Error Analysis and Compensation Method on the Mechanical Structure of the Hydraulic Control System

Luo Yanyan, Liu Jun, Zhang Li and Zhang Haining
Qingan Group Co., LTD., Xian, China

Abstract. Mechanical deformation of mechanical transmission part in hydraulic control system directly affects the loading accuracy of the system. For improving the mechanical properties of the system, the force analysis and motion analysis of mechanism are simulated based on the four-bar linkage structure (FLS), and kinematics simulation is designed by using Matlab program, then came to a system error bar graph. The system error was calculated accurately according to the results of the structural mechanics simulation made by Solidworks motion module. The structure of the system will be modified when systematic errors exceed the required limit values until it reach the required value.

1 Introduction
The electro-hydraulic servo method is widely used in hydraulic control system which has high efficiency, accurate control, easily design control law, etc. Especially, the accuracy of control is improved greatly in the case of rapid development of sensor technology. Because of the force deformation of system organization and tiny errors in system installation, will cause the change of hydraulic load torque, also it difficult to measure through the pressure sensor or displacement sensor in the system [1]. While the system error caused by the deformation is greater than the system error caused by sensor accuracy, the using of high precision sensor becomes meaningless.

In this essay, the mathematic model of FLS will be established, and the error caused by deformation will be analysed which is widely happened in many conditions of electro-hydraulic experiment. By using Matlab and Solidworks, the system error of FLS caused by mechanical deformation will be calculated. Finally, compensation algorithm was proposed.

2 Structure analysis of the hydraulic loading system

2.1 Analysis of force balance
The schematic diagram of the FLS is shown in Fig. 1. The OA lever represents the loaded parts, such as crane booms, mechanical arm, and aircraft door; the BC1D lever is the loading cylinder; point O, point C and point D are fixed point, and yet point BC1 is the hinge point of hydraulic cylinder interacting with CB lever; OA lever, CB lever, BC1D lever circles respectively point O, point C, point D; the swing Angle $\alpha$ of OA lever is decided by the elongation of actuator. While the specific load torque (known its output characteristic) force to the loaded parts (OA lever), hydraulic cylinder operated and output hydraulic pressure, and then FLS will transmit the hydraulic torque to cabin door [2], [3]. Take some reason such as site structure in to consideration, the swing angle $\alpha$ is the only angle feedback quantity which can be obtained.

$$\begin{align*}
T_{oa} &= F_{oa} \cdot l_{oa} \cdot \sin(\phi_{oa}) \\
T_{cb} &= F_{cb} \cdot l_{cb} \cdot \sin(\phi_{cb}) \\
T_{ch} &= F_{ch} \cdot l_{ch} \cdot \sin(\phi_{ch})
\end{align*}$$

where $T_{oa}$ is the torque of OA lever shaft with point O; $T_{cb}$ is the torque of CB lever shaft with point C; $F_{oa}$ is...
the stress suffered of two-force bars; \( F_{bcld} \) is the hydraulic pressure of actuator; \( \phi \) is the angle between the two-force bars and the loaded parts; \( \phi_{ab} \) is the angle between the two-force bars and the big rocker; \( \phi' \) is the angle between actuator and the loaded parts [4].

### 2.2 Axis mathematical model

A coordinate system is established for the X axis in the horizontal direction, as shown in Fig. 1. The coordinates of point O, point C and point D are known in the design of an aircraft test bench structure, and the angle between the limit position of OA lever and horizontal direction is 20.16°. Coordinate of point A, point B, point BC1 as follows [5]:

\[
\begin{align*}
X_a &= l_{oa} \cdot \cos(\alpha) + l_{oa} \cdot \cos(\alpha + 20.16) \\
Y_a &= l_{oa} \cdot \sin(\alpha) + l_{oa} \cdot \sin(\alpha + 20.16) \\
\phi_{ca} &= \arctan \left( \frac{X_a - X_c}{Y_a - Y_c} \right) \\
A_{cb} &= \arccos \left( \frac{l_{cb}^2 + l_{ac}^2 - l_{bc}^2}{2l_{ac} \cdot l_{bc}} \right) \\
\phi_{ab} &= \phi_{ca} + \phi_{cb} \\
X_b &= X_a + l_{ab} \cdot \cos(\phi_{ab}) \\
Y_b &= Y_a + l_{ab} \cdot \sin(\phi_{ab}) \\
X_{bc1} &= \frac{X_a + X_b}{2} + \sqrt{\left( \frac{l_{ab}}{2} \right)^2 + \Delta l^2 \cdot \cos(\phi_{ab} + 90)} \\
Y_{bc1} &= \frac{Y_a + Y_b}{2} + \sqrt{\left( \frac{l_{ab}}{2} \right)^2 + \Delta l^2 \cdot \sin(\phi_{ab} + 90)}
\end{align*}
\]

where \( X_{x/cb1}, Y_{x/cb1} \) are respective the horizontal / vertical coordinates of the point A, point B, point C ; \( \phi_{ca} \) is the angle between the connection of point A and point C with the x-axis; \( \phi_{cb} \) is the angle between the connection of point B and point C with the x-axis.

The trajectory of point A, point B, mid-point of CB lever and point BC1 are drawn by using Matlab software, as shown in Fig. 2.

### 2.3 Calculating the relative error

According to the axis mathematical model (equation (2), (3), (4), (5)) and torque balance equation (1), these parameters can be obtained, such as coordinates of each point, angles, system of force, torque of each lever and force of each point in torque balance equation. The torque curve of OA could be calculated as follows:

\[
T_{oa} = T_{bc1} \cdot \frac{l_{bc1}}{l_{ab} \cdot \sin \phi} \cdot \frac{l_{ab}}{l_{oa} \cdot \sin \phi}
\]

After calculation, torque curve of OA lever was drawn with angle of OA lever increased from 0° to 122.8 ° (fully open), on the assumption that the system error is zero, output pressure of actuator is 50000 N, as shown in Fig. 3.

![Figure 2. Trajectory of mechanism node.](image)

![Figure 3. Torque curve of OA lever.](image)

The system error was produced since the load mechanism (point C, point D) is skewing caused by force. The system error of point D was analysed firstly. Point D can offset from any direction to point \( D' \) under stress, while BC1D lever rotate to different directions. The coordinates of Point \( D' \) as shown in equation (7): \( \Delta l \) is the offset.

\[
\begin{align*}
X_{d'} &= X_d + \Delta l \cdot \cos(\theta) \\
Y_{d'} &= Y_d + \Delta l \cdot \sin(\theta)
\end{align*}
\]

![Figure 4. Course deviation of point D.](image)

While the deviation of point D changes, we get a new parameter of the angle \( \phi' \) by formula (8), (9) and (10):

\[
\phi_{dcb} = -\phi_{cb} + \Delta \phi
\]
\[
\phi_{hle} = \sin^{-1}\left(\frac{Y_d - Y_{hcl}}{\sqrt{(Y_d - Y_{hcl})^2 + (X_d - X_{hcl})^2}}\right)
\]  

(9)

\[
\phi' = \frac{\pi}{2} - \phi_{hle} + \frac{\pi}{2} + \phi_{hcl}
\]  

(10)

By substitute the new \(\phi'\) into formula 6, we can calculate a new torque of the OA lever.

Relative error of the system can be calculated with excursion of point D along x-axis, under the assumption that excursion of point D is 10 mm in the horizontal direction, as shown in equation (11).

\[
\Delta W = \frac{T_f - T_0}{T_0} \times 100\%
\]  

(11)

where \(T_f\) is the torque of actual system; \(T_0\) is the torque of theory system.

3 The simulation result

The results of relative error are shown in Fig. 5 after simulation and calculation.

![Figure 5. The relative error of system.](image)

The system error was produced only consider the force deformation when point O and point C were skewing synchronously. The error matrix \(W[\alpha \theta]\) was established with the position deviation of point D extension to 360°. Where the \(\alpha\) is swing angle of OA lever; the \(\theta\) is swing angle of point \(D'\); the values of error matrix is the relative error. As shown in Fig. 6.

After analysis, the relative error can be produced only in the offset of point D is 10 mm. The relative error is 0.62% while the swing angle of OA lever is 10.9° and the deviation angle of point D is 324°.

The matrix \(W[\alpha \theta \theta']\) was established while the deviation was occurred synchronously between point C and point D, wherein the \(\theta'\) is swing angle of point \(C'\). The maximum relative error is 2.08% in the deviation of point C and point D is 10 mm. At this time, the swing angle of OA lever is 123°, the deviation angle of point \(D'\) is 145° and the deviation angle of point \(C'\) is 113°.

According to the above methods, the maximum relative error was calculated respective with the deviation value \(\Delta l\) range for 0.1, 0.5, 1, 5, 10, 50, 100. The semi-logarithmic graph of deviation-relative error was drawn using least-squares fitting method, as shown in Fig. 7.

![Figure 7. The curve between the maximum error and the force deformation.](image)

In the above figure, the horizontal axis represents \(\log(\Delta l)\) and the vertical axis represents the relative error. With the increase of the offset ratio, the relative error also will increase. Based on the curve data to make the relative error is less than 0.01, the deviation of loading point needs to be controlled within 5 mm. However, how to calculate the deviation of loading point in theory? Solidworks researched by Dassault company make it easy and solve the problem quickly.

4 Structural calculations of solidworks

Structural mechanics of hinge point of FLS were calculated by using the motion simulation module of Solidworks.

4.1 Establish 3D model of the structure

The precise mechanism model was drawn by using 3D modeling of Solidworks. The structure model of
system take root at point D was drawn, as shown in Fig. 8. The single hanger pedestal welded on the 20# square steel tube.

![Figure 8. 3D model of hinge point (point D).](image)

### 4.2 Choose material and load of the structure

After model was established, the setting of simulation can be started by clicking on the options of SolidWorks motion. At the same time, the materials and load of the structure can be set up. The simulation results as shown in Fig. 9.

![Figure 9. The simulation result of force deformation.](image)

In the above figure, the deviation of the load point is 0.4 mm which meets the requirements of less than 5 mm. At this point, the relative error of the system is 0.08% with the output of error calculation program.

## 5 Conclusion

The deviation of load point’s position caused by force deformation is inevitable for the design of hydraulic loading system. Therefore, the method provided in this paper can greatly improve the reliability of structure. It also can solve the problem of system control precision in the basic theory design and provide the data when needs to consider the accuracy deviation compensation in design control system.

## References