

# Fourth-order Perturbed Eigenvalue Equation for Stepwise Damage Detection of Aeroplane Wing

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**Abstract.** Perturbed eigenvalue equations up to fourth-order are established to detect structural damage in aeroplane wing. Complete set of perturbation terms including orthogonal and non-orthogonal coefficients are computed using perturbed eigenvalue and orthonormal equations. Then the perturbed eigenparameters are optimized using BFGS approach. Finite element model with small to large stepwise damage is used to represent actual aeroplane wing. In small damaged level, termination number is the same for both approaches, while rms errors and termination d-norms are very close. For medium damaged level, termination number is larger for third-order perturbation with lower d-norm and smaller rms error. In large damaged level, termination number is much larger for third-order perturbation with same d-norm and larger rms error. These trends are more significant as the damaged level increases. As the stepwise damage effect increases with damage level, the increase in stepwise effect leads to the increase in model order. Hence, fourth-order perturbation is more accurate to estimate the model solution.

## 1 Introduction

To solve physical problem without exact solution, perturbation theory is established to obtain their approximate solution. A general approach is carried out using various perturbation order expansions on its equation. Adopting the advanced modeling method of element stiffness matrix modification, the order of the structure stiffness matrix was kept invariable in establishing the model of intact and damaged structures by Yu et al. [1]. First-order eigenvalue perturbation theory is introduced to obtain the eigenvalues and eigenvectors of the damaged structure. In the first-order perturbed eigenvalue equation of Wang et al. [2], modal strain energy was used as damage indicator in the damage detection of simply supported beam. Second-order perturbation technique was developed by Chen et al. [3] for damage detection of simply supported beam. When the extent of damage was small, first-order perturbation equation was adopted. Meanwhile the extent was large, second-order perturbation equation was employed to improve the identification precision.

A general order perturbation method involving multiple perturbation parameters was developed by Wong et al. [4] for eigenvalue problems with changes in the stiffness parameters. Symmetric perturbation solutions and eigenparameter sensitivities of different orders were derived. The perturbed eigenvalue problem was established from the perturbations of stiffness matrix, eigenvector, and eigenvalue. Then stiffness parameters of fixed-fixed modular beam was estimated from this equation using the DFP approach [5]. The perturbed

orthonormal equation was generated from the perturbation of the eigenvectors and eigenvalues to obtain the  $k$ -th skew-symmetric coefficients[6].

In this effort, third and fourth-order perturbation equations are established. Complete set of perturbation terms including orthogonal and non-orthogonal coefficients are computed using perturbed eigenvalue and orthonormal equations. Then the perturbed eigenparameters are optimized using BFGS approach. Limited number of eigenparameters are used to detect different level stepwise damages within aeroplane wing. Finite element model with variational elastic moduli is used to represent the aeroplane wing.

## 2 Derivation of fourth-order perturbation equation

The stiffness parameters of the healthy structure prior to any damage are denoted by  $G_{hi}$  ( $i=1,2,\dots,m$ ), where  $m$  is the number of the stiffness parameters. Structural damage is characterized by reductions in the stiffness parameters. The estimated stiffness parameters of the damaged structure before each iteration are denoted by  $G=[G_1, G_2, \dots, G_m]^T$ , and its stiffness matrix, which depends linearly on  $G_i$ , is denoted by  $K=K(G)$ . The eigenvalue equation of the structure with stiffness parameters  $G_i$  is

$$K\phi^k = \lambda^k M\phi^k \quad (1)$$

where  $M$  is the constant mass matrix, and  $\lambda^k, \phi^k$  are the  $k$ -th eigenvalue and mass-normalized eigenvector

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respectively. It is noted that  $\lambda_k = \omega_k^2$ , where  $\omega_k$  is the  $k$ -th natural frequency of the structure. The normalized eigenvectors of Eq.(1) satisfy the orthonormality relations

$$(\phi^k)^T M \phi^u = \delta_{ku}, \quad (\phi^k)^T K \phi^u = \lambda^k \delta_{ku} \quad (2)$$

where  $1 \leq u \leq N$  and  $\delta_{ku}$  is the Kronecker delta. Before the first iteration, the initial stiffness parameters of the damaged structure are assumed to be  $G_i^{(0)} = \sigma_i G_{hi}$  ( $i=1,2,\dots,m$ ), where  $0 < \sigma_i \leq 1$ , and the eigenvalue equation in Eq.(1) corresponds to that of the structure with stiffness parameters  $G_i^{(0)}$ . If there is no prior knowledge of the integrity of the structure, one can start the iteration from the stiffness parameters of the ‘‘healthy’’ structure and set  $\sigma_i = 1$ . Let  $G_{di}$  ( $i=1,2,\dots,m$ ) denote the stiffness parameters of the damaged structure. The eigenvalue equation of the damaged structure is

$$K_d \phi_d^k = \lambda_d^k M \phi_d^k \quad (3)$$

where  $K_d = K(G_d)$  is the damaged stiffness matrix, and  $\lambda_d^k$ ,  $\phi_d^k$  are the  $k$ -th eigenvalue and mass-normalized eigenvector respectively. As  $K$  is a linear function of  $G_d$ , first-order Taylor expansion of  $K_d$  is

$$K_d = K + \sum_{i=1}^m \frac{\partial K}{\partial G_{di}} \delta G_{di} \quad (4)$$

where  $\delta G_{di} = G_{di} - G_i$  ( $i=1,2,\dots,m$ ) are the changes in the stiffness parameters. Note that based on its finite element model, the system stiffness matrix of a distributed structure satisfies Eq.(4) because the higher-order derivatives of the stiffness matrix with respect to the elemental stiffness parameters vanish.

Let the  $k$ -th eigenvalue and mass-normalized eigenvector of the damaged structure be related to  $\lambda^k$  and  $\phi^k$  through

$$\lambda_d^k = \lambda^k + \sum_{i=1}^m \lambda_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m \lambda_{(2)ij}^k \delta G_i \delta G_j + \dots + \sum_{i=1}^m \sum_{j=1}^m \dots \sum_{t=1}^m \lambda_{(p)ij\dots t}^k \delta G_i \delta G_j \dots \delta G_t + \varepsilon_\lambda^k \quad (5)$$

$$\phi_d^k = \phi^k + \sum_{i=1}^m z_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m z_{(2)ij}^k \delta G_i \delta G_j + \dots + \sum_{i=1}^m \sum_{j=1}^m \dots \sum_{t=1}^m z_{(p)ij\dots t}^k \delta G_i \delta G_j \dots \delta G_t + \varepsilon_\phi^k \quad (6)$$

where  $\lambda_{(1)i}^k, \lambda_{(2)ij}^k \dots \lambda_{(p)ij\dots t}^k$  are the coefficients of the first, second, ..., and  $p$ -th order perturbations for the eigenvalue.  $z_{(1)i}^k, z_{(2)ij}^k \dots z_{(p)ij\dots t}^k$  are the coefficient vectors of the first, second, ..., and  $p$ -th order perturbations for the eigenvector.  $\varepsilon_\lambda^k$  and  $\varepsilon_\phi^k$  are the residuals of order  $p+1$ . Note that the numbers in the parentheses in the subscripts

of the coefficients and coefficient vectors indicate the orders of the terms. By the Taylor expansion, one has for any  $p \geq 1$ ,

$$p! \lambda_{(p)ij\dots t}^k = \frac{\partial^p \lambda^k}{\partial G_i \partial G_j \dots \partial G_t}$$

$$p! z_{(p)ij\dots t}^k = \frac{\partial^p \phi^k}{\partial G_i \partial G_j \dots \partial G_t} \quad (7)$$

By Eq.(7),  $\lambda_{(p)ij\dots t}^k$  and  $z_{(p)ij\dots t}^k$  are symmetric in the  $p$  indices,  $i, j, \dots$ , and  $t$ . The right-hand sides of Eq.(7) are the  $p$ -th order sensitivities of the eigenvalues and eigenvectors with respect to the stiffness parameters. Using the normalization relations of the eigenvectors,  $\phi^k$  and  $\phi_d^k$ , and symmetry of the coefficient vectors in Eq.(6), as indicated earlier, one obtains the perturbed orthonormal equation:

$$1 = (\phi_d^k)^T M \phi_d^k$$

$$= (\phi^k + \sum_{i=1}^m z_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m z_{(2)ij}^k \delta G_i \delta G_j + \dots + \underbrace{\sum_{i=1}^m \sum_{j=1}^m \dots \sum_{t=1}^m z_{(p)ij\dots t}^k \delta G_i \delta G_j \dots \delta G_t}_{p \text{ summations}} \delta G_t + \dots)^T$$

$$M (\phi^k + \sum_{i=1}^m z_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m z_{(2)ij}^k \delta G_i \delta G_j + \dots + \underbrace{\sum_{i=1}^m \sum_{j=1}^m \dots \sum_{t=1}^m z_{(p)ij\dots t}^k \delta G_i \delta G_j \dots \delta G_t}_{p \text{ summations}} \delta G_t + \dots)$$

$$= 1 + \sum_{i=1}^m [(\phi^k)^T M z_{(1)i}^k + (z_{(1)i}^k)^T M \phi^k] \delta G_i +$$

$$\sum_{i=1}^m \sum_{j=1}^m \frac{1}{R_{ij}} \{2!(\phi^k)^T M z_{(2)ij}^k + [(z_{(1)i}^k)^T M z_{(1)j}^k + (z_{(1)j}^k)^T M z_{(1)i}^k] + 2!(z_{(2)ij}^k)^T M \phi^k\} \delta G_i \delta G_j +$$

$$\sum_{i=1}^m \sum_{j=1}^m \sum_{l=1}^m \frac{1}{R_{ijl}} \{3!(\phi^k)^T M z_{(3)jil}^k + 2![(z_{(1)i}^k)^T M z_{(2)jl}^k + (z_{(1)j}^k)^T M z_{(2)il}^k + (z_{(1)l}^k)^T M z_{(2)ij}^k] +$$

$$2![(z_{(2)jl}^k)^T M z_{(1)i}^k + (z_{(2)il}^k)^T M z_{(1)j}^k + (z_{(2)ij}^k)^T M z_{(1)l}^k] +$$

$$+ 3!(z_{(3)jil}^k)^T M \phi^k\} \delta G_i \delta G_j \delta G_l + \dots + \underbrace{\sum_{i=1}^m \sum_{j=1}^m \dots \sum_{t=s}^m}_{p \text{ summations}}$$

$$\frac{1}{R_{ij\dots st}} \{p!(\phi^k)^T M z_{(p)ij\dots st}^k + (p-1)![(z_{(1)i}^k)^T M z_{(p-1)jl\dots st}^k + (z_{(1)j}^k)^T M z_{(p-1)il\dots st}^k + \dots + (z_{(1)l}^k)^T M z_{(p-1)ij\dots st}^k] +$$

$$2!(p-2)![(z_{(2)ij}^k)^T M z_{(p-2)l\dots st}^k + (z_{(2)il}^k)^T M z_{(p-2)j\dots st}^k + \dots + (z_{(2)st}^k)^T M z_{(p-2)ij\dots r}^k] + \dots + (p-2)!2!$$

$$\begin{aligned} & [(z_{(p-2)l \dots st}^k)^T Mz_{(2)ij}^k + (z_{(p-2)j \dots st}^k)^T Mz_{(2)il}^k + \dots \\ & + (z_{(p-2)ij \dots r}^k)^T Mz_{(2)st}^k] + (p-1)! [(z_{(p-1)jl \dots st}^k)^T Mz_{(1)i}^k + \\ & (z_{(p-1)il \dots st}^k)^T Mz_{(1)j}^k + \dots + (z_{(p-1)ij \dots s}^k)^T Mz_{(1)t}^k] + \\ & p! (z_{(p)ij \dots st}^k)^T M\phi^k \delta G_i \delta G_j \dots \delta G_s \delta G_t + \dots \end{aligned} \quad (8)$$

where the superscript  $T$  denotes transpose. Expanding  $z_{(3)ijl}^k$  using normalized eigenvectors of Eq.(1) as basis vectors, one has

$$z_{(3)ijl}^k = \sum_{u=1}^n P_{(3)ijlu}^k \phi^u \quad (9)$$

where  $P_{(3)ijlu}^k$  is the coefficient of the  $u$ -th term and the number in the parentheses in its subscript corresponds to that of  $z_{(3)ijl}^k$ . Equating the coefficients of the  $\delta G_i \delta G_j \delta G_l$  terms in Eq.(8) and using symmetry of  $M$  and  $z_{(3)ijl}^k$  yields

$$P_{(3)ijlk}^k = -\frac{2!}{3!} \left[ (z_{(1)i}^k)^T Mz_{(2)jl}^k + (z_{(1)j}^k)^T Mz_{(2)li}^k + (z_{(1)l}^k)^T Mz_{(2)ij}^k \right] \quad (10)$$

Expanding  $z_{(4)ijlr}^k$  using normalized eigenvectors of Eq.(1) as basis vectors, one has

$$z_{(4)ijlr}^k = \sum_{u=1}^n P_{(4)ijlru}^k \phi^u \quad (11)$$

where  $P_{(4)ijlru}^k$  is the coefficient of the  $u$ -th term and the number in the parentheses in its subscript corresponds to that of  $z_{(4)ijlr}^k$ . Equating the coefficients of the  $\delta G_i \delta G_j \delta G_l \delta G_r$  terms in Eq.(8) and using symmetry of  $M$  and  $z_{(4)ijlr}^k$  yields

$$\begin{aligned} P_{(4)ijlr}^k = & -\frac{1}{48} \left\{ 6[(z_{(1)i}^k)^T Mz_{(3)jlr}^k + (z_{(1)j}^k)^T Mz_{(2)ilr}^k + \right. \\ & (z_{(1)l}^k)^T Mz_{(3)ijr}^k + (z_{(1)r}^k)^T Mz_{(3)ijl}^k] + 8[(z_{(2)ij}^k)^T Mz_{(2)lr}^k + \\ & (z_{(2)il}^k)^T Mz_{(2)jr}^k + (z_{(2)ir}^k)^T Mz_{(2)jl}^k + (z_{(2)jl}^k)^T Mz_{(2)ir}^k + \\ & (z_{(2)jr}^k)^T Mz_{(2)il}^k + (z_{(2)lr}^k)^T Mz_{(2)ij}^k] + 6[(z_{(3)jlr}^k)^T Mz_{(1)ir}^k + \\ & (z_{(3)ilr}^k)^T Mz_{(1)jr}^k + (z_{(3)ijr}^k)^T Mz_{(1)l}^k + (z_{(3)ijl}^k)^T Mz_{(1)r}^k] \left. \right\} \end{aligned} \quad (12)$$

Substituting Eqs.(4-6) into Eq.(3) yields the perturbed eigenvalue equation:

$$\left\{ K + \sum_{i=1}^m \frac{\partial K}{\partial G_i} \delta G_i \right\} \left\{ \phi^k + \sum_{i=1}^m z_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m z_{(2)ij}^k \delta G_i \delta G_j + \sum_{i=1}^m \sum_{j=1}^m \sum_{l=1}^m z_{(3)ijl}^k \delta G_i \delta G_j \delta G_l + \dots \right\}$$

$$\begin{aligned} = & \left\{ \lambda^k + \sum_{i=1}^m \lambda_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m \lambda_{(2)ij}^k \delta G_i \delta G_j + \right. \\ & \left. \sum_{i=1}^m \sum_{j=1}^m \sum_{l=1}^m \lambda_{(3)ijl}^k \delta G_i \delta G_j \delta G_l + \dots \right\} M \\ & \left\{ \phi^k + \sum_{i=1}^m z_{(1)i}^k \delta G_i + \sum_{i=1}^m \sum_{j=1}^m z_{(2)ij}^k \delta G_i \delta G_j + \right. \\ & \left. \sum_{i=1}^m \sum_{j=1}^m \sum_{l=1}^m z_{(3)ijl}^k \delta G_i \delta G_j \delta G_l + \dots \right\} \end{aligned} \quad (13)$$

Equating the coefficients of the  $\delta G_i \delta G_j \delta G_l$  terms in Eq.(13) to obtain the third-order coefficient equation. Pre-multiplying by  $(\phi^k)^T$  and using Eqs.(1,2) yields

$$\begin{aligned} \lambda_{(3)ijl}^k = & \frac{2!}{3!} (\phi^k)^T \left[ \frac{\partial K}{\partial G_i} z_{(2)jl}^k + \frac{\partial K}{\partial G_j} z_{(2)li}^k + \frac{\partial K}{\partial G_l} z_{(2)ij}^k \right. \\ & \left. - \lambda_{(1)i}^k Mz_{(2)jl}^k - \lambda_{(1)j}^k Mz_{(2)li}^k - \lambda_{(1)l}^k Mz_{(2)ij}^k \right] \end{aligned} \quad (14)$$

Pre-multiplying the coefficient equation by  $(\phi^v)^T$ , where  $1 \leq v \leq N$  and  $v \neq k$ , and using Eqs.(1,2,9) yields

$$\begin{aligned} P_{(3)ijlv}^k = & \frac{2!}{3! (\lambda^k - \lambda^v)} (\phi^v)^T \left[ \frac{\partial K}{\partial G_i} z_{(2)jl}^k + \right. \\ & \frac{\partial K}{\partial G_j} z_{(2)li}^k + \frac{\partial K}{\partial G_l} z_{(2)ij}^k - \lambda_{(1)i}^k Mz_{(2)jl}^k \\ & - \lambda_{(1)j}^k Mz_{(2)li}^k - \lambda_{(1)l}^k Mz_{(2)ij}^k - \lambda_{(2)il}^k Mz_{(1)j}^k \\ & \left. - \lambda_{(2)li}^k Mz_{(1)i}^k - \lambda_{(2)ij}^k Mz_{(1)l}^k \right] \end{aligned} \quad (15)$$

By Eqs.(10,15) we have determined  $z_{(3)ijl}^k$ . Equating the coefficients of the  $\delta G_i \delta G_j \delta G_l \delta G_r$  terms in Eq.(13) to obtain the fourth-order coefficient equation. Pre-multiplying by  $(\phi^k)^T$  and using Eqs.(1,2) yields

$$\begin{aligned} \lambda_{(4)ijlr}^k = & \frac{1}{4!} (\phi^k)^T \left[ 3! \left( \frac{\partial K}{\partial G_i} z_{(3)jlr}^k + \frac{\partial K}{\partial G_j} z_{(3)ilr}^k + \right. \right. \\ & \frac{\partial K}{\partial G_l} z_{(3)ijr}^k + \frac{\partial K}{\partial G_r} z_{(3)ijl}^k - 3! (\lambda_{(1)i}^k Mz_{(3)jlr}^k + \lambda_{(1)j}^k Mz_{(3)ilr}^k + \\ & \lambda_{(1)l}^k Mz_{(3)ijr}^k + \lambda_{(1)r}^k Mz_{(3)ijl}^k) - 2! 2! (\lambda_{(2)ij}^k Mz_{(2)lr}^k + \\ & \lambda_{(2)il}^k Mz_{(2)jr}^k + \lambda_{(2)ir}^k Mz_{(2)jl}^k + \lambda_{(2)jl}^k Mz_{(2)ir}^k + \lambda_{(2)jr}^k Mz_{(2)il}^k \\ & \left. \left. + \lambda_{(2)lr}^k Mz_{(2)ij}^k) - 2! 2! (\lambda_{(2)lr}^k Mz_{(2)ij}^k + \lambda_{(2)il}^k Mz_{(2)jr}^k + \right. \right. \\ & \left. \left. \lambda_{(2)il}^k Mz_{(2)ir}^k + \lambda_{(2)ir}^k Mz_{(2)jl}^k + \lambda_{(2)il}^k Mz_{(2)jr}^k + \lambda_{(2)ij}^k Mz_{(2)lr}^k) \right. \right. \end{aligned} \quad (16)$$

Pre-multiplying the coefficient equation by  $(\phi^v)^T$ , where  $1 \leq v \leq N$  and  $v \neq k$ , and using Eqs.(1,2,11) yields

$$\begin{aligned}
 P_{(4)ijlrv}^k &= \frac{1}{4!(\lambda^k - \lambda^v)} (\phi^v)^T \left[ 3! \left( \frac{\partial K}{\partial G_i} z_{(3)jlr}^k + \right. \right. \\
 &\left. \left. \frac{\partial K}{\partial G_j} z_{(3)ilr}^k + \frac{\partial K}{\partial G_l} z_{(3)ijr}^k + \frac{\partial K}{\partial G_r} z_{(3)ijl}^k \right) - 3! (\lambda_{(1)i}^k Mz_{(3)jlr}^k + \right. \\
 &\left. \lambda_{(1)j}^k Mz_{(3)ilr}^k + \lambda_{(1)l}^k Mz_{(3)ijr}^k + \lambda_{(1)r}^k Mz_{(3)ijl}^k) - 2!2! \right. \\
 &\left. (\lambda_{(2)ij}^k Mz_{(2)lr}^k + \lambda_{(2)il}^k Mz_{(2)jr}^k + \lambda_{(2)ir}^k Mz_{(2)jl}^k + \lambda_{(2)jl}^k Mz_{(2)ir}^k \right. \\
 &\left. + \lambda_{(2)jr}^k Mz_{(2)il}^k + \lambda_{(2)lr}^k Mz_{(2)ij}^k) - 3! (\lambda_{(3)jlr}^k Mz_{(1)i}^k + \lambda_{(3)ilr}^k Mz_{(1)j}^k \right. \\
 &\left. + \lambda_{(3)ijr}^k Mz_{(1)l}^k + \lambda_{(3)ijl}^k Mz_{(1)r}^k) \right] \quad (17)
 \end{aligned}$$

Using Eqs. 12,17 we have determined  $z_{(4)ijlrv}^k$ . To investigate the applicability on different orders perturbed eigenvalue equations, various stepwise damaged cases are introduced to wing model. Then these equations are optimized by BFGS algorithm[7].

### 3 Geometric dimension of aeroplane wing

XX-XXX series aeroplane belongs to air-jet type passenger aeroplane. It is a short distance twin turbo-engine aeroplane. It meets the requirements of middle and short distance flights, reliable, simple and fast, with economical operational and maintenance characteristics. Passenger capacity is around 120, while maximum flight distance is 5648 miles. Main geometric dimensions of aerodynamic wing are discussed here. Wing span  $W_s$  is the length between left and right aeroplane wings. Wing chord is the chord length along the body. Besides the rectangular wing, the wing chord is different at different position. At wing tip, wing chord is  $b_0$ , while at wing root, wing chord is  $b_1$ . Fig.1 shows the ordinarily dimension used in wing chord is the mean geometric chord length  $b_m$  calculated by  $b_m = (b_0 - b_1)/2$ .

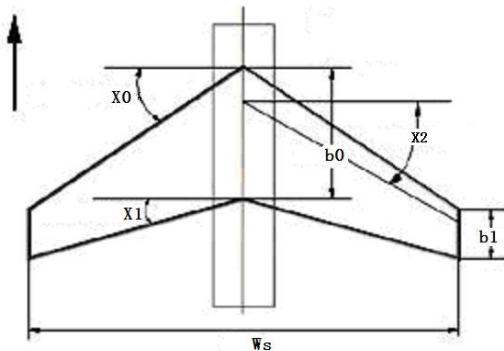


Figure 1. Span-chord dimensions of aeroplane wing.

Another geometric dimension is the span-chord aspect ratio. Indicated as  $A_{sc}$ , its calculation formula is  $A_{sc} = W_s/b_m$ . Simultaneously, aspect ratio can be

represented as the ratio between square of wing span and area of wing surface. When  $A_{sc}$  increases, wing lift force coefficient increases, and its drag force increases simultaneously. Therefore, small  $A_{sc}$  is utilized in high speed aeroplane wing.

### 4 Stepwise damage detection of variational wing model

Finite element model of the wing with variational chord sections is used to model its planar transverse vibrations. Length of the model is  $L_i = 15.97m$ . Linear variations are imposed on the dimensions of different wing chords by this length. Thus  $4.65 \leq b_i \leq 2.91m$ ,  $0.937 \leq h_i \leq 0.234m$  where  $i$  is the chord number. For example, at the middle chord, width  $b_m = 3.78m$ , thickness  $h_m = 0.63m$ , area moment of inertia  $I_m = b_m h_m^3/12 = 7.86 \times 10^{-2} m^4$ , mass density  $\rho = 2715 kg/m^3$  and healthy Young's modulus  $E_m = 1.0 \times 10^{10} N/m^2$ . Then its healthy elastic modulus is  $W_{c_m} = E_m I_m$ . The wing is divided into  $N_e = 6$  elements with length of each element being  $l_e = L_i/N_e$ , and there are  $N_e + 1$  nodes. First five eigenparameter pairs are used with  $N = 5$ . The force moment diagrams of the wing subjected to shear force and bending moment are illustrated in Fig.2. From the diagram, one can the region around wing root and aeroengine takes up the peak loads. This accounts for the peak damage level at first three chords. Thus the extra random damage range of 0.2-0.3 is situated at first three chords, while at second three chords extra random damage range of 0-0.2 is assumed. They are namely small, medium and large damaged levels for 0.1, 0.4 and 0.7 respectively.

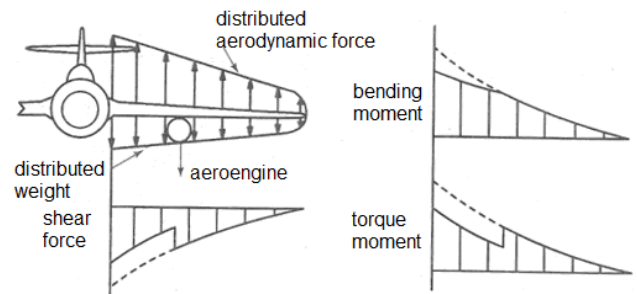
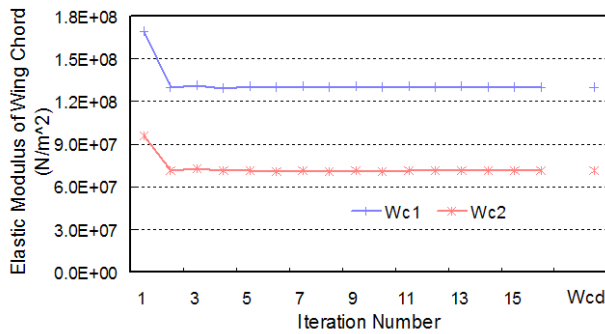


Figure 2. Load curves of variational wing model.

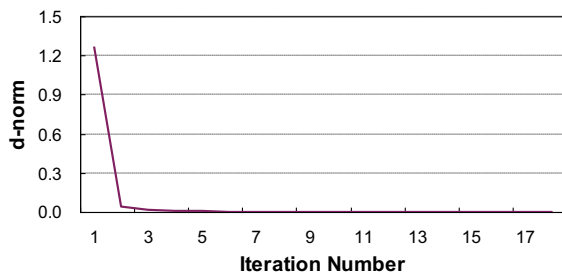
Three damaged levels are implemented to cater for the whole stepwise damaged range. The first case illustrates the small stepwise damaged level of  $0.1 < \sigma_i \leq 0.4$ . Using fourth-order perturbation, the d-norm drops rapidly from  $2.92 \times 10^{-1}$  to  $2.66 \times 10^{-2}$  in second iteration. Afterwards, it converges gradually to the  $2.13 \times 10^{-5}$  termination level. For the elastic moduli of wing chords,  $W_{c_1}$  and  $W_{c_2}$  change rapidly in two to three iterations as shown in Fig.3. The other chords  $W_{c_3}$  to  $W_{c_6}$  follow the similar patterns. Then they converge smoothly to the correct values in 16<sup>th</sup> iteration with rms error of

0.174%. Similar iteration patterns with same termination number are observed for the third-order perturbation. However, it's rms error is slightly larger at 0.178%, while it's termination d-norm is slightly higher at  $2.26 \times 10^{-5}$ .



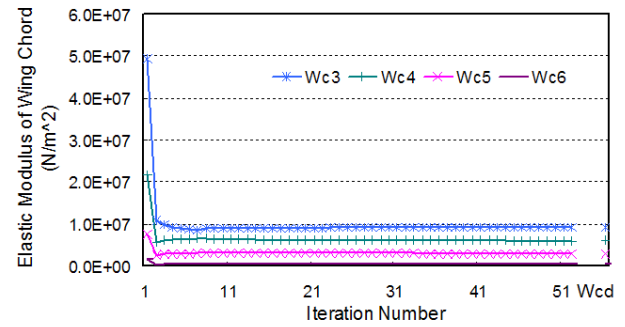
**Figure 3.** Elastic moduli of wing model in small stepwise damage detection ( $0.1 < \sigma_i \leq 0.4$ ).

Medium stepwise damaged level of  $0.4 < \sigma_i \leq 0.7$  is shown in second case. Convergence rate is faster for the fourth-order perturbation case, in which termination number is 18. From Fig.4, the d-norm drops rapidly from 1.26 to  $4.12 \times 10^{-2}$  in second iteration. Then it converges gradually to the  $2.03 \times 10^{-5}$  termination level. In the initial region, the Wcs change rapidly. Gradual convergence patterns are observed until 18<sup>th</sup> iteration. It's rms error is 0.192%. Larger termination number of 21 is recorded for third order perturbation case, while its iteration patterns are similar with lower d-norm of  $1.26 \times 10^{-5}$ . Slightly smaller rms error of 0.180% is resulted.



**Figure 4.** d-norm of wing model in medium stepwise damage detection ( $0.4 < \sigma_i \leq 0.7$ ).

Third case illustrates the large stepwise damaged level of  $0.7 < \sigma_i \leq 1.0$ . For fourth-order perturbation, the d-norm drops rapidly from 19.2 to 3.44 in second iteration. Afterwards, it converges to the  $1.87 \times 10^{-5}$  % termination level gradually. Considering the elastic moduli of wing chords,  $Wc_3$  to  $Wc_6$  change rapidly in two to three iterations (Fig.5). Similar patterns are obtained for  $Wc_1$  and  $Wc_2$ . Then they converge smoothly to the correct values in 52<sup>th</sup> iteration with rms error of 0.265%. In the third order perturbation, termination number is larger at 76. It's rms error is larger at 0.465%, while it's termination d-norm is exactly the same.



**Figure 5.** Elastic moduli of wing model in large stepwise damage detection ( $0.7 < \sigma_i \leq 1.0$ ).

## 5 Summary

Finite element model of aeroplane wing is constructed. Third and fourth-order perturbation equations with orthogonal and non-orthogonal coefficients are computed using perturbed eigenvalue and orthonormal equations. In small stepwise damaged level, termination number is the same for both approaches, while rms error and termination d-norm are very close. For medium stepwise damaged level, termination number is larger for third-order perturbation with lower d-norm and smaller rms error. In large damaged level, termination number is much larger for third-order perturbation with same d-norm and larger rms error. This trend is more significant as the damaged level increases. This indicates that the increase in stepwise effect leads to the increase in model order. Hence, fourth-order perturbation is more accurate to approximate the model solution.

## Acknowledgements

This research is supported by the National Natural Science Foundation of China under grant numbers 50905028 and 51275554.

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