimal control synthesis problem: it is necessary to determine the vector of generalized forces $\hat{\mathbf{Q}}$ as a function of the generalized coordinates, as well as the corresponding trajectory providing a minimum of functional (5) under the constraints (3).

3 The stochastic synthesis of the estimating filter of the controlled system state based on the maximum condition of the generalized power function

It was found in [5] that the minimum of functional (5) is determined by the maximum condition of the generalized power function

$$\Phi = \max_{\mathbf{Q}\in G_Q} \sum_{s=1}^{n}\left[\lambda Q_s(\mathbf{q},\dot{\mathbf{q}}) - R_{\xi ss}^{-1}\frac{\partial h_s}{\partial \hat{q}_s}\left[y_s(t) - h_s(\hat{\mathbf{q}},t)\right]\right]\dot{q}_s, \tag{6}$$
$$\lambda = \text{const}.$$

Thus the structure of equations for the controlled system model is as follows:

$$\ddot{\hat{q}}_s = \frac{1}{\lambda a_{ss}}\left[-\frac{|\dot{\hat{q}}_s|\dot{\hat{q}}_s}{L_s|\hat{q}_s|} + R_{\xi ss}^{-1}\frac{\partial h_s}{\partial \hat{q}_s}\left[y_s - h_s(\hat{\mathbf{q}},t)\right]\right], s = \overline{1,n}; \tag{7}$$

where λ is the undetermined Lagrange multiplier, $L_s \geq 0$ is a constant of the switching curve [5]. We can rewrite eq. (7) in the form:

$$\ddot{\hat{q}}_s = \frac{-|\dot{\hat{q}}_s|\dot{\hat{q}}_s}{a_{ss}L_s\lambda|\hat{q}_s|} + \frac{R_{\xi ss}^{-1}}{\lambda a_{ss}}\frac{\partial h_s}{\partial \hat{q}_s}\left[y_s - h_s\right] = f(q_s,\dot{q}_s) + \frac{R_{\xi ss}^{-1}}{\lambda a_{ss}}\frac{\partial h_s}{\partial \hat{q}_s}\left[y_s - h_s\right]. \tag{8}$$

The sequential estimating problem [9] can be formulated as follows: for the random observation $y(t)$ we need to find estimation $q(t)$, $t \in [t_0, t_1]$ according the criterion of the minimum of the mean square error with t_1 increasing.

For $n=1$ we write down the model of the controlled system (8) movement as a vector differential first-order equation [1]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}\boldsymbol{\eta}(t), \quad \mathbf{x}(t_0) = \mathbf{x}^0, \tag{9}$$

where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_1 = \hat{q}$, $x_2 = \dot{\hat{q}}$, and

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_2 \\ \dfrac{-|x_2|x_2}{\lambda a L|x_1|} \end{bmatrix} \tag{10}$$

is a vector-function of the system, $\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{\lambda a R_\xi}\dfrac{\partial h}{\partial \hat{q}} \end{bmatrix}$,

$\boldsymbol{\eta}(t)$ is a vector-function of the stochastic process centered with respect to x_1 with the intensity matrix $\mathbf{R}_\eta$.

Thus the problem of synthesis of the filter of state estimating is reduced to the problem to obtain the estimation $\hat{\mathbf{x}}$ from the minimum condition of the extended objective functional [9]

$$J_2 = J + \frac{1}{2}\int_{t_0}^{t_1} \boldsymbol{\eta}^T(t)\mathbf{R}_\eta^{-1}\boldsymbol{\eta}(t)\,\mathrm{d}t. \tag{11}$$

According [9] the corresponding estimation formally satisfies the two-point boundary value problem

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{f}(\hat{\mathbf{x}}) - \mathbf{G}\mathbf{R}_\eta \mathbf{G}^T \boldsymbol{\eta}(t), \tag{12}$$

$$\dot{\boldsymbol{\eta}}(t) = \frac{\partial \mathbf{h}^T(\hat{\mathbf{x}}(t),t)}{\partial \hat{\mathbf{x}}}\mathbf{R}_\xi^{-1}[\mathbf{y}(t) - \mathbf{h}(\hat{\mathbf{x}}(t),t)] - \frac{\partial \mathbf{f}^T(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}}\boldsymbol{\eta}(t), \tag{13}$$

With boundary conditions

$$\hat{\mathbf{x}}(t_0) = \mathbf{x}^0 + \mathbf{P}^0\boldsymbol{\eta}(t_0), \quad \boldsymbol{\eta}(t_1) = 0, \tag{14}$$

where $\mathbf{P}^0$ is an analogue for the covariance matrix of the filtering errors [1, 9], $\mathbf{x}^0$ is an initial value of estimation.

Using the invariant imbedding procedure [9] we obtain the following equation:

$$\begin{gathered} \dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{P}\frac{\partial \mathbf{h}^T(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}}\mathbf{R}_\xi^{-1}[\mathbf{y}(t) - \mathbf{h}(\hat{\mathbf{x}},t)], \\ \dot{\mathbf{P}} = \mathbf{P}\frac{\partial \mathbf{f}^T(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}} + \frac{\partial \mathbf{f}(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}}\mathbf{P} + \mathbf{G}\mathbf{R}_\eta\mathbf{G}^T - \\ - \mathbf{P}\frac{\partial}{\partial \hat{\mathbf{x}}}\left[\frac{\partial \mathbf{h}^T(\hat{\mathbf{x}},t)}{\partial \hat{\mathbf{x}}}\mathbf{R}_\xi^{-1}[\mathbf{y}(t) - \mathbf{h}(\hat{\mathbf{x}},t)]\right]\mathbf{P}, \\ \mathbf{P}(t_0) = \mathbf{P}^0, \quad \hat{\mathbf{x}}(t_0) = \hat{\mathbf{x}}^0. \end{gathered} \tag{15}$$

4 The evaluation of the synthesized filter efficiency

The evaluation of the efficiency of the developed filter was carried out based on the mathematical simulation of the process of estimating of the stochastic process state with non-stationary perturbation.

The state equation (8) is as follows:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}\boldsymbol{\eta}(t), \quad \mathbf{x}(t_0) = \mathbf{x}^0. \tag{16}$$

The observation equation (4) has the form:

$$y = \mathbf{H}\mathbf{x} + \xi, \tag{17}$$

where $\mathbf{H} = [1 \quad 0]$, ξ is white Gaussian noise of observations.

According (15) the equations of the suggested algorithm can be written in the form:

$$\begin{gathered} \dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{P}\mathbf{H}^T R_\xi^{-1}[y - \mathbf{H}\hat{\mathbf{x}}], \\ \dot{\mathbf{P}} = \mathbf{P}\frac{\partial \mathbf{f}^T(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}} + \frac{\partial \mathbf{f}(\hat{\mathbf{x}})}{\partial \hat{\mathbf{x}}}\mathbf{P} + \mathbf{G}\mathbf{R}_\eta^{-1}\mathbf{G}^T - \\ - \mathbf{P}\mathbf{H}^T R_\xi^{-1}\mathbf{H}\mathbf{P}, \\ t_0 = 0, \mathbf{P}(0) = \mathbf{P}^0, \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}^0, \end{gathered} \tag{18}$$

where

$$\frac{\partial \mathbf{f}(\hat{\mathbf{x}},t)}{\partial \hat{\mathbf{x}}} = \begin{bmatrix} 0 & 1 \\ \dfrac{|\hat{x}_2|\hat{x}_2 sign(\hat{x}_1)}{\lambda a L|\hat{x}_1|^2} & \dfrac{-sign(\hat{x}_2)\hat{x}_2 - |\hat{x}_2|}{\lambda a L|\hat{x}_1|} \end{bmatrix}. \tag{19}$$

The mathematical simulation is performed using dimensionless units. Initial parameters are assumed as $\hat{x}(0) = x(0) = 0, \dot{\hat{x}}(0) = \dot{x}(0) = 0, \ddot{\hat{x}}(0) = \ddot{x}(0) = 0.$ The mean-square deviation of the noise of observations is 100. The simulated perturbation is

$$x(t) = 1000h((t-70)(80-t)), \tag{20}$$

where $h(\cdot)$ is the Heaviside step function.

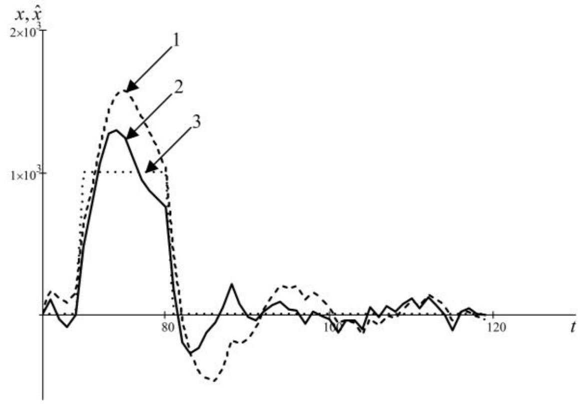

Figure 1. The process with nonstationary perturbation and its estimation.

Fig. 1 illustrates mathematic modelling results for estimating using the new and traditional algorithms. The results are: the curve 1 is the Kalman filter estimation with traditional model [1]; the curve 2 is the developed algorithm (18) estimation; the curve 3 is the process for estimation.

This computational experiment shows that the use of new estimating filter (18) for controlled systems in comparison with the Kalman filter with the traditional model [1] under conditions of regular perturbations provides increase the estimation accuracy by 10% on average. In addition, there is effect of increasing the limit of stability in the case of increasing perturbation frequency. The elementary operations count shows that application of the algorithm requires less computational effort when the operations are addition, subtraction, multiplication and division.

5 Conclusion

This model developed based on the methodology of the unified principle of maximum differs from those obtained using the forming filter by less dimensionality and by non-linear structure. Its use as a base for synthesis procedure on the basis of the invariant imbedding method leads to a new filter for estimation of the controlled systems state. It decreases the estimation errors in comparison with the classical algorithm and demands less computational resources.

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