

Detection of emergent situations in complex systems represented by algebras of transformations

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Abstract. In this paper we will investigate emergent situations in complex systems represented by algebras of transformations. Though algebras of transformations seem to be rather distanced from a modeled complex system we show that it is representation very effective and for our application very appropriate. The paper continues in recent published works where the detection of an emergent situation was done by indication of violence of so called structural invariants. In this paper is used only one type of structural invariant Matroid and Matroid Bases (M, BM). A simple calculus for the emergent situation appearance computation is introduced. The application of the presented approach and computation method is demonstrated for the case of a possible collapse of the selected ecosystem.

1 Introduction

The paper continues in topics and technologies introduced in papers [1], [2], [3] and [4]. Especially papers [2] and [4] are needed for the effective reading and the understanding this paper. Similarly as in paper [1] we start here with 5 working hypotheses:

Hypothesis 1(H1): The emergent phenomenon is induced by a sharp change of complex system structure (“jump on the structure”). ♦

Hypothesis 2(H2): In case that we accept *H1*, the eventuality of an emergent phenomenon appearance may be detected as a sharp violence of complex system structure (or the violence of Structural Invariants respectively). ♦

Hypothesis 3(H3): The appearance of emergent situation is detected as the possibility of extension (reduction) of the basis of the matroid (matroid formed on the complex system) by at least one element. ♦

Hypothesis 4(H4): One of possibilities how to represent detection of appearance of emergent situation by extension of a matroid basis is to anticipate the increase of number of interacting elements in so called basic group (compartment). ♦

Hypothesis 5(H5): In order to attain an emergent phenomenon (by an extension (reduction) of a matroid basis) the complex system increases the number of elements (transformations, properties, processes) in *basic group* (compartment) by a minimum number of elements (transformations, properties, processes). ♦

Notes to organization of the paper. In section 2 there are introduced some works that are relevant to the topic of our paper. The description of the theory and computation method is introduced in section 3. In section 4, is a case

study for application of the method for a possible collapse of the selected ecosystem.

2 Related works

The proposed paper is associated especially with results published in works [1, 2, 3] and [4]. In these papers is presented as well theoretical material for computing with emergent situations as application of the developed method in phenomena of macro-world ($10^{-2}\text{m}\div 10^2\text{m}$) and also in phenomena of supermicro-world ($10^{-10}\text{m}\div 10^{-15}\text{m}$).

General background for conceptual constructions in the field of Complex Systems are published in [5, 6, 7]. Work [10] forms the background for application of our method in the field of ecosystems. It is fair to introduce attempts for the description of irregularities in the ecosystem quantifying them by means of some special variables, e.g., fractal dimension, [11, 12]. The application of the proposed detection method for the considered ecosystem (Trebou basin in South Bohemia, Czech Republic) is easy to deploy for the super ecosystem of the planet and compare the results with existing results acquired from quantitative models (e.g., [13]).

A (small) common negative point of all cited works (including this proposed paper) is an absence of a chaotic phase and self-organizing phenomenon from expression (1).

3 Background for the detection of emergent situations

The simple scheme for the detection of emergent situations by means of algebras of transformations [2] is

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the following one:

$$G1 \rightarrow (G1 \oplus g) \rightarrow \text{Chaotic phase} \rightarrow \text{EP} \rightarrow \text{SOP} \rightarrow G2, \quad (1)$$

where $G1, G2$ are algebras of transformations characterizing complex system in situations of balances and g is a set of transformations that extends (reduces) kernel part of $G1$. (In case that is used matroid approach the kernel part will be the basis of the matroid.) Symbol \oplus has no specific significance and depends on a real case of method application. EP symbolizes “emergent phenomenon” and SOP is “self organizing process”.

In paper [1] were discussed problems of “chaotic phases” and “self-organizing processes” that need more detailed investigation however not necessarily introduced here.

For the description of the discussed complex system will be applied state model. Transformations transfer the system from one state to another state and the only problem is to detect a possible appearance of an emergent situation. For it is used a common decision rule:

$$\text{IF } (\#g \geq \min \Delta f(\text{RN})) \Rightarrow \text{THEN (PAES)}, \quad (2)$$

where “ $\min \Delta f(\text{RN})$ ” is a minimal difference between further and actual Ramsey number and PAES is “a possible appearance of an emergent situation”.

According to *H1* (from Introduction) – the emergent phenomenon is induced by a sharp change of complex system structure, i.e., by “a jump on the structure” and the eventuality of an emergent phenomenon appearance may be detected as a sharp violence of complex system structure (or the violence of so called Structural Invariants respectively – *H2*).

There have been tested the following violences of SIs that induce the appearance of Emergent Situation (EMS).

- (Matrix, Rank of Matrix) – the addition of linearly independent row or column.
- (Matroid, Bases of Matroid) – the addition of at least one element to basis.
- (Dulmage-Mendelsohn Decomposition, Tree Ordering) – violence of tree ordering.
- (Hasse Diagram, Set of Associated Rules) – violence of set of rules,
- (Set of Situations, Algebra of Transformations) - the deformation of group operation and as a consequence - construction a false element into the group carrier.

In the following sub-section we will deal only with one SI - (Matroid, Bases of Matroid) – (M, BM).

3.1. Structural Invariant (B, BM) and its violence

At first a few general facts:

Matroid has the following pleasant properties:

- It is possible to construct it for each set of elements (carrier of the system) when we have the relation of independence or when they are given independent sets.
- If we have relation of independence, there are investigated all elements (of the system) with regard to relation of independence.

- If the relation of independence exists it is easy (in most cases) to associate it with a semantic content (according to real conditions).
- Matroid is usually introduced as the following structure

$$M = \langle X, \text{IND}, \{N_1, N_2, \dots, N_n\} \rangle = \langle X, B \rangle, \quad (3)$$

where X is the ground set of elements (components), IND is a relation of independence, N_1, N_2, \dots, N_n are independent sets and B is a set of matroid bases. Matroid bases are maximum (according to cardinality) independent sets.

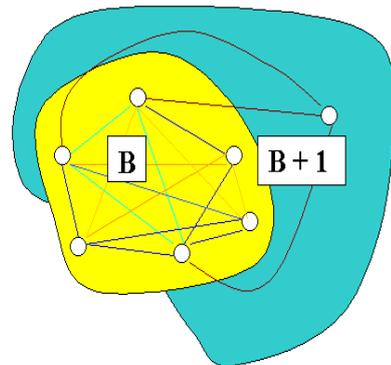


Figure 1. Extension of a matroid basis by one element.

The violence of this SI is considered (in this paper) as an extension of a matroid basis at least by one element (and it will be considered as an indicator of emergent situation appearance).

In case that relation IND is considered as a *binary relation* it is possible to use the following consequences:

The bases (B) will be constructed as perfect sub-graphs (in a perfect graph on X).

The independent and dependent elements in the perfect graph are easy constructed by coloring the edges by *two* colors and the formalism of Ramsey numbers – $R(\#B, \#Y), B \in B$ is offered to be used.

The extension of one of bases $B \in B$ by one element is illustrated in Fig.1. The perfect graph in Fig.1 has six nodes and 15 (brown) edges. Coloring the edges by green and blue colors, there appears at least one perfect sub-graph with 3 nodes and 3 edges (basis B) – for example the green one. For extension of B by one element (into $B+1$ with 4 nodes) we need to add at least 3 elements (that are invisible here).

Historical note: Till now there are known only some Ramsey numbers (RNs), e.g.: $R(3, 3) = 6, R(3, 4) = 9, R(3, 5) = 14, R(3, 6) = 18, \dots, R(3, 15) = [73, 78], \dots, R(4, 4) = 18, R(4, 11) = [96, 191], \dots, R(6, 10) = [177, 1171], \dots, R(10, 10) = [798, 23\ 556], \dots, R(19, 19) \geq 17\ 885$. The brackets $[., .]$ denote intervals of integer numbers. (In computations in section 4 will be used known table quantities from [9].)

Note 3.1: According to hypothesis *H4* (from the Introduction) the system selects as optimal those Ramsey numbers for which is needed to add the minimum elements for the extension of basis by one element.

Example 3.1.: #X = 1600. (#B = 11 for #X ≥ 1597) and for one element extension of Basis (#B = 12 for #X ≥ 1637) is needed to add at least 40 elements.

4 Detection of a possible collapse of an ecosystem

In paper [10] has been modeled the situation of the violence of so called Short Water Cycle in a selected ecosystem (Trebos basin in South Bohemia, Czech Republic). The Short Water Cycle (SWC) refers to the behavior of the local ecosystem (e.g., the Trebos region),

in which the volume of water that comes into the ecosystem is evaporated and falls back into this system. In the Trebos ecosystem, the evaporated water rises quickly inside the transport zone and does not have time to recondense before it is transported outside the ecosystem to the distant mountains, where it condenses spontaneously in the rising air streams. (Due to the enormous volumes of vapor that are transported, the condensation is very dynamic and sometimes leads to torrential downpours).

The ecosystem was described (for the conditions of the violence of SWC) by 19 states – Table. 1.

Table 1. States of the modeled ecosystem.

Field of observation	Content of the state
<i>Name of the state</i>	
Air humidity and water	
S1	<i>Low local humidity</i>
S2	<i>Medium local humidity</i>
S3	<i>High local humidity</i>
S4	<i>Local fog</i>
S5	<i>Regional fog (covering an area greater than 20 km²)</i>
S6	<i>High volume of water absorbed in the soil</i>
S7	<i>Local floods</i>
S8	<i>Violation of the small water cycle (SWC)</i>
Weather	
S9	<i>Rain</i>
S10	<i>Snow</i>
S11	<i>Longtime local dry atmosphere, (arid soil)</i>
S12	<i>Semi-clear weather</i>
S13	<i>Very cloudy weather and overcast</i>
S14	<i>Strong wind</i>
S15	<i>Storm</i>
Evaporation	
S16	<i>High evaporation (nearly no water goes back into the ecosystem)</i>
S17	<i>Medium evaporation – (some of the evaporated water returns back)</i>
S18	<i>Low evaporation</i>
S19	<i>Diminished evapotranspiration</i>

Note 4.1: The number of states could be substantially larger however after a few experiments we recognized that substantially larger number of states do not bring substantially greater amount of information.

In the introduced set of states has been discovered (with help of a qualified expert) the set of transformations – Table. 2 (“1” transformation possible, “0” transformation impossible, “*” irrelevant question).

Table 2. Matrix of transformations in considered ecosystem.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19
S1	1	1	0	0	0	0	*	*	*	*	1	*	*	*	*	1	1	1	1
S2	0	1	1	1	0	0	*	0	1	0	0	0	1	*	0	0	1	1	0
S3	0	1	1	1	1	1	1	0	*	*	0	*	*	*	*	1	1	0	0
S4	0	0	1	1	1	*	*	*	1	1	0	1	1	*	0	0	1	1	0
S5	0	1	1	1	1	*	*	0	1	1	0	1	1	*	1	0	0	1	0
S6	0	1	1	1	0	1	1	0	*	*	0	1	1	*	*	0	1	1	0
S7	0	1	1	1	0	1	1	0	1	0	0	1	1	1	1	0	0	1	0
S8	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	1	0	0	1
S9	0	1	1	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0
S10	0	0	0	0	0	1	1	0	1	1	0	1	1	0	0	0	0	1	0
S11	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	1
S12	*	*	*	0	0	1	*	*	1	1	0	1	1	0	0	0	1	1	0
S13	*	*	*	0	0	*	*	0	0	0	0	1	1	1	*	0	1	1	0
S14	*	*	*	0	0	*	*	*	*	*	0	1	1	1	1	1	0	0	0
S15	0	0	1	1	0	0	1	0	1	0	0	0	0	0	1	0	0	1	0
S16	1	1	0	0	0	0	0	1	0	0	1	1	0	*	1	1	1	0	1
S17	1	1	0	1	1	1	0	0	1	1	0	1	0	*	1	0	1	1	0
S18	1	0	0	0	0	0	*	1	0	0	1	1	0	*	1	0	0	1	0
S19	1	0	0	0	0	0	0	1	0	0	1	*	*	*	*	1	0	0	1

Now we apply the theory from section 3. The number of transformations in Table.2 is **141**. Using Ramsey numbers from [9] we find a few possible results for basis of matroid with 141 elements: $\#B \in \{4, 5, 7, 10, 14\}$. ($R(4, 10) \in [80, 149]$, $R(4, 14) \in [141, 349]$, $R(5, 7) \in [80, 143]$, $R(5, 10) \in [141, 442]$.) The minimal number of transformations that is necessary to add to X (in order to attain an emergence phenomenon by extension of the matroid basis by one element) is **12** ($R(4, 14) \in [141, 349]$, $R(4, 15) \in [153, 417]$). We may find in Table.2 additional transformations (τ_{ai}), not discovered still, that could be really *dangerous* for the ecosystem:

- $\tau_{a1} = S1 \rightarrow S14,$ (4)
- $\tau_{a2} = S1 \rightarrow S15,$ (5)
- $\tau_{a3} = S2 \rightarrow S14,$ (6)
- $\tau_{a4} = S2 \rightarrow S15,$ (7)
- $\tau_{a5} = S1 \rightarrow S16,$ (8)
- $\tau_{a6} = S14 \rightarrow S18,$ (9)
- $\tau_{a7} = S12 \rightarrow S19,$ (10)
- $\tau_{a8} = S5 \rightarrow S11,$ (11)
- $\tau_{a9} = S11 \rightarrow S6,$ (12)
- $\tau_{a10} = S3 \rightarrow S11,$ (13)
- $\tau_{a11} = S3 \rightarrow S14,$ (14)
- $\tau_{a12} = S12 \rightarrow S7.$ (15)

The description of the introduced ecosystem by 19 states represents rather an approximation of a state model. The needed number of states could be for about 100 (and as a consequence it induces large number of transformations). Nevertheless even in our case has already been seen that

the higher number of elements in the carrier of transformation algebra brings

- higher variability of emergent phenomenon (here registered only by a change of elements and by number of elements in the matroid basis),
- higher number of elements that are necessary to add to matroid carrier for the extension of matroid basis.

The second item could be understood as a reason why the standard ecosystems are relatively stable and *sufficiently distanced from collapse situations*.

State description that has been used is a qualitative knowledge based description. (Expert used a knowledge based concepts as “Local floods”, “Semi-clear weather”, etc. that are on the used ontology understandable however they have rather complicated internal structure. This is a great advantage of such qualitative description not to be obliged to consider interaction of an enormous number of elements of ecosystem.)

Another direction for the discussion could be done by question “why was not used technology of Markov chains or fuzzy logic?”. Well-they could be used. But we are not interested in state trajectories of the system. We are interested in sharp structural changes of the system that we consider as emergent phenomena or appearance of emergent situations. From this point of view Markov chains technology and also fuzzy logic would not bring any new information.

Detection of emergent situations in ecosystems is a very actual field of research especially in relation to climate changes. However it requires clear and simple methods.

The presented approach is a small contribution to this trend.

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5 Conclusions

In continuation of paper [2] there was applied in this paper the detection method for emergent situations appearances in complex systems. The method introduced in [1] was reduced here only for computation with numbers of matroid bases (without relations to the power of emergent phenomena). The proposed method provides only a necessary condition for an emergent situation appearance. The sufficient condition depends on self-organizing process that forms an external view of emergent situation (and it is not discussed here).

The proposed method has been applied for a selected ecological system. The model of this system was developed as an expert knowledge based state model. This state model decreased complexity of the original ecosystem (without necessity to consider all relevant elements and bonds). For such a model was easy to apply proposed detection method. (The same approach has been used for ant colonies and bee communities though the elements of these systems seemed to be more attainable.)

Acknowledgement: The development of this paper has been supported by Research Grant SGS12/177/OHK2/3T/12. This support is very gratefully acknowledged.

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