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# Efficiency Estimation of MIMO Communication System Channel Coefficients Based on Markov Model

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**Abstract.** Based on Markov model of MIMO data transmission system channel coefficients from moving object held synthesis and analysis of channel coefficients error estimation. The model takes into account distribution configuration of reflectors in area, reception center and moving object location at a given time, kind of trajectory and speed of the moving object. A theory discrete Markov processes linear filtration is used by the development of estimation algorithm. Posteriori dispersions matrix is calculated. The simulation of filtering algorithm for different signal-interference situations is carried out.

## 1 Introduction

Due to the expansion of applications of mobile objects to accumulate various types of information arises the problem of transmitting information from mobile object to receiving station. Transmission of optical images and radar high-resolution images in real-time shows increased requirements to channel capacity. Implementation of required bandwidth causes difficulties due to multipath propagation of radio waves from mobile object to receiving station. In these conditions data transmission systems based on MIMO technology [1] are widely used. To realize MIMO channel potential performance it is necessary to measure channel coefficients from each transmit antenna to each receive antenna with high accuracy. The channel estimation errors has a significant impact on channel capacity [2–4]. The reduction of channel coefficients estimation errors can be realize by the most complete account of their statistical properties, as well as development of optimal estimation algorithm. The statistical description of channel coefficients most frequently uses Gaussian probability density distribution. However, the usage of correlation properties of channel coefficients and their dependence on speed of object movement in the estimation algorithm design is not enough to allow potential accuracy. Determination of the optimal mode of channel coefficients matrix estimation is considered in [5], but the possibility of accounting channel coefficients correlation in their estimation also not implemented.

The purpose of the article is to increase the MIMO data transmission system capacity by reduction of channel coefficients estimation error, which is achieved by using the optimal algorithm for filtering channel coefficients based on the Markov model.

## 2 Statement of the Problem

To describe the signal propagation in MIMO channel a geometric one-ring scattering model (Figure 1) [6] is used. Complex channel coefficients between antennas form a channel matrix

$$\underline{\mathbf{H}} = \{h_{nm} = a_{nm} e^{j\psi_{nm}}, n = 1,...,N_R,\ m = 1,...,N_T\},$$

where $a_{nm}$ – signal amplitude transmission coefficient, $\psi_{nm}$ – phase shift of signal, $N_R$, $N_T$ – numbers of receive and transmit antennas respectively. Terrestrial antenna system operates in receive mode, and contains $N_R$ elements of the coordinates $x_n$, $y_n$ in the coordinate system $O_1$, associated with terrestrial antenna system. The antenna system of moving object operates in transmit mode, and contains $N_T$ elements of the coordinates $x_m, y_m$, $m = 1,...,N_T$, in the coordinate system $O_2$, associated with the moving object. The coordinate system $O_2$ is located at point $(S, H)$, which characterizes the position of the moving object, and its dependence on time sets its trajectory. Each of the $N$ scatterers located on a circle of radius $R$ centered at the point $O_2$; coordinates of scatterers are: $x_n = R\cos\phi_n$, $y_n = R\sin\phi_n$. Angular scatterers coordinate $\phi_n = n\dfrac{2\pi}{N}$, $n = 1,...,N$, specify to the uniform distribution of the scatterers along the circumference.

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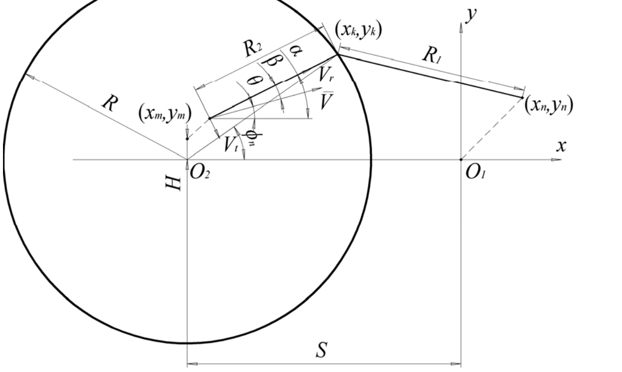


**Figure 1.** A geometric one-ring scattering model

In accordance with the model, channel matrix coefficient is determined by the following expression:

$$\underline{h}_{nm}(t)=\frac{1}{\sqrt{N}}\sum_{k=1}^{N}a_{nm}e^{i2\pi\frac{R_{nk}(t)}{\lambda}}e^{i2\pi\frac{R_{mk}(t)}{\lambda}}e^{i[2\pi f_{d,k}(t)t+\theta_k]}=\frac{1}{\sqrt{N}}\sum_{k=1}^{N}\underline{A}_k(t)e^{i\theta_k}$$

where $e^{i2\pi\frac{R_{mk}(t)}{\lambda}}$ – the coefficient describes the phase shift of the signal from the transmitting antenna to one of scatterers; $e^{i2\pi\frac{R_{nk}(t)}{\lambda}}$ – coefficient describes the phase shift of the signal from one of the scatterers to the receiving antenna, $R_{nk}(t)=\sqrt{(x_k(t)-x_n(t))^2+(y_k(t)-y_n(t))^2}$ – distance from the receiving antenna to one of scatterers, $R_{mk}(t)=\sqrt{(x_k(t)-x_m(t))^2+(y_k(t)-y_m(t))^2}$ – distance from the transmitting antenna to one of scatterers, $\lambda=c/f_0$ – wavelength. Similarly, we write channel transmission rate at time $t+\tau$: $\underline{h}_{nm}(t+\tau)=\frac{1}{\sqrt{N}}\sum_{k=1}^{N}\underline{B}_k(t+\tau)e^{i\theta_k}$. The Doppler frequency shift is determined by radial component of the moving object speed in the direction of the terrestrial antenna system, $f_{dk}=f_{\max}\cos\beta_k$ – the value of the Doppler shift for the $k$-th scatterer, $\cos\beta_k=\frac{(x_k-x_m)\cos\theta-(y_k-y_m)\sin\theta}{\sqrt{(x_k-x_m)^2+(y_k-y_m)^2}}$, $f_{\max}=Vf_0/c$ – the maximum Doppler frequency, $V$ – the speed of moving object, $c=3\times10^8$ m/s – the speed of wave propagation. Phase shift $\theta_k$, introduced by each scatterer, are independent and identically distributed random variables with uniform distribution law in the interval $[0,2\pi)$, and it is assumed to be constant when the calculation of channel coefficients. Attenuation of the signal amplitude when passing from the transmitter to the scatterer, from the scatterer to the receiver, and the reflection coefficient is not take into account when the calculation of the normalized correlation function.

The behavior of channel coefficients in time is important when estimating channel coefficients and capacity of MIMO data transmission system. In general, changes are specified correlation function of each of coefficients, which reflects the rate of change over time and especially changes of module and argument of complex channel coefficients:

$$\underline{K}_{nm}(t,\tau)=\mathbf{M}\{\underline{h}_{nm}(t)\underline{h}_{nm}^{*}(t+\tau)\}=$$
$$=\frac{1}{N}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}\underline{A}_{k_1}(t)\underline{B}_{k_2}^{*}(t+\tau)\mathbf{M}\{e^{i(\theta_{k_1}-\theta_{k_2})}\}.$$

Because phase shift introduced by each scatterer, are independent and identically distributed random variables with uniform distribution law in the interval $[0,2\pi)$, have the equality:

$$\underline{K}_{nm}(t,\tau)=\frac{1}{N}\sum_{k=1}^{N}\underline{A}_k(t)\underline{B}_k^{*}(t+\tau). \quad (1)$$

## 3 Development of Channel Coefficients Model And Filtering

Values of the MIMO communication system channel coefficients matrix correlation $\underline{K}_{nm}(t,\tau)$, $n=1,...,N_R$, $m=1,...,N_T$, obtained in accordance with (1). Figure 2 shows the dependence of channel coefficient correlation function at $m=n=1$, the radius of scatterer ring $R=50$ m, the sampling time interval $\Delta t=1$ μs, the number of scatterers $N=5000$, center frequency of transmitter $f_0=2{,}4$ GHz, the initial distance from the transmitting antenna to reception of the axes $S=1000$ m, $H=100$ m, the velocity of moving object $V=10...40$ m/s. This parameters are typical characteristics of moving objects for urban environment and indoor as well. Represent channel coefficients model in the form of a Markov sequence

$$\mathbf{\underline{H}}_k=\{\underline{H}_{ki}=\underline{h}_{nm}(k),\, i=(n-1)N_R+m\}\rightarrow\mathbf{\underline{x}}_k,\ k=1,2,... .$$

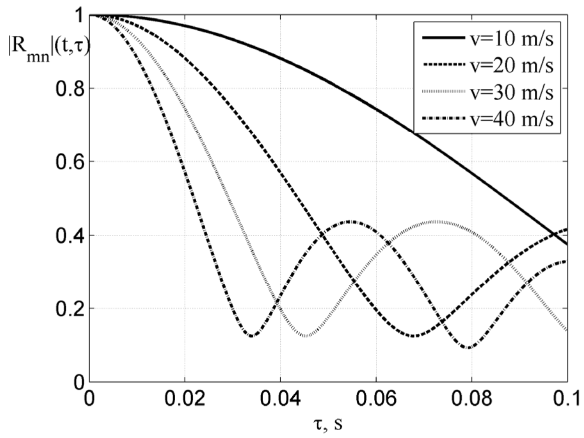


**Figure 2.** Channel coefficient correlation function

Given $M$ correlation matrix values $\mathbf{\underline{K}}_j$, $j=0,...,M$, state vector of the dynamic system $\mathbf{\underline{x}}_k$, $k=1,2,..$, forming all channel coefficients. Dimension of channel coefficients vector is equal $L=N_TN_R$. We also believe that relocation of mobile object is insignificantly during the filtration interval, which suggests that channel coefficients are stationary. The state vector is formed by means of transformation of input sequence in the form of discrete uncorrelated noise $\underline{\mathbf{w}}_k$ with a diagonal intensities matrix $\mathbf{M}\{\underline{\mathbf{w}}_k \underline{\mathbf{w}}_k^{\mathrm{H}}\} = \mathbf{I}$:

$$\underline{\mathbf{x}}_{k+1} = \underline{\mathbf{A}}\underline{\mathbf{x}}_k + \underline{\mathbf{G}}\underline{\mathbf{w}}_k, \quad k = 1,2,..... \qquad (2)$$

To ensure the independence of channel coefficients the size of discrete noise column vector $\underline{\mathbf{w}}_k$ is equal to the number of different channel coefficients $N_{TX}N_{RX}$. Submission process being evaluated in the form of (2) is widely used in optimal filtration theory [7-9].

Matrix of forming filter of size $L \times L$ can be calculated from measurements of correlation matrices with different delays into time [10,11]: $\underline{\mathbf{A}} = \left( \sum_{j=0}^{M-1} \underline{\mathbf{K}}_{j+1} \underline{\mathbf{K}}_j^{\mathrm{H}} \right) \times \left( \sum_{j=0}^{M-1} \underline{\mathbf{K}}_j \underline{\mathbf{K}}_j^{\mathrm{H}} \right)^{-1}$. Assuming a stationary random process matrix $\underline{\mathbf{G}}$ is solution of the following equation $\underline{\mathbf{K}}_0 = \underline{\mathbf{A}}\underline{\mathbf{K}}_0\underline{\mathbf{A}}^{\mathrm{H}} + \underline{\mathbf{G}}\underline{\mathbf{G}}^{\mathrm{H}}$ by Cholesky decomposition: $\underline{\mathbf{G}} = \mathrm{Chol}\left(\underline{\mathbf{K}}_0 - \underline{\mathbf{A}}\underline{\mathbf{K}}_0\underline{\mathbf{A}}^{\mathrm{H}}\right)$. Thus completely determined stochastic differential equation (2), specifies time variation of channel coefficients.

Priory correlation matrix of stationary channel coefficients is equal [10] $\underline{\mathbf{K}}_j = \underline{\mathbf{A}}^j \underline{\mathbf{P}}$, where priory variances matrix determined from the linear algebraic equation:

$$\underline{\mathbf{P}} = \underline{\mathbf{A}}\underline{\mathbf{P}}\underline{\mathbf{A}}^{\mathrm{H}} + \underline{\mathbf{G}}\underline{\mathbf{Q}}\underline{\mathbf{G}}^{\mathrm{H}} = \underline{\mathbf{A}}\underline{\mathbf{P}}\underline{\mathbf{A}}^{\mathrm{H}} + \underline{\mathbf{G}}\underline{\mathbf{G}}^{\mathrm{H}}.$$

Single out the terms containing the matrix posteriori variances:

$$\underline{\mathbf{P}} - \underline{\mathbf{A}}\underline{\mathbf{P}}\underline{\mathbf{A}}^{\mathrm{H}} = \underline{\mathbf{M}}\underline{\mathbf{P}} = \left\{ \underline{P}_{ij} - \sum_{k_1=1}^{L}\sum_{k_2=1}^{L} \underline{A}_{ik_2} \underline{P}_{k_2 k_1} \underline{A}_{jk_1},\ i,j = 1,...,L \right\}. \quad (3)$$

To solve this equation, it is advisable to introduce a matrices $\underline{\mathbf{P}}$, $\underline{\mathbf{A}}$ in vector form in rows:

$$\underline{\mathbf{P}}^{\mathrm{V}} = \left\{ \underline{P}_k^{\mathrm{V}} = \underline{P}_{ij},\ i,j = 1,...,L,\ k = j + (i-1)L \right\},$$

$$\underline{\mathbf{A}}^{\mathrm{V}} = \left\{ \underline{A}_k^{\mathrm{V}} = \underline{P}_{ij},\ i,j = 1,...,L,\ k = j + (i-1)L \right\}.$$

Represent equation (3) in a vectorized form: $\underline{\mathbf{M}}^{\mathrm{V}}\underline{\mathbf{P}}^{\mathrm{V}} = \left(\underline{\mathbf{G}}\underline{\mathbf{G}}^{\mathrm{H}}\right)^{\mathrm{V}}$, where vectorized matrix is the Kronecker product $\underline{\mathbf{M}}^{\mathrm{V}} = \underline{\mathbf{A}} \otimes \underline{\mathbf{A}}^{\mathrm{H}}$. Solution vectorized equation is: $\underline{\mathbf{P}}^{\mathrm{V}} = \left(\mathbf{M}^{\mathrm{V}}\right)^{-1}\left(\underline{\mathbf{G}}\underline{\mathbf{G}}^{\mathrm{T}}\right)^{\mathrm{V}}$.

For optimal criterion of minimum mean squared error channel coefficients estimates used the method of Markov filtering of discrete random sequences [7-9], using developed dynamic model of channel coefficients in the form of a stochastic difference equation (2). The measured values $\underline{\mathbf{Y}}_k = \underline{\mathbf{H}}_k + \underline{\mathbf{E}}_k$ of channel coefficients at each discrete point in time contain measurement error $\underline{\mathbf{E}}_k$, which is assumed to be uncorrelated Gaussian random variable with correlation matrix $\mathbf{M}\{\underline{\mathbf{E}}_k \underline{\mathbf{E}}_k^{\mathrm{H}}\} = \underline{\mathbf{R}} = D_R\mathbf{I}$. Recursive filtering algorithm is given by [8]:

$$\hat{\underline{\mathbf{x}}}_k = \underline{\mathbf{A}}\hat{\underline{\mathbf{x}}}_{k-1} + \underline{\mathbf{K}}_k\left(\underline{\mathbf{Y}}_k - \underline{\mathbf{A}}\hat{\underline{\mathbf{x}}}_{k-1}\right), \qquad (4)$$

where a posteriori variance of channel coefficients estimation determined by the difference equation:

$$\underline{\mathbf{V}}_k = (\mathbf{I} - \underline{\mathbf{K}})\tilde{\underline{\mathbf{V}}}_{k-1},\ \tilde{\underline{\mathbf{V}}}_k = \underline{\mathbf{A}}\underline{\mathbf{V}}_{k-1}\underline{\mathbf{A}}^{\mathrm{H}} + \underline{\mathbf{G}}\underline{\mathbf{Q}}\underline{\mathbf{G}}^{\mathrm{H}}. \quad (5)$$

Expressions (4) and (5) compose optimal filtering algorithm, and matrix $\underline{\mathbf{V}}_k$ sets the channel coefficients posteriori dispersion, and its diagonal coefficients are estimation error variance.

## 4 Analysis of Channel Coefficients Estimating Errors

If channel coefficients are formed by difference equation (2), the channel coefficient estimation error variance determined by diagonal matrix elements $\underline{\mathbf{V}}_k$. Variances matrix of channel coefficient estimation errors can also be obtained by means of statistical modeling filtration equation (4) and averaging over $N_S$ realizations:

$$\underline{\mathbf{D}}_{ps\_x} = \frac{1}{N_S}\sum_{k=1}^{N_S}\left(\underline{\mathbf{x}}_k - \hat{\underline{\mathbf{x}}}_k\right)\left(\underline{\mathbf{x}}_k - \hat{\underline{\mathbf{x}}}_k\right)^{\mathrm{H}}.$$

For more comprehensive taking into account the properties of channel coefficients can, it is necessary to use values $\underline{\mathbf{H}}$ for recording the measured values at discrete time $\underline{\mathbf{Y}}_k = \underline{\mathbf{H}}_k + \underline{\mathbf{E}}_k$, and equation (4) when the simulation. Matrix of posteriori variances

$$\underline{\mathbf{D}}_{ps\_H} = \frac{1}{N_S}\sum_{k=1}^{N_S}\left(\underline{\mathbf{H}}_k - \hat{\underline{\mathbf{H}}}_k\right)\left(\underline{\mathbf{H}}_k - \hat{\underline{\mathbf{H}}}_k\right)^{\mathrm{H}}.$$

is obtained by the simulation and it is more accurate than $\underline{\mathbf{D}}_{ps\_x}$, since more fully take into account the form of the correlation function (1).

When technical implementation of channel coefficients estimation algorithms (4) and (5) must perform a certain number of arithmetic operations in real time. This requires computing power of embedded processor: computing speed and amount of memory. Equations (5) can be solved in advance and results stored in memory, the volume of which is equal $L(L-1)/2$ complex words. To compute channel coefficients vector estimates (4) is necessary to make $L(2L+1)$ additions and $2L^2$ multiplications of complex words. Bit depth representation of result would provide a digital representation error is not more of optimal estimation error. Processor performance is determined by time of formation of a valid channel coefficients estimation, which for modern communication systems is 0.1...100 µs.

Considering the example of MIMO system with $N_{TX} = N_{RX} = 2$ and 8-core TMS320C6678 signal processor with 1 GHz clock speed, it can be shown that necessary computing time can be 68 ns. Comparing this time with time interval necessary to update channel coefficients (Figure 1), which is about 50 μs, we conclude that technical implementation of proposed algorithm is possible.

## 5 Conclusion

Investigated Markov model of MIMO data transmission system channel coefficients from mobile object, taking into account different conditions of radio waves propagation and reflections. Application of channel coefficients filtering algorithm provides potential estimation accuracy. However, for implementation of optimal estimation algorithm need to know correlation properties of channel coefficients that as shown in this work depend on specific practical situations. Computational costs for technical implementation of proposed channel coefficients estimating algorithm are realized with modern signal processors.

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## References

1. I. Emre Telatar. *Capacity of multi-antenna Gaussian channels*, European Transactions on Telecommunications, **vol. 10**, No 6, (1999), pp.585–595.
2. V.T. Ermolayev, A.G. Flaksman, I.P. Kovalyov, I.M. Averin. *Weight error loss in MIMO systems using eigenchannel technique*. Proceedings of the 1st International Conference on Antenna Theory and Techniques (ICATT'03), Sevastopol, Ukraine, (2003), pp. 333–336.
3. D.D.N Bevan., V.T. Ermolayev, A.G. Flaksman. *Analysis of weight error loss with a multichannel beamformer processor*. IEE Proceedings - Radar, Sonar and Navigation, (1998), **vol. 145**, No 1, pp. 63–72.
4. D.D.N Bevan., V.T.Ermolayev, A.G. Flaksman. *Coherent multichannel reception of binary modulated signals with dependent Rician fading*. IEE Proceedings Communications, (2001), **vol. 148**, No 2, pp. 105–111.
5. Yu.N.Parshin, V.I. Kudryashov. *Analysis of information transmission channel capacity from unmanned aerial vehicle with incorrect channel matrix*. Journal of Ryazan State Radio Engineering University, (2015), No 52, pp. 19–24.
6. M. Patzold, B. O. Hogstad. *A space-time channel simulator for MIMO channels based on the geometrical one-ring scattering model*. Wireless Communications and Mobile Computing. Special Issue on Multiple–Input Multiple–Output (MIMO) Communications, (2004), **vol. 4**, No 7, pp. 727–737.
7. M.S. Yarlykov. *Application of Markov nonlinear filtering theory in radio*. – M .: Sov. radio, (1980). – 360 p.
8. A.P.Sage, J.L. Melse. *Estimation Theory with Application to Communication and Control*. – N.–Y.: McGraw-Hill, (1972). – 496 p.
9. Yu.G. Sosulin. *Detection and Estimation Theory of Stochastic Signals.* – M.: Sov. radio, (1978). – 320 p.
10. R.I. Ivanovsky. *Theory of Probability and Mathematical Statistics. Basics, Practical Aspects with Examples and Problems in MathCad Environment.* – SPb: BHV–Petersburg, (2008) – 528 p.
11. Kuznetcov D.F. Numerical Simulation of Stochastic Differential Equations and Stochastic Integrals. – SPb .: Science, 1999. – 458 p.