Research on Dynamics and Stable Tracking Model of Six-axis Simulation Turntable

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Abstract. A new type of six-axis simulation turntable used to simulate ship swaying is proposed in the paper, and the dynamics and stable tracking model of six-axis simulation turntable are researched. Firstly, according to the design indexes of six-axis simulation/testing turntable, the dynamics of six-axis simulation turntable was established by Lagrange method. Then the dynamics was simulated by software Matlab, which can provide data and theoretical basis for dynamics compensation and driving motors selection of six-axis simulation turntable. Secondly, the stable tracking model of six-axis simulation turntable was established by geometric analysis method, and simulation and reverse checking computation results show that the stable tracking model is correct.

1 Introduction

Simulation turntable executing semi-physical simulation and test in the marine, aviation, aerospace and defense domain is one of key guarantee equipments, so it plays an extremely important role in the development process of related equipments [1-5]. Simulation turntable can realistically simulate the attitude of moving vehicle, and reproduce the dynamic characteristics in the runtime, which can help designer improve the related equipments [6-10]. Furthermore, simulation turntable can be used as a stable platform which can effectively isolate the motion of moving vehicle and ensure the attitude stabilization of equipments mounted on the turntable. Simulation turntable can be used for all types of moving vehicle as well as some handheld and ground-mounted applications, such as Hubble Space Telescope, motion simulation system, military system, tracking system and the handheld camera and so on [5].

When the six-axis simulation turntable is running, the dynamic factor and interaction between the frame will change, which resulting in decreased performance of anti-interference, speed stability, dynamic characteristics and control precision [11]. Therefore, if we want to achieve high-precision control for six-axis simulation turntable, it must be to compensate for its dynamic characteristics. Dynamics is the precondition achieving compensation of dynamic characteristics. In addition, dynamics is the precondition of driving motor selection in the design process of the six-axis simulation turntable. On the other hand, the ship will be swaying on the sea due to the action of sea wave, which causes the precision equipments on ship not work accurately [12]. So, it must be to research the compensation model of ship swaying for ensuring the normal work of the precision equipments on ship.

The above mentioned importance prompts the study of dynamics and stable tracking of six-axis simulation turntable. According to the process of research, the rest of this paper is organized as follows. In Section 2, we present the structure of six-axis simulation/testing turntable and establish its dynamics. Then the stable tracking model of six-axis simulation/testing turntable is researched in Section 3. Therefore, the simulation and a short conclusion are drawn in Section 4.

2 Six-axis simulation turntable and dynamics

Six-axis simulation turntable is mainly composed of machinery body, motor servo control system, angle measurement system and computer control system. Machinery body is composed of upper body and lower body. The lower body is a three-axis turntable which simulates the rocking motion of a ship on the sea, and the upper body which simulates the stable and tracking motion of platform is also a three-axis turntable. They are both U-O-O type structure, which inner frame and middle frame are both O-type structure, and outer frame is U-type structure. The upper and lower body can be used alone or in combination. When they are used in combination, the upper body is mounted on the outer frame of the lower body, and the six-axis simulation turntable is shown in Fig. 1.

The Denavit-Hartenberg method is used to establish the following coordinates (shown in Fig. 2): (1) Fixed
coordinate system \((o_0-x_0y_0z_0z_0)\), the intersection point between outer frame axis of lower body and ground surface is selected as the origin \(o_0\), the direction of outer frame axis of lower body is selected as the axis-\(o_0x_0\) and the upward direction is its positive direction, the axis-\(o_0y_0\) is perpendicular to the axis-\(o_0x_0\) and the outward direction is its positive direction, the six-axis \(o_0y_0\) meets the right-hand rule with the axis-\(o_0x_0\) and axis-\(o_0z_0\). (2) Frame coordinate system \((o_i-x_iy_iz_i)\) \(i=\) 1, 2, . . . .6; \(i=\) 1, 2, 3 are used in outer frame, middle frame and inner frame of lower body respectively, \(i=\) 4, 5, 6 are used in outer frame, middle frame and inner frame of upper body respectively. When the \(i\)-th frame rotates, the frame coordinate system \(o_i-x_iy_iz_i\) moves with it. The intersection point of three frames of lower body is selected as the origin \(o_i\), where \(i=\) 1, 2, 3. The intersection point of three frames of upper body is selected as the origin \(o_i\), where \(i=\) 4, 5, 6. The rotational axis direction of the \(i\)-th frame is the direction of axis-\(oix_i\), where \(i=\) 1, 2, . . . .6. The normal direction of the \(i\)-th frame is the direction of axis-\(oix_i\), where \(i=\) 1, 2, 4, 5. The middle frame axis of lower body and middle frame axis of upper body are the direction of axis-\(oix_i\) and axis-\(oix_i\) respectively, and the axis-\(oix_i\), the axis-\(oy_i\) and the axis-\(oz_i\) all meet the right-hand rule.

![Figure 1. Machinery body of six-axis simulation turntable.](image1)

The link parameters of six-axis simulation turntable can be seen in Table 1, and the homogeneous transformation matrix between the coordinate system \(o_i-x_iy_iz_i\) and coordinate system \(o_{i-1}-x_{i-1}y_{i-1}z_{i-1}\) can be written as:

\[
^i_1T = \begin{bmatrix}
^i_1R & ^i_1P \\
0_{1	imes 3} & 1
\end{bmatrix}
\]

where

\[
^i_1R = \begin{bmatrix}
c\theta_i & -s\theta_i & 0 \\
s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} \\
s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1}
\end{bmatrix}
\]

\[
^i_1P = \begin{bmatrix}
a_{i-1} \\
-s\alpha_{i-1}d_i \\
c\alpha_{i-1}d_i
\end{bmatrix}^T
\]

\(s\theta_i\), \(c\theta_i\), \(s\alpha_{i-1}\), \(c\alpha_{i-1}\) and \(\alpha_{i-1}\) are the abbreviation of \(\sin(\theta_i)\), \(\cos(\theta_i)\), \(\sin(\alpha_{i-1})\) and \(\cos(\alpha_{i-1})\) respectively.

### Table 1. Link parameters of six-axis simulation turntable.

<table>
<thead>
<tr>
<th>(i)</th>
<th>Link rotation angle (\alpha_{i-1})</th>
<th>Link length (a_{i-1})</th>
<th>Offset (d_i)</th>
<th>Joint angle (\theta_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(L_1)</td>
<td>(\theta_1)</td>
</tr>
<tr>
<td>2</td>
<td>-(\pi/2)</td>
<td>0</td>
<td>0</td>
<td>(\theta_2)</td>
</tr>
<tr>
<td>3</td>
<td>-(\pi/2)</td>
<td>0</td>
<td>0</td>
<td>(\theta_3)</td>
</tr>
<tr>
<td>4</td>
<td>-(\pi/2)</td>
<td>0</td>
<td>(L_2)</td>
<td>(\theta_4)</td>
</tr>
<tr>
<td>5</td>
<td>-(\pi/2)</td>
<td>0</td>
<td>0</td>
<td>(\theta_5)</td>
</tr>
<tr>
<td>6</td>
<td>-(\pi/2)</td>
<td>0</td>
<td>0</td>
<td>(\theta_6)</td>
</tr>
</tbody>
</table>

All middle frame gravity centers and inner frame gravity centers are coincident with their geometric centers, and all outer frame gravity centers are in the revolving axle of outer frame. The homogeneous coordinates of gravity center of each frame in the respective frame coordinate system are as follows:

\(\mathbf{C}_1=[0, 0, b_1, 1]^T\), \(\mathbf{C}_2=[0, 0, 0, 1]^T\), \(\mathbf{C}_3=[0, 0, 0, 1]^T\), \(\mathbf{C}_4=[0, 0, 0, 1]^T\), \(\mathbf{C}_5=[0, 0, 0, 1]^T\) and \(\mathbf{C}_6=[0, 0, 0, 1]^T\). So, the homogeneous coordinates of gravity center of each frame in the fixed coordinate system can be calculated as follows:

\[
\begin{align*}
^0\mathbf{C}_1 &= [0, 0, b_1 + L_1, 1]^T \\
^0\mathbf{C}_2 &= [0, 0, L_1, 1]^T \\
^0\mathbf{C}_3 &= [0, 0, L_1, 1]^T \\
^0\mathbf{C}_4 &= \begin{bmatrix}
(b_2 + L_2)(s\theta_2 s\theta_3 + c\theta_2 s\theta_3 c\theta_3) \\
(b_2 + L_2)(-c\theta_2 s\theta_3 + s\theta_2 s\theta_3 c\theta_3) \\
(\theta_2 + L_2)c\theta_2 c\theta_3 + L_1 \\
1
\end{bmatrix} \\
^0\mathbf{C}_5 &= \begin{bmatrix}
L_2(s\theta_3 s\theta_4 + c\theta_3 s\theta_4 c\theta_4) \\
L_2(-c\theta_3 s\theta_4 + s\theta_2 s\theta_3 c\theta_4) \\
L_2 c\theta_3 c\theta_4 + L_4 \\
1
\end{bmatrix} \\
^0\mathbf{C}_6 &= \begin{bmatrix}
L_2(s\theta_3 s\theta_4 + c\theta_3 s\theta_4 c\theta_4) \\
L_2(-c\theta_3 s\theta_4 + s\theta_2 s\theta_3 c\theta_4) \\
L_2 c\theta_3 c\theta_4 + L_4 \\
1
\end{bmatrix}
\end{align*}
\]

Setting the gravitational acceleration \(g=9.8m/s^2\), and the weight vector \(\mathbf{M}=[m_1, m_2, m_3, m_4, m_5, m_6]\). According
to the equation (2), the total potential energy can be calculated and written as:

\[
E_p = g(m_1 b_1 + m_2 b_2 + m_3 b_2 + m_4 b_2 + m_5 b_2 + m_6 b_2) + (m_1 L_1 + m_2 L_2 + m_3 L_2 + m_4 L_2 + m_5 L_2 + m_6 L_2) c \theta_2 c \theta_2
\]

By substituting the equation (1) into equation (2), the velocity vectors of gravity center of each frame in the fixed coordinate system can be calculated by differential method as follows:

\[
\begin{align*}
\dot{v}_{c1} &= 0 \\
\dot{v}_{c2} &= (b_1 + L_1) A \left[ \begin{array}{c}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 
\end{array} \right] \\
\dot{v}_{c3} &= (b_2 + L_2) A \left[ \begin{array}{c}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 
\end{array} \right]
\end{align*}
\]

where

\[
A = \begin{bmatrix}
c \theta_2 \theta_3 - s \theta_2 s \theta_3 c \theta_3 & c \theta_2 s \theta_2 c \theta_3 & s \theta_2 s \theta_2 c \theta_3 - c \theta_2 s \theta_2 s \theta_3 \\s \theta_2 s \theta_2 c \theta_3 & c \theta_2 s \theta_2 c \theta_3 & s \theta_2 s \theta_2 c \theta_3 - c \theta_2 s \theta_2 s \theta_3 \\0 & -s \theta_2 c \theta_3 & c \theta_2 c \theta_3
\end{bmatrix}
\]

Because of \( i = 1 \), the angular velocity vector of each frame in the respective frame coordinate system can be displayed as:

\[
\begin{align*}
\omega_{c1} &= \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 
\end{bmatrix} \\
\omega_{c2} &= \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 
\end{bmatrix} \\
\omega_{c3} &= \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 
\end{bmatrix}
\end{align*}
\]

By substituting the equation (4) and equation (5) into \( E_p = \sum_{i=1}^{6} \frac{1}{2} m_i (\dot{v}_{ci})^2 + \frac{1}{2} (\omega_i)^T I_i \omega_i \), and the total kinetic energy of six-axis simulation turntable can be calculated, where \( I_i \) is the inertia matrix of the i-th frame in the respective frame coordinate system.

Let:

\[
\begin{align*}
\theta &= \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 
\end{bmatrix}^T \\
\dot{\theta} &= \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 
\end{bmatrix}^T
\end{align*}
\]

By substituting \( E_p \) and \( E_k \)

\[
\tau = \frac{d}{dt} \frac{\partial E_k}{\partial \theta} - \frac{\partial E_p}{\partial \dot{\theta}} + \frac{\partial E_k}{\partial \dot{\theta}}
\]

We can get the driving torque - \( \tau \) of each frame.

### 3 Stable tracking model

A tracking antenna is mounted on the plane of inner frame of upper body, and the task is that the antenna tracks the moving target in the sky. The steps to achieve the task as follows:

Step 1. Calculating the rotation angle-\( \theta_3 \) where the outer frame of upper body rotates about its axis to achieve that the azimuth angle of antenna is equal to the azimuth angle of target in the fixed coordinate system.

Step 2. Calculating the rotation angle-\( \theta_3 \) where the middle frame of upper body rotates about its axis to achieve that the inner frame axis is parallel to the ground surface.

Step 3. Calculating the rotation angle-\( \theta_3 \) where the inner frame of upper body rotates about its axis to achieve that the antenna tracks to the target in the sky.

The unit reference vector of antenna in the frame coordinate system can be written as: \([1 0 0 0]^T\). When three frames of lower body and outer frame of upper body rotate, the unit reference vector of antenna in the fixed coordinate system can be calculated as:

\[
\begin{bmatrix}
x_{01} \\
y_{01} \\
z_{01}
\end{bmatrix} = \begin{bmatrix}
0 \\
2^T y' \\
0 \\
0
\end{bmatrix}
\]

By substituting the equation (1) into equation (6), the unit reference vector of antenna in the fixed coordinate system can be written as:

\[
\begin{align*}
x_{01} &= (c \theta_2 c \theta_2 c \theta_3 - s \theta_2 s \theta_3) s \theta_4 + c \theta_2 s \theta_3 c \theta_4 \\
y_{01} &= (s \theta_2 c \theta_2 c \theta_3 + c \theta_2 s \theta_3) s \theta_4 + s \theta_2 s \theta_3 c \theta_4 \\
z_{01} &= c \theta_2 c \theta_3 - s \theta_2 c \theta_3
\end{align*}
\]

The azimuth angle and elevation angle of target are \( A \) and \( E \) in the fixed coordinate system respectively. The azimuth angle of antenna is equal to the azimuth angle of target in the fixed coordinate system, which can get the following condition:
By substituting the equation (7) into equation (8), the rotation angle-$\theta_4$ can be calculated as:

$$\tan \theta_4 = \frac{s\theta_2 c \theta_4 c \theta_1 + c \theta_2 s \theta_4}{s \theta_2 s \theta_4 c \theta_1}$$

(9)

The middle frame of the upper body rotates about its axis so that the inner frame axis is parallel to the ground surface after the azimuth angle of the antenna is equal to the azimuth angle of the target in the fixed coordinate system, the unit reference vector of axis-$\mathbf{o}_0\mathbf{z}_0$ in the frame coordinate system can be displayed as:

$$\begin{bmatrix}
  x_{20} \\
  y_{20} \\
  z_{20}
\end{bmatrix} = \begin{bmatrix}
  1 & T & T & T & T & T \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} \mathbf{B}$$

(10)

By substituting the equation (1) into equation (10), $x_{20}$ can be written as:

$$x_{20} = (s \theta_2 c \theta_4 c \theta_1 + c \theta_2 s \theta_4) s \theta_1 + s \theta_2 s \theta_4 c \theta_1$$

(11)

If and only if $x_{20} \equiv 0$, the inner frame axis can be parallel to the ground surface. Therefore, according to the equation (11), the rotation angle-$\theta_4$ can be calculated as:

$$\tan \theta_4 = \frac{-s \theta_2 s \theta_4}{s \theta_2 c \theta_4 + c \theta_2 s \theta_4}$$

(12)

The difference value between the elevation angle of the antenna and the elevation angle of the target can be calculated as:

$$\sin \Delta E = z_{20} / 1$$

(13)

And then the rotation angle-$\theta_6$ can be calculated as:

$$\theta_6 = E - \Delta E = E - \arcsin(c \theta_2 c \theta_4 - s \theta_2 s \theta_4 s \theta_1)$$

(14)

4 Simulation and conclusion

Dynamics and stable tracking simulation studies were conducted on Matlab. The dynamics simulation was carried out with the angles $\theta_1=30^\circ \sin 1.256t$, $\theta_2=15^\circ \sin 0.628t$, $\theta_3=15^\circ \sin 0.628t$, $\theta_4=40^\circ \sin 1.256t$, $\theta_5=20^\circ \sin 0.628t$, $\theta_6=20^\circ \sin 0.628t$, the weights $m_1=200kg$, $m_2=125kg$, $m_3=63kg$, $m_4=25kg$, $m_5=20kg$, $m_6=10kg$ and the moment of inertia of the $i$-th frame in the respective frame coordinate system $J_1=217kgm^2$, $J_2=75kgm^2$, $J_3=24kgm^2$, $J_4=2.9kgm^2$, $J_5=2.9kgm^2$, $J_6=1.1kgm^2$. According to the dynamics, the driving torque of each frame can be calculated shown in Fig. 3, which can provide data and theoretical basis for dynamics compensation and driving motors selection of six-axis simulation turntable.

Figure 3. Driving torque of each frame.

When the simulation turntable is running in the stable tracking mode, the rotation angle, azimuth angle and elevation angle of the target are $\theta_1=10^\circ \sin 0.785t$, $\theta_2=15^\circ \sin 0.785t$, $\theta_3=20^\circ \sin 0.628t$, $A=50^\circ$ and $E=25^\circ$ respectively. According to the stable tracking model, the rotation angle $\theta_4$, $\theta_5$ and $\theta_6$ shown in the figure 4 can be calculated, which achieves the tracking task. Furthermore, the angle $\theta_4$, $\theta_5$ and $\theta_6$ being known conditions are substituted into the kinematics model, which calculates the azimuth angle $A'$ and elevation angle $E'$ of the antenna. Calculating the error $\Delta A=A'-A$ and $\Delta E=E'-E$ shown in the figure 5, which shows the stable tracking model is correct.

Figure 4. Compensatory angle of stable tracking model.

Figure 5. Calculation error of stable tracking model.

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