Mathematical Modeling the Hydrological Properties of Soil for Practical Use in the Land Ecological Management

Vitaly Terleev¹, Ekaterina Petrovskaya^{1,}, Aleksandr Nikonorov¹, Vladimir Badenko¹, Yulia Volkova¹, Sergey Pavlov¹, Natalia Semenova², Kirill Moiseev³, Alex Topaj³, and Wilfried Mirschel⁴*

¹Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya, 29, St. Petersburg, 195251, Russia

²St. Petersburg State University, Universitetskaya naberezhnaya, 7/9, St. Petersburg, 199034, Russia

³Agrophysical Research Institute, Grazhdanskii pr. 14, St. Petersburg, 195220, Russia

⁴Leibniz-Centre for Agricultural Landscape Research, Eberswalder Straße 84, Müncheberg, 15374, Germany

Abstract. An original and convenient (from a practical point of view) method to estimate the supply of productive moisture in the soil is offered. The method is based on a physically adequate mathematical model of the soil hydrological properties considering the hysteresis of the water-retention capacity. The computation of the irrigation rates, which is based on such estimates, minimizes the water wastage if the excess of the gravitational water is formed and this water percolates out of the moisturized soil profile under watering conditions. The practical applying of the method is able to optimize the crop irrigation techniques, eliminates any inefficient losses of irrigation water and nutrients (and other agricultural chemicals), promotes the rational usage of the water resources as well as provides developing effective solutions of urgent problems of the land ecological management.

1 Introduction

The ecological management of lands is rather relevant under the climate change and the intensification of anthropogenic impact [1-4]. The assessments of weak soils and drained lands are quite important [5-7]. The researches on the use of water intakes and protecting designs of hydraulic structures, as well as for the protection of areas from floods, are very urgent [8-10]. Limitation of natural resources, in particular water resources, raises the question of their protection and rational use of the designers of ameliorative systems, as well as the specialists operating. The problem of preventing admission of nutrients and other agriculture chemicals to the natural groundwater and surface water applies to the topical issues of the environment management.

* Corresponding author: kate-petro@yandex.ru

The economic and ecological significance of this problem is obvious: the removal of nutrients and other agricultural chemicals out of the root zone of the soil by means of a downward water flux creates unproductive losses of mineral fertilizers, pesticides and land reclamation chemicals. This poses a threat of eutrophication and pollution of natural waters. For this reason, in the construction of water management objects as well as at the irrigation agriculture there is a challenge to develop energy-saving technologies of crops irrigation, ensuring the protection of natural waters and the rational water resources usage. The development of such technologies involves the use of a significant amount of data on the reclaimed lands in particular – data on hydrological properties of the soils. The direct measurements of the soil hydrological indicators are the rather labor-consuming procedures. Therefore, for the indirect quantitative estimation of these indicators and for prediction of the state of ecosystems it is widely applied mathematical modeling techniques [11-18].

Consider one of such techniques on the example for water-retention capacity (WRC) of the soil. This property is usually described in the form of the functional dependence of the volumetric water content θ [$\text{cm}^3 \cdot \text{cm}^{-3}$] on the capillary pressure (capillary-sorption potential) of the soil moisture ψ [$\text{cm H}_2\text{O}$] [19-21]. The $\theta(\psi)$ dependence is widely used in the calculations of the soil moisture dynamics [12, 22]. To the characteristic features of soil water-retention capacity a phenomenon of hysteresis refers. In the world literature quite a lot of research in relation to this phenomenon is represented. They revealed multiple significant differences between the wetting and drying branches of the WRC hysteretic loop [23-26]. Due to hysteresis, one ψ value corresponds to an infinite number of θ values, which, depending on the «background» of the wetting-draining cycles of the soil, may differ essentially. For complete information on WRC, it is required the direct measurements of $\theta(\psi)$ on a set of all possible wetting-draining «scenarios» of the soil. This requirement is impossible practically.

2 Theory and method

An identification of θ for a given Ψ is a rather urgent task, for example when irrigation rates are calculating. In most practical situations this calculation is reduced to determining the potential supply of the productive moisture in the soil. As is known, such supply is determined by the difference between the two so-called soil hydrological indices (SHIs), namely: «field capacity» (FC is equal to θ value at $pF \approx 2.5$) and «permanent wilting point» (WP is equal to θ value at $pF \approx 4.2$). Recall that pF is the decimal logarithm of the absolute ψ value measured in $\text{cm H}_2\text{O}$.

The numerical values of the SHIs are usually obtained by direct measurements of the main drying branch of $\theta(\psi)$ dependence. In practical irrigation agriculture agronomists are trying to prevent drying of the soil till WP, leading to plant death. Therefore, irrigation is implemented at slightly moist soil, and irrigation rates are calculated as the difference between FC and value of the present soil water content, that is controlled. As the allowable lower border of θ value, it is accepted SHI, which is called «the moisture of capillary break» (CB is equal to θ value at $pF \approx 3.48$). Usage of CB as such is explained due to the fact that this index characterizes the transition between two categories of the soil moisture: «readily-available for plants water» and «difficult-available for plants water». The numerical value of CB is usually obtained by direct measurements of WRC main drying branch.

Note that the availability of water for plants is not determined by its specific content per unit volume of the soil, but is characterized by the energy state of water, which depends on the action of capillary sorption forces that retain moisture in the soil. As the quantitative characteristics of the energy state of water in the soil, ψ and pF are used usually. However irrigation rate should be expressed in units of volume (mass) of water. Therefore, it is now widely practiced, the following approach to the calculation of irrigation rates. If the present soil water content is θ_{in} , hence the amount of irrigation water per cm^3 of the soil V is calculated by the formula: $V = FC - \theta_{in}$. But this seemingly obvious approach leads to an overestimation of irrigation rates and, as a result, unproductive losses of irrigation water. We prove this statement.

It is known that for a given pF any θ value on the main drying branch of WRC exceeds all relevant values of volumetric water content on other branches of the hysteresis loop. In particular, for $pF \approx 2.5$ FC index, which characterizes θ on the main drying branch of WRC, is the maximum of all the possible θ values on the remaining branches of the hysteretic $\theta(\psi)$. Furthermore, if we compare the branches of WRC hysteresis loop using pF value, corresponding to $\theta = FC$, the maximum of the pF value refers to the main drying branch of $\theta(\psi)$ dependence. Consequently, the application of the irrigation rate $V = FC - \theta_{in}$ causes that soil «goes» along the WRC wetting branch to condition, which is characterized by pF value that is less than pF, corresponding to the measured FC on the main drying branch of the hysteresis loop. This means that an excess of free moisture, which the soil is not able to retain, will flow out by gravity from the analyzed soil layer. It should be noted that, in addition to the unnecessary loss of irrigation water, some undesirable phenomena, such as leaching of nutrients and other agricultural chemicals from the root zone of the soil as well as their percolation into ground water with concomitant negative consequences of eutrophication of natural waters on the areas of the watershed, occur.

Of course, the hysteresis phenomenon of soil WRC is well-known to specialists of hydromelioration. But accounting of this phenomenon in calculating the irrigation rates with usage of data on WRC direct measurement is not possible for the following reason. Suppose, for the soil, on which the irrigated crops are cultivated, the main drying branch of WRC has been measured, as well as FC and pF value, which corresponds to FC, have been identified. For example, taking into account the peculiarities of the physiology of cultivated plants, the pF maximum threshold value (for instance, corresponding CB) has been determined as well the primary wetting branch of WRC has been measured from this pF value to pF that corresponds to FC. With these data, it would be possible to use fixed cycles of wetting and drying of the soil using the well-defined irrigation rates.

However, in real conditions of agricultural field such an «ideal» scenario will likely be disturbed by atmospheric precipitations. Under such conditions, the soil can «go» along the wetting branch of WRC, about which there are no data; further it will change on the drying branch of WRC, which may not coincide with the main drying branch of the hysteretic $\theta(\psi)$ dependence. The intensity of the precipitation and the duration of their falling may differ essentially at different vegetation periods, so it is not possible to measure the diversity of «configurations» for WRC hysteresis loop. For this reason, accounting of the hysteresis phenomenon in the calculating the irrigation rates is possible only with use of physically adequate mathematical models. Of course, the parameters of such models have to be identifiable using of a limited data set of $\theta(\psi)$ direct measurements, for example – data about the main drying branch as well as about one of the primary wetting branches of

the hysteresis loop. This condition is satisfied if the model parameters have a physical meaning.

3 Results and discussion

A model of water-retention capacity of soil as a capillary-porous media is described in [27]. For the parameters of the model, it is proposed the physical interpretation in some concepts about interphase abiotic interactions in soil with involvement of water as a liquid, as well as about the configuration space of soil pores. A computer program «Hysteresis», which allows carrying out computational experiments using the mathematical model of hysteretic $\theta(\psi)$, is developed at the laboratory of mathematical modeling of agroecosystems (Agrophysical Research Institute, Russia).

The experiments with the oscillations of the capillary pressure of moisture within the various ranges of values showed no negative «pump effect». Fig. 1 shows the result of calculation of WRC hysteresis loop using the model parameters typical for loamy soil: $n_d = 2.8$; $n_w = 2.8$; $\theta_s = 0.6$ [cm³·cm⁻³]; $\theta_r = 0.1$ [cm³·cm⁻³]; $\psi_{ae} = -20$ [cm H₂O]; $\psi_{we} = 150$ [cm H₂O]; $\alpha_d = 0.0020$ [cm H₂O⁻¹]; $\alpha_w = 0.0025$ [cm H₂O⁻¹]. The vertical dashed line intersects several drying and wetting branches of hysteretic WRC at $\psi = -330$ [cm H₂O], corresponding to FC.

Fig. 1. A possible set of the drying and wetting branches of the hysteretic WRC.

Fig. 2 shows the results of calculating the irrigation rates in two variants. In the calculations it is used the set of typical for loamy soil SHI, such as: FC = 0.495 [cm³·cm⁻³] at $\psi = -330$ [cm H₂O]; CB = 0.165 [cm³·cm⁻³] at $\psi = -1000$ [cm H₂O]. In the a) variant after the first watering free gravitational water flows out of the analyzed soil layer, and the

capillary pressure of soil moisture takes the value $\psi = -330$ [cm H₂O], corresponding to the «new FC», which takes the value: $\theta = 0.415$ [cm³·cm⁻³]. The loss of water in this case is $0.495 - 0.415 = 0.08$ [cm³·cm⁻³]. In the *b*) variant there is not any loss of irrigation water. The second watering in the *a*) variant begins at $\theta = 0.165$ [cm³·cm⁻³], while the «new CB» at $\psi = -1000$ [cm H₂O] takes the value: $\theta = 0.135$ [cm³·cm⁻³]. The second watering in the *b*) variant begins at $\theta = 0.135$ [cm³·cm⁻³]. As a result of watering the volumetric water content of soil takes the value 0.270 [cm³·cm⁻³] $\psi = -330$ [cm H₂O]. In this case, it is additionally used the readily available for plants moisture, which is characterized by the difference: $0.165 - 0.135 = 0.03$ [cm³·cm⁻³].

Besides, whole irrigation water is used without any loss. In the *a*) variant the irrigation rates for the second watering and for subsequent ones are 2.44 times higher than in the *b*) variant $\{(0.495 - 0.165)/(0.270 - 0.135) = 2.44\}$. Moreover, in the *a*) variant the watering is consumed only at 75.8% $\{(0.415 - 0.165)/(0.495 - 0.165) \cdot 100 = 75.8\}$, whereas in the *b*) variant the irrigation rate is fully used. Unproductive losses of irrigation water can be rather impressive: to increase water content in the soil layer by 30 cm thickness (with the above parameters) from the value $\theta = 0.165$ [cm³·cm⁻³] till a value $\theta = 0.495$ [cm³·cm⁻³] such loss for every watering can reach about 240 [m³·ha⁻¹].

Along with the above example of the advantages of the proposed model, which is the basis for the computer program «Hysteresis», earlier there was described an example of the practical using the model in [27]. To justify the irrigation rates for agricultural crops the specific hydrophysical indices, which are typical of the loamy chernozem (FC = 0.35 [cm³·cm⁻³], CB = 0.26 [cm³·cm⁻³]), were applied. The maximum effective supply of productive moisture in this soil (that is initially fully saturated) after draining the gravitational water out of the water-saturated soil profile is characterized by the difference $FC - CB = 0.09$ [cm³·cm⁻³]; such water supply for the soil profile with the depth of 1 m corresponds to 90 mm of the H₂O layer. The hysteretic WRC of the soil under consideration has the following parameters: $\theta_s = 0.54$ [cm³·cm⁻³]; $\theta_r = 0.13$ [cm³·cm⁻³]; $\psi_{ae} = -5.0$ cm H₂O; $\psi_{we} = 0.0$ cm H₂O; $\alpha_d = 0.0029$ [cm H₂O⁻¹]; $\alpha_w = 0.0032$ [cm H₂O⁻¹]; $n_d = 2.7$; $n_w = 2.7$. For this example, the following θ values correspond to ψ value at FC (-330 [cm H₂O]): $\theta_d(-330) = FC = 0.35$ [cm³·cm⁻³] and $\theta_w(-330) = 0.32$ [cm³·cm⁻³]. The difference between the mentioned θ values: $\theta_d(-330) - \theta_w(-330) = 0.03$ [cm³·cm⁻³] fully agrees with the measured data for chernozem [28].

Hence, to minimize the percolation of the gravitational water out of the moisturized soil profile under watering conditions, it is preferable to compute the irrigation rates as follows. For this purpose, it should be used the difference between the prospective values of volumetric water content in soil on the wetting curve of hysteretic WRC (that correspond to the water capillary pressure at FC) and the real pre-watering values of volumetric water content in soil on the drying curve of hysteretic WRC. However, in practice the mentioned values of volumetric water content in soil on the wetting curve of hysteretic WRC are not always available; therefore, the irrigation rates are computed from the difference $FC - CB$ or from the difference between FC and the pre-watering value of volumetric water content in soil. It is obvious that such practice results in the formation of the excess of gravitational water.

Fig. 2. *a)* Variant: two cycles of «drying-wetting» of the soil with watering from CB to FC, which are defined by the «classical» WRC curve (without hysteresis); *b)* Variant: two cycles of «drying-wetting» of the soil with watering from ψ , corresponding to CB, till ψ , corresponding to FC (taking into account the hysteresis).

The prevention of this negative process and the minimization of water wastage seem to be quite realizable at the presence of reliable data concerning the hysteretic WRC. However, to obtain such data, some rather labor-intensive soil-hydrophysical investigations (that consider the great set of drying-wetting variants) should be carried out. In this paper it was considered the adequate model of hysteretic WRC with physically interpreted parameters which can be identified using the limited set of experimental data (for example, with usage of SHIs and the data on the wetting curve of hysteretic WRC) as an alternative to labor-intensive measurements.

4 Summary

The original mathematical model of the soil hydrological properties has been considered. This model has been developed in the framework of physical concepts on the phenomenon of the hysteresis of soil water-retention capacity. The «Hysteresis» software has been employed for carrying out the computer experiments with this model. Usage of this software allows minimizing the time consuming calculations and improving of their accuracy. This is especially important for reclamation issues solving and for rational using of water resources. The approach to calculate the supply of productive moisture and the irrigation rates has been proposed.

The calculation is based on the values of capillary pressure at FC and CB, taking into account the hysteretic properties of the soil. The rates of watering, which are computed by facilities of this approach, are able to minimizing the losses of irrigation water, mineral fertilizers, pesticides and land reclamation chemicals caused by the percolation of the excess of free water by gravity out of the moisturized soil profile. Practical usage of the approach proposed here allows solving the urgent challenges of the land ecological management. This approach favors the optimization of crop irrigation techniques, as well as provides the rational usage of water resources.

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