A Fast Kurtogram Demodulation Method in Rolling Bearing Fault Diagnosis

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Abstract. Targeting at the problem of finding the best demodulation band when applying envelope analysis in rolling bearing fault diagnosis, this paper proposes a novel Fast Kurtogram Demodulation Method (FKDM) to solve the problem. FKDM is established based on the theory of spectrum kurtosis and the short-time Fourier Transform. It determines the best demodulation band firstly, which is also known as the central frequency and frequency resolution. Then, the fault signals can be demodulated in the obtained frequency band by using envelope demodulation algorithm. The FKDM method ensures the fault diagnosis correction by solving the problem of demodulation band selection. Applied FKDM in rolling bearing fault diagnosis and compared with conventional envelope analysis, the results demonstrate FKDM can achieve a better performance.

1. Introduction

Rolling bearing fault is difficult to diagnose. This is because of the high incidence of rolling bearing fault generation due to its complexity and poor working conditions. Also, the bearing signals are usually drowned by noises during operation, which makes the fault diagnosis difficult [1]. When a fault occurs in rolling bearing, it will interact with other rolling element surfaces, which makes the vibration signals to appear with the feature of amplitude modulation [2]. To diagnose this fault, the envelope analysis is one of the most commonly used methods, which mainly includes the Hilbert Transform and Teager energy operator demodulation [3]. Besides, wavelet transform and empirical mode decomposition have also been widely applied [4].

In the process of forming the envelope signals in envelope analysis method, the central frequency of demodulation band is selected based on prior experiences. However, this selection step is totally neglected in some cases, which posts a great influence on analysis results. In consideration of this problem, a novel Fast Kurtogram Demodulation Method (FKDM) is proposed.

The Fast Kurtogram is a method based on Spectral Kurtosis (SK) which is systematically defined by J. Antoni [5]. SK is used to measure the peak of square envelope. Moreover, SK is very sensitive to non-stationary patterns in a signal, and is able to indicate exactly the frequency at which those patterns occur. However, SK is only a theoretical concept and not so practical in application. By combining band-pass filter and short-time Fourier transform (STFT) with SK[6], Fast Kurtogram is introduced which is more practical and less complex while processing signals. With Fast Kurtogram, the best demodulation band can be located in advance, and signals can be filtered at specific frequency before obtaining the envelope. This method is known as FKDM.

2. The spectral kurtosis

In this section, Spectral kurtosis, the foundation of Fast Kurtogram, will be elaborated in detail.

2.1 Signal modeling

For rolling element bearings, many incipient faults produce a series of repetitive short transients, which in turn excite some structural resonances. They appear in forms of shocks which are not periodic and subjected to modulations by rotating relative frequency [7]. Hence, the signal modeling of rolling bearings $Y(t)$ can be presented as following:

$$Y(t) = X(t) + N(t) \tag{1}$$

where $X(t)$ is fault signal and $N(t)$ is additive noise. Furthermore, $X(t)$ can be presented as:

$$X(t) = \sum_k X_k h(t - \tau_k) \tag{2}$$

where $h(t)$ is the impulse response results from a single impact and where $\{X_k\}_{k\in Z}$ and $\{\tau_k\}_{k\in Z}$ are sequences of random variables which account for possible amplitudes and random occurrence of impact.

2.2 Wold-cramer decomposition

Wold-Cramer decomposition deals with fault signals expressed as a time-varying form, i.e.

$$Y(t)=\int_{-\infty}^{+\infty} e^{j2\pi ft} H(t,f)dX(f) \tag{3}$$

where $H(t,f)$ is the Fourier transform of impulse response, $h(t,s)$ and $dX(f)$ is the spectral process derived from $X(t)$. Also, $e^{j2\pi ft}$ $H(t,f)$ $dX(f)$ is the output at time t of an infinitely narrow-band filter central on frequency f.

2.3 Spectral moment and spectral kurtosis

The Wold-Cramer decomposition introduces $H(t,f)$ to describe fault signals. It will be accessed statistically by spectral moment, which will be defined as：

$$S_{2nY}(f) \triangleq E\{|H(t,f)dX(f)|^{2n}\}/df = E\{|H(t,f)^{2n}|\cdot S_{2nX} \tag{4}$$

$S_{2nY}(f)$ is also known as 2n-order instantaneous moment. In this way, the strength of the energy of $H(t,f)$ at frequency f can be measured.

Based on spectral moment, spectral cumulant i.e. combinations of several spectral moments of different orders can be achieved to characterize fault signals. Specifically, the fourth-order spectral cumulant is used:

$$C_{4Y}(f)=S_{4Y}(f)-2S_{2Y}^2(f), f\neq 0 \tag{5}$$

It can be shown that the larger the deviation of signals, the larger its fourth-order cumulant. Therefore, the energy-normalised fourth-order spectral cumulant will measure the peakiness of probability density function at frequency f. The definition of SK is given as:

$$K_Y(f) \triangleq \frac{C_{4Y}(f)}{S_{2Y}^2(f)} = \frac{S_{4Y}(f)}{S_{2Y}^2(f)} - 2, f\neq 0 \tag{6}$$

SK can overcome the shortage of kurtosis as a global indicator which is not appropriate to measure fault signals masked by strong noise, and also can be applied locally in different frequency bands. Generally, SK can be seen as the kurtosis computed at the output of a perfect filter-bank at each frequency f.

3. Fast kurtogram demodulation method

In this section, the algorithm of Fast Kurtogram and its application to rolling bearing fault diagnosis will be illustrated.

3.1 STFT-based estimation of spectral kurtosis

$Y(n)$ is used to replace $Y(t)$ by changing the sampling period to 1. The STFT of fault signal at a given analysis window $w(n)$ of length N_w and a given step P can be defined as

$$Y_w(kP,f) \triangleq \sum_{n=-\infty}^{\infty} Y(n)w(n-kP)e^{-j2\pi nf} \tag{7}$$

Therefore, spectral moment can be written as

$$\widehat{S}_{2nY}(f) \triangleq \left\langle |Y_w(kP,f)|^{2n} \right\rangle_k \tag{8}$$

It should be noted that the spectral moment are functions of window $w(n)$ and step P, so that the STFT-based estimator of spectral kurtosis can be defined as

$$\widehat{K}_Y(f) = \frac{\widehat{S}_{4Y}(f)}{\widehat{S}_{2Y}^2(f)} - 2, |f-\text{mod}(1/2)| > \frac{1}{N_w} \tag{9}$$

3.2 Kurtogram

To address the question mentioned in section 2.3 of finding perfect filter-bank, a perfect band-pass filter should be designed to maximize the kurtosis of the envelope of the filtered signal. This problem is strictly equivalent to finding the central frequency f and the frequency resolution Δf which maximizes the STFT-based SK over all possible choices.

3.3 Fast Kurtogram[8]

The kurtogram is an appealing exploratory tool for rolling bearing fault signals. However, the complete exploration of (f , Δf) plane is a formidable task.

In this section, a fast algorithm for computing Kurtogram will be proposed, which is known as the Fast Kurtogram.

The algorithm is described in tree structure from binary to 1/3-bianry. Two quasi-analysis low-pass and high-pass filter $h_0(n)$ and $h_1(n)$ in the frequency bands[0;1/4] and [1/4;1/2] are constructed:

$$h_1(n)=h(n)e^{j3\pi n/4}=h(n)[cos(3\pi n/4)+j\sin(3\pi n/4)] \tag{10}$$

where $h(n)$ is a low-pass FIR filter with cut-off frequency $f_c = 1/8+\varepsilon, \varepsilon>0$ .This process is iterated from level $k=0$, where $c_0(n)\equiv x(n)$ down to level

$k-1$. Noteworthy is the multiplication by $(-j)^n$ in order to turn the high-pass sequences into low-pass sequences. As a consequence, coefficient $c_k^i(n)$ can be interpreted as the complex envelope of signal $x(n)$ positioned on the central frequency. $f_i = (i+2^{-1})2^{-k-1}$

And with bandwidth (frequency resolution) $(\Delta f)_k = 2^{-k-1}$ The Kurtogram is finally estimated by computing the kurtosis of all sequences $c_k^i(n)$ $i = 0,\cdots,2^k-1$, $k = 0,\cdots,K-1$:

$$K_Y(f,\Delta f) = \frac{\left\langle \left| c_k^i(n) \right|^4 \right\rangle}{\left\langle \left| c_k^i(n) \right|^2 \right\rangle^2} - 2 \tag{10}$$

The Fast Kurtogram representation at nodes $\{f_i;(\Delta f)_k\}$ of the (f , Δf) plane is illustrated in Fig.1(c), in which 2^K-1 kurtosis values are compounded.

After all the description from section 1 to 4, here we can put forward the specific process of rolling bearing fault diagnosis based on Fast Kurtogram i.e. FKDM. With Fast Kurtogram, best demodulation band can be determined by scientific theory which also overcomes the main shortage of conventional envelope analysis. The process has been illustrated in Fig. 1.

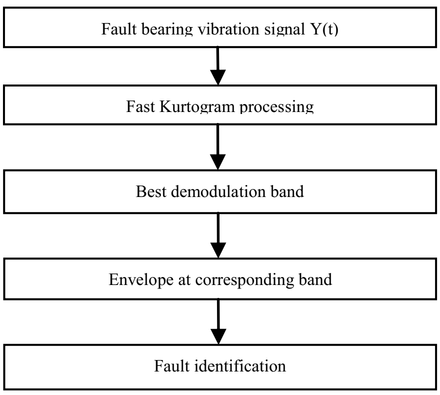

Figure 1. Flow chart of FKDM

4. Application of FKDM

In this section, FKDM will be applied to rolling bearing faults diagnosis. The results are presented and compared to conventional envelope demodulation method.

Rolling bearing fault signals are obtained from the Case Western Reserve University Bearing Data Center. Date was collected from bearing test stand which consists of a 2hp motor, a torque transducer, a dynamometer and control electronics. The rolling bearings selected are 6205-2, deep groove ball bearing. The size of the bearings is shown in Table 1. The sampling frequency is 12000Hz, with motor speed of 1797Rpm and rotating frequency fr=29.95Hz. Meanwhile, the fault frequency of the bearings can be calculated and the result is shown in Table 2.

Table 1. Size of Deep Groove Ball Bearing

Inside Diameter	Outside Diameter	Contact Angel	Ball Diameter	Number of Balls
25.00mm	52.00mm	0	8.20mm	9

Table 2. Fault Frequency of Different Fault Location in Rolling Elements

Fault location	Fault frequency
Inner race	fi=162Hz
Outer race	fo=107Hz
Ball	fb=141Hz

In the first case, ball fault signal is processed under both FKDM and envelope analysis. Result of envelope analysis is displayed in Fig. 2(a), and ball fault frequency of 141Hz can be found with certain magnitude and many unidentified frequency components appear in the envelope. However, the result of FKDM is different. As shown in Fig. 3(a), central frequency and frequency resolution (f , Δf)=(2875, 250) can be obtained, which is the best demodulation band. The corresponding envelope in Fig. 4(a) clearly reveals that the ball fault frequency of 141Hz is modulated by rotation speed of 29.95Hz.

In the second case, outer race fault signal is processed the same as above. Result of envelope analysis is displayed in Fig. 2(b), where rotation speed of 29.95Hz obtains the maximum amplitude of the whole envelope, which makes the outer race fault frequency not obvious. Meanwhile, FKDM not only reveals the best demodulation band (f , Δf)=(3375, 750) as shown in Fig. 3(b), but also clearly exhibits the outer race fault frequency of 107Hz and its harmonics in Fig. 4(b).

In the last case, inner race fault signal is processed the same way. Result of envelope analysis is displayed in Fig. 2(c), inner race fault frequency of 162Hz and its harmonics dominate the whole envelope. On the other hand, FKDM obtains the same result as shown in Fig.4(c).

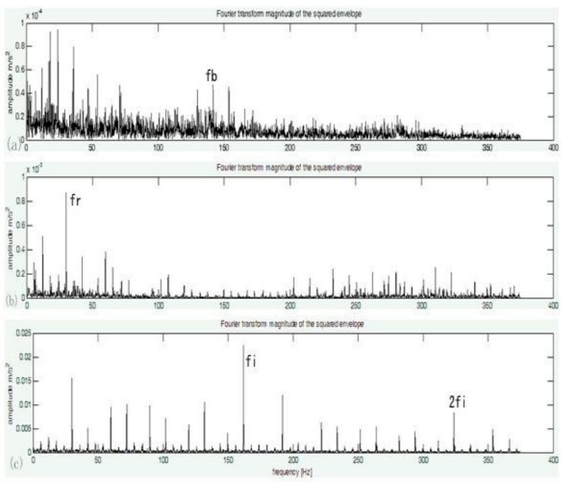

Figure 2. Envelope of (a) ball fault (b) outer race fault (c) inner race fault bearing signal with conventional envelope demodulation method

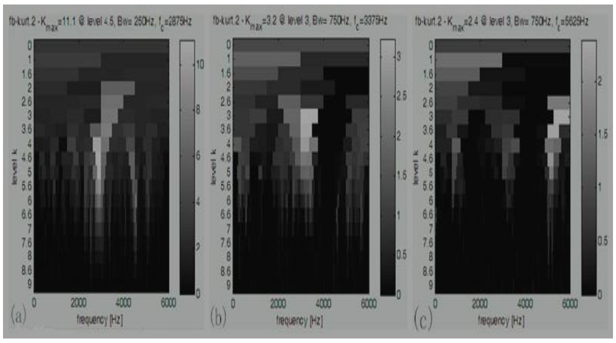

Figure 3. Fast Kurtogram of (a) ball fault (b) outer race fault (c) inner race fault bearing signal

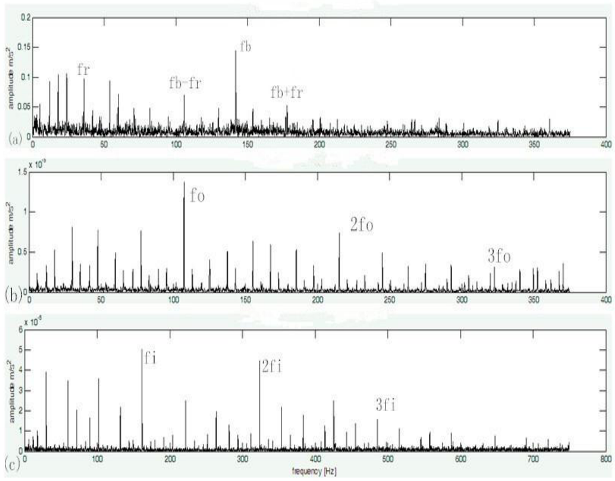

Figure 4. FKDM results of (a) ball fault (b) outer race fault (c) inner race fault bearing signal

5. Conclusion

As shown in section 4, in the diagnosis of outer race fault and ball fault rolling bearing, the FKDM is more advanced compared to conventional envelope analysis. Fault features are much more obvious when applying FKDM. In the case of inner race fault rolling bearing diagnosis, the two methods have both obtained certain fault feature, however, FKDM has obtained the best demodulation band, which would be useful in further research.

By delicately combining Fast Kurtogram with envelope analysis, FKDM not only keeps the advantage of envelope analysis for processing modulation signal, but also overcomes the shortage of finding optimum demodulation band. Moreover, the application result has also indicated that FKDM has a better performance.

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