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# A Simple Dual Decomposition Method for Resource Allocation in Telecommunication Networks

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**Abstract.** We consider a problem of optimal resource allocation in a wireless communication network divided into zones (clusters). The network manager aims to distribute some homogeneous resource (bandwidth) among users of several zones in order to maximize the total network profit, which takes into account payments from users and implementation costs. As a result, we obtain a convex optimization problem involving capacity and balance constraints. By using the dual Lagrangian method with respect to the capacity constraint, we reduce the initial problem to a suitable one-dimensional problem, so that calculation of its cost function value leads to independent solution of zonal problems, treated as two-side auction models with one trader. We show that solution of each zonal problem can be found exactly by a simple arrangement type algorithm even in the case where the trader price is not fixed. Besides, we suggest ways to adjust the basic problem to the case of moving nodes. Some results of computational experiments confirm the applicability of the new method.

## 1 Introduction

The current development of telecommunication systems creates a number of new challenges of efficient management mechanisms for efficient allocation of limited communication networks resources. In fact, despite the existence of powerful processing and transmission devices, increasing demand of different communication services and its variability lead to serious congestion effects and inefficient utilization of network resources (e.g., bandwidth and batteries capacity), especially in wireless telecommunication networks. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. [1-3]. The problem is to suggest such models and to develop suitable solution methods. Usually, the decision making processes are based on solutions of the corresponding optimization problems. At the same time, experience of dealing with these very complicated and spatially distributed systems usually shows that these problems have to utilize a proper decomposition/clustering approach, which can be based on zonal, time, frequency and other attributes of nodes/units; see e.g. [4, 5].

In this paper, we consider one of such problems, i.e. optimal allocation of a homogeneous resource in a zonal telecommunication network such that the income received from users payments is maximized and the implementation costs of the network operator are minimized; see [6, 7]. The network manager problem consists in optimal distribution of the resource shares among zones in order to maximize the total network profit. As a result, we obtain a convex optimization problem involving capacity and balance constraints. By using the dual Lagrangian method with respect to the capacity constraint, we reduce the initial problem to a suitable one-dimensional problem, so that calculation of its cost function value leads to independent solution of zonal problems, treated as two-side auction models with one trader. We show that solution of each zonal problem can be found exactly by a simple arrangement type algorithm even in the case where the trader price is not fixed. In such a way we develop a new dual decomposition approach for solution finding, whose implementation is simpler essentially in comparison with the methods from [6, 7]. We present results of computational experiments which confirm the applicability of the new method.

## 2 Notation and the problem statement

Let us consider a telecommunication network with nodes attributed to users (consumers) which is divided into zones (clusters). The problem of a manager of the network is to find the optimal allocation of a limited homogeneous network resource among the zones. That is, the optimal shares should maximize the value of the total profit containing the total income received from consumers' fees and negative resource implementation costs.

Let us use the following notation:

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- $n$ is the number of zones;
- $I_k$ is the index set of users (currently) located in zone $k$ ($k=1,\dots,n$);
- $B$ is the total resource supply (the total bandwidth) for the system (network);
- $x_k$ is an unknown quantity of the resource allotted to zone $k$ with the upper bound $b_k$ and $f_k(x_k)$ is the cost of implementation of this quantity of the resource for zone $k$ ($k=1,\dots,n$);
- $y_i$ is the resource amount received by user $j$ with the upper bound $a_i$ and $\varphi_i(y_i)$ is the charge value paid by user $j$ for the resource value $y_i$.

The problem of the network manager can be written as follows:

$$\max \to \mu(x,y)=\sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-f_k(x_k)\right], \qquad (1)$$

subject to

$$\sum_{i\in I_k} y_i = x_k,\ k=1,\dots,n; \qquad (2)$$

$$0\le y_i \le a_i,\ i\in I_k,\ k=1,\dots,n; \qquad (3)$$

$$\sum_{k=1}^{n} x_k \le B; \qquad (4)$$

$$0\le x_k\le b_k,\ k=1,\dots,n. \qquad (5)$$

That is, (2) provides the balance for demand and supply in each zone, (3) and (5) are capacity constraints for users and network supply values in each zone, respectively, and (4) gives the upper bound for the total resource supply. The goal of the network manager is to maximize the total network profit subject to all these constraints.

In what follows we assume that there exists at least one feasible point satisfying conditions (2)–(5), each function $f_k(x_k)$ is convex and differentiable, and all the functions $\varphi_i(y_i)$ are affine, i.e.

$$\varphi_i(y_i)=\alpha_i y_i+\beta_i,\ \alpha_i>0,\ i\in I_k,\ k=1,\dots,n. \qquad (6)$$

This means that the prices (marginal utilities) $\alpha_i$ of the users are fixed, but the manager can vary the prices depending on volumes, so that each zonal price is a non-increasing function.

## 3 Dual solution method

Under the basic assumptions of the previous section, (1)–(5) is a differentiable convex optimization problem, which has a solution since its feasible set is bounded. Hence it can be found by a great number of iterative methods; see e.g. [8, 9]. However, the problem of selection of an efficient decomposition method here is not trivial task since problem (1)–(5) has $n+1$ functional constraints (2) and (4) and many box type ones. For instance, utilization of the standard duality approach which is based on defining the Lagrangian function with respect to all the functional constraints leads to a non-smooth dual convex optimization problem in $n+1$ dual variables, whose solution may cause certain difficulties. For this reason, we intend to apply a special dual method, which takes into account peculiarities of this problem and does not require hard implementation procedures.

Let us first define the Lagrange function of problem (1)–(4) as follows:

$$L(x,y,\lambda)=\mu(x,y)-\lambda\left(\sum_{k=1}^{n}x_k-B\right),$$

i.e. we insert only the term corresponding to the the upper bound constraint for the total resource supply (4) with the Lagrangian multiplier $\lambda$. At the same time, we keep the zonal balance constraints (2) as well as the capacity constraints (3) and (5).

Hence, we can write the one-dimensional dual problem:

$$\min_{\lambda\ge 0} \to \psi(\lambda), \qquad (7)$$

where

$$\psi(\lambda)=\sup_{(x,y)\in W} L(x,y,\lambda)$$
$$=\sup_{(x,y)\in W}\sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-f_k(x_k)-\lambda x_k\right]$$
$$+\lambda B;$$

where

$$W=\left\{(x,y)\ \middle|\ \begin{array}{l}\sum_{i\in I_k} y_i = x_k,\\ 0\le y_i\le a_i,\ i\in I_k,\\ 0\le x_k\le b_k,\ k=1,\dots,n\end{array}\right\}.$$

By duality (see e.g. [8, 9]), problems (1)–(5) and (7) have the same optimal value. However, solution of (7) can be found by one of well-known one-dimensional optimization algorithms based on calculation of values of $\psi(\lambda)$. We now discuss this problem in more detail. The main element in calculation of $\psi(\lambda)$ is a solution of the problem:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i)-f_k(x_k)-\lambda x_k\right] \qquad (8)$$

subject to

$$\sum_{i\in I_k} y_i = x_k,\ 0\le y_i\le a_i,\ i\in I_k,$$

$$0\le x_k\le b_k,\ k=1,\dots,n.$$

However, this problem decomposes into $n$ independent zonal convex programming problems

$$\max \to \left[ \sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - \lambda x_k \right] \tag{9}$$

subject to

$$\sum_{i\in I_k} y_i = x_k,\ 0 \le y_i \le a_i,\ i \in I_k,$$

$$0 \le x_k \le b_k,$$

for $k = 1,\dots,n$. Hence, we have to suggest a simple and efficient algorithm for the basic problem (9).

Set $y(k) = (y_i)_{i\in I_k}$,

$$W_k = \left\{ (x_k, y(k)) \left| \begin{array}{l} \sum_{i\in I_k} y_i = x_k, \\ 0 \le y_i \le a_i,\ i \in I_k, \\ 0 \le x_k \le b_k \end{array} \right. \right\},$$

then

$$W = \prod_{k=1}^{n} W_k.$$

The necessary and sufficient optimality condition for problem (9) in view of (6) is written in the form of the variational inequality: find $(\bar{x}_k, \bar{y}(k)) \in W_k$ such that

$$\begin{array}{l} (f'_k(\bar{x}_k) + \lambda)(x_k - \bar{x}_k) - \sum_{i\in I_k} \alpha_i (y_i - \bar{y}_i) \ge 0 \\ \forall (x_k, y(k)) \in W_k. \end{array} \tag{10}$$

This is a two-sided auction type equilibrium problem with one seller and several buyers; see e.g. [10, 11]. It is equivalent to the problem of finding a feasible vector $(\bar{x}_k, \bar{y}(k)) \in W_k$ and a cutting price $\bar{p}_k$ such that

$$f'_k(\bar{x}_k) + \lambda \begin{cases} \ge \bar{p}_k & \text{if } \bar{x}_k = 0, \\ = \bar{p}_k & \text{if } \bar{x}_k \in (0, b_k), \\ \le \bar{p}_k & \text{if } \bar{x}_k = b_k, \end{cases} \tag{11}$$

and

$$\alpha_i \begin{cases} \le \bar{p}_k & \text{if } \bar{y}_i = 0, \\ = \bar{p}_k & \text{if } \bar{y}_j \in (0, a_i), \quad i \in I_k. \\ \ge \bar{p}_k & \text{if } \bar{y}_j = a_i, \end{cases} \tag{12}$$

Since buyers' prices are fixed, we can re-arrange them to be non-increasing and then find easily an intersection point of the staircase-wise inverse common demand and offer price $f'_k(x_k) + \lambda$ lines; see also [11]. Therefore, an exact solution of problem (10) or (9) (hence (8)) can be found explicitly by simple ordering type algorithms, although (9) contain a non-linear function in general. In other words, calculation of values of $\psi(\lambda)$ can be accomplished by several independent simple ordering type algorithms. Notice that the re-arrangement of bid prices $\alpha_i$ in each zone should be made only one time that reduces the computational expenses essentially in comparison with the general duality approach. So, having the optimal value $\lambda^*$ of problem (7), we can find a solution of problem (1)–(5) by solving problem (8) with $\lambda = \lambda^*$, i.e. it is accomplished within the main calculation process for (7).

## 4 Adjustment for the case of moving nodes

In the above model it was assumed that users locations are fixed. We now intend to suggest some adjustments of the above model to networks with more complex and non-stationary behavior of users (nodes), which is typical for various modern wireless telecommunication systems; see e.g. [2, 12].

We consider the above problem of the network manager for some time slot. In this case we need some additional information about the behavior of users (nodes). It was suggested by I. Konnov (see e.g. [13]) to treat each moving node in a wireless network as a separate Markovian chain.

In order to create such a model, we determine a suitable grid $\mathsf{G}$ covering the domain of the network so that $\mathsf{G}_k$ denotes the index set of all the cells belonging to zone $k$. Next, we consider the discrete time model and suppose that, given a user (node) $j$, we can determine the starting probability vector $\pi^{j,(0)}$, whose components $\pi_\sigma^{j,(0)}$ give its probabilities to be in cell $\sigma \in \mathsf{G}$ by time slot (stage) 1, and the probability $\tilde{\pi}_{\sigma\tau}^{j}$ (for the simplicity of exposition, it is supposed to be independent of time) of the one stage transition $\sigma \to \tau$ for each pair $\sigma, \tau \in \mathsf{G}$. Knowing the starting and transition probability vectors for each node $j$, we can calculate its probability $\pi_\sigma^{j,(t-1)}$ to be in cell $\sigma \in \mathsf{G}$ by a selected slot $t$ via the standard Markovian chain technique (see e.g. [14]). Afterwards we calculate the value

$$\tilde{p}_k^{j,(t-1)} = \sum_{\sigma\in \mathsf{G}_k} \pi_\sigma^{j,(t-1)}$$

for each zone $k$ and assign user $j$ to zone $l$ where the probability $\tilde{p}_l^{j,(t-1)}$ is maximal, i.e. then $j \in I_l$.

Therefore, we can solve the same problem (7)–(9) with this assignment and obtain the desired resource allocation for time slot $t$ in the case of moving nodes.

Similarly, if

$$\lim_{m\to\infty} (\Pi^j)^m = \overline{\Pi}^j \tag{13}$$

for each probability matrix $\tilde{\Pi}^j = (\tilde{\pi}_{\sigma\tau}^{j})_{\{\sigma,\tau\in \mathsf{G}\}}$, behavior of each user is stable and we can evaluate the optimal resource allocation for a long-time stationary period by calculation of the limit probabilities

$$\overline{\pi}_{\tau}^{j} = \sum_{\sigma \in G} \pi_{\sigma}^{j,(0)} \overline{\pi}_{\sigma\tau}^{j} \quad for \ \tau \in G$$

and set

$$\overline{p}_{k}^{j} = \sum_{\sigma \in G_k} \overline{\pi}_{\sigma}^{j}$$

for each $j$. Then we can assign user $j$ to zone $l$ where the probability $\overline{p}_{l}^{j}$ is maximal, i.e. then $j \in I_l$ and solve problem (7)–(9) with this assignment and obtain the long-time resource allocation strategy.

However, this approach can not be used if the limit in (13) does not exist. Then we can apply the statistical approach and calculate the probabilities on-line as it was suggested in [15]. After $t$ time slots we can determine the value

$$p_{k}^{j,t} = s_{j,k}(t)/t,$$

for each user $j$ and for each zone $k$, where $s_{j,k}(t)$ denotes the number of time slots when user $j$ was in zone $k$. It is treated as some approximation of the probability of user $j$ to be in zone $k$. We set

$$\overline{p}_{k}^{j} = p_{k}^{j,t}$$

if

$$\left\{ \sum_{j \in I} \sum_{k=1}^{n} (p_{k}^{j,t} - p_{k}^{j,t-1})^2 \right\}^{1/2} \leq \delta$$

for $\delta > 0$ small enough, where $I$ denotes the index set of all the users. Then we utilize the values $\overline{p}_{k}^{j}$ as above in order to assign each user $j$ to some zone $l$. Solution of problem (7)–(9) with this assignment gives the long-time resource allocation strategy.

## 5 Numerical experiments

In order to evaluate efficiency of the new method we made several series of computational experiments. Since the case of moving nodes yields the same mathematical model (1)–(5) we restricted ourselves with the fixed case.

We utilized the golden section method for solving the single-dimensional optimization problem (7). The programs were coded in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable $\lambda$ were taken as [0,1000]. Values of $b_k$ were chosen by trigonometric functions in $[1,11]$, values of $a_i$ were chosen by trigonometric functions in $[1,2]$. The functions $f_k(x_k)$ were chosen to be convex quadratic, all the coefficients of $f_k(x_k)$ and $\varphi_i(y_i)$ were chosen with the help of trigonometric functions. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution. The processor time and number of iterations, which gave an approximate solution of problem (7) within the same accuracy, were not significantly different for these two cases of distributions. We made calculations with 1000 test examples for each set of parameters, their average values are indicated in each row of the tables.

Further we report the results of tests, which include the time and number of iterations needed to find a solution of problem (7) within some accuracies. Let $\varepsilon$ denote the desired accuracy of finding an approximate solution of problem (7). Let $J$ denote the total number of users, $N_\varepsilon$ the number of upper iterations in $\lambda$, $T_\varepsilon$ the total processor time in seconds. The results of computations are given in Tables 1–3. In Table 1, we vary the accuracy $\varepsilon$, in Tables 2 and 3 we vary the total number of users and the number of zones, respectively.

**Table 1.** Results of testing with $J = 510$, $n = 70$.

| $\varepsilon$ | $N_\varepsilon$ | $T_\varepsilon$ |
|---|---|---|
| $10^{-1}$ | 20 | 0.0003 |
| $10^{-2}$ | 24 | 0.0004 |
| $10^{-3}$ | 29 | 0.0008 |
| $10^{-4}$ | 34 | 0.0007 |

**Table 2.** Results of testing with $n = 70$, $\varepsilon = 10^{-2}$.

| $J$ | $N_\varepsilon$ | $T_\varepsilon$ |
|---|---|---|
| 210 | 24 | 0.0001 |
| 310 | 24 | 0.0003 |
| 410 | 24 | 0.0004 |
| 510 | 24 | 0.0004 |
| 610 | 24 | 0.0008 |
| 710 | 24 | 0.0012 |
| 810 | 24 | 0.0011 |
| 910 | 24 | 0.0014 |
| 1010 | 24 | 0.0018 |

**Table 3.** Results of testing with $J = 510$, $\varepsilon = 10^{-2}$.

| $n$ | $N_\varepsilon$ | $T_\varepsilon$ |
|---|---|---|
| 5 | 24 | 0.0001 |
| 15 | 24 | 0.0002 |
| 25 | 24 | 0.0001 |
| 35 | 24 | 0.0001 |
| 45 | 24 | 0.0002 |
| 55 | 24 | 0.0002 |
| 65 | 24 | 0.0002 |
| 75 | 24 | 0.0002 |
| 85 | 24 | 0.0003 |
| 95 | 24 | 0.0004 |
| 105 | 24 | 0.0002 |

From the results we can conclude that the performance of the new method is satisfactory for applications.

## 6 Conclusions

Equations should be centred and should be numbered with the number on the right-hand side. In this work, we considered a problem of managing limited resources in a zonal wireless telecommunication network and gave its constrained convex optimization problem formulation. We proposed a new dual decomposition method, which reduces the initial problem to a sequence of simple zonal convex optimization problems, they can be solved by efficient ordering type algorithms despite the nonlinear cost network functions. The results of the numerical experiments confirmed the rapid convergence of these methods. We also suggested ways to adjust the problem to the case of moving nodes.

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