

Mathematical Modeling of Temperature Field of Multilayer Enclosure Structures

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Abstract. The mathematical model of heat exchange process through multilayer enclosure structure is constructed. Herein infiltration and heat-conducting inclusions are taken into account. Heat exchange process modeling in the stationary and non-stationary conditions is considered. The method of temperature field constructing based on the suggested model is offered. The three-dimensional boundary problem of non-stationary heat conductivity is solved numerically using the implicit method of alternating directions. The temperature field constructing of the enclosure structures in the non-stationary condition allows to track dynamics of thermal flows through enclosing structures and to receive the thermal resistance considering influence of heat-conducting inclusions.

1 Introduction

There are problems with indoor microclimate when using the thermally insulated cladding panel with aluminum alloy skin.

Modeling of heat exchange processes in the panel is required to work out the premises design with the required positive temperature indoors at a negative external air temperature. Moreover the inside temperature differentials should not exceed the set norms.

Work [1] offers the heat exchange model in aluminum panels at the following assumptions: stationary condition of the process, constancy of thermophysical properties of materials, independence of heat exchange process on mass transfer. This model represents a system of equations describing heat exchange on cladding panel internal and external skins and through panel joints.

It is possible to calculate temperature distribution on the panel skin by a two-dimensional boundary problem of stationary heat conductivity.

The stationary model is insufficient for accurate thermal structural calculations when operating the factory-assembled buildings in environments with large diurnal temperature variation and strong winds. During the time required to establish stationary state both the outside air temperature and the indoor heat balance may change.

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2 Modeling of heat exchange processes in the enclosure structures in non-stationary conditions

The physical and mathematical model is based on the following assumptions. Heat is transferred from indoor air to the inner skin plate, then through the insulation layer to the outer siding and then into the atmosphere. Besides there is a heat flow through the "thermal bridge" having a high thermal conductivity. It should be noted that the standard panel thickness is an order less than its height and length.

Thermophysical properties of materials are represented by functions depending on time and coordinates. Mass transfer is not taken into account.

The temperature distribution on internal and external panel skin surface is characterized by heat exchange at a panel skin surface. The equations describing the distribution of temperature on an internal and external panel skin surface are derived from the equation of thermal balance, and contain heat and moisture exchange coefficients which are defined by the intensity of a convective heat transfer, radiation heat exchange of internal and external enclosure structure surfaces.

Temperature on a side plate is not calculated. Heat exchange through panel joints is taken into account when formulating boundary conditions. The thermal flow arriving to a joint along the plate consists of the heat loss flow through heat-conducting inclusion from the premises into environment, and the cold air infiltration flow heating conditioned by more preheated panel.

The thermal flow in a thermal insulation layer is considered directed strictly perpendicular to the panel plane as near the panel edge the flow through insulation is small enough in comparison with the flow through a metal panel skin.

Let us introduce a three-dimensional Cartesian coordinate system with the origin at the center of the internal panel skin size $2a \times 2b \times d$. The direction of the axis Z from the inner panel skin to the outer one is then $-a \leq x \leq a, -b \leq y \leq b, 0 \leq z \leq d$.

To derive the equation describing the temperature distribution t_1 on the inner panel skin surface, let us consider the surface element $dx \times dy \times \delta$, where δ is the panel skin thickness.

Let us express the heat flowing into surface element along the axes X, Y, Z respectively at time τ by $Q_x(\tau, x, y)$ $Q_y(\tau, x, y)$ $Q_z(\tau, x, y, z)$. Based on Fourier's law mathematical expression of these flows is as follows:

$$\begin{aligned} Q_x(\tau, x, y) &= -\lambda_a \cdot \delta \cdot dy \frac{\partial t_1(\tau, x, y)}{\partial x} \cdot d\tau, \\ Q_y(\tau, x, y) &= -\lambda_a \cdot \delta \cdot dx \frac{\partial t_1(\tau, x, y)}{\partial y} \cdot d\tau, \\ Q_z(\tau, x, y, z) \Big|_{z=0} &= \alpha_1(t_w(\tau) - t_1(\tau, x, y, z)) dx \cdot dy \cdot d\tau. \end{aligned} \quad (1)$$

The expressions for the resulting heat flow can be written as follows:

$$\begin{aligned}
 Q_x(\tau, x + \Delta x, y) &= -\lambda_a \cdot \delta \cdot dy \frac{\partial t_1(\tau, x + \Delta x, y)}{\partial x} \cdot d\tau; \\
 Q_y(\tau, x, y + \Delta y) &= -\lambda_a \cdot \delta \cdot dx \frac{\partial t_1(\tau, x, y + \Delta y)}{\partial y} \cdot d\tau; \\
 Q_z(\tau, x, y, z)|_{z=\delta} &= -\lambda_s \cdot dx \cdot dy \frac{\partial t_s(\tau, x, y, z)}{\partial z} \Big|_{z=\delta} \cdot d\tau.
 \end{aligned}
 \tag{2}$$

where λ_a, λ_s are panel skin and insulation thermal conductivity, α_1 is heat exchange coefficient on an internal panel skin, t_s is isolation temperature; t_w is average air temperature indoors.

In the non-stationary condition the heat balance equation is:

$$\begin{aligned}
 &\left(Q_x(\tau, x, y) + Q_y(\tau, x, y) + Q_z(\tau, x, y, z)|_{z=0} \right) - \\
 &- \left(Q_x(\tau, x + \Delta x, y) + Q_y(\tau, x, y + \Delta y) + Q_z(\tau, x, y, z)|_{z=\delta} \right) = dH,
 \end{aligned}
 \tag{3}$$

where dH is the enthalpy change of the panel skin element

$$dH = c_a \frac{\partial t_1(\tau, x, y, z)}{\partial \tau} \cdot d\tau \cdot dV,
 \tag{4}$$

where c_a is thermal capacity of panel skin unit volume, dV - panel skin element volume

After rearrangement the thermal balance equation (3) with regard to (1), (2), (4) will change to:

$$\begin{aligned}
 &\lambda_a \left(\frac{\partial^2 t_1(\tau, x, y)}{\partial x^2} + \frac{\partial^2 t_1(\tau, x, y)}{\partial y^2} \right) + \frac{\lambda_s}{\delta} \frac{\partial t_s(\tau, x, y, z)}{\partial z} \Big|_{z=\delta} + \\
 &+ \frac{\alpha_1}{\delta} (t_w(\tau) - t_1(\tau, x, y)) = c_a \frac{\partial t_1(\tau, x, y)}{\partial \tau}, \\
 &-a < x < a, -b < y < b,
 \end{aligned}
 \tag{5}$$

The equation describing thermal balance for the panel external surface is derived similarly and has a form:

$$\begin{aligned}
 &\lambda_a \left(\frac{\partial^2 t_2(\tau, x, y)}{\partial x^2} + \frac{\partial^2 t_2(\tau, x, y)}{\partial y^2} \right) - \frac{\lambda_s}{\delta} \frac{\partial t_s(\tau, x, y, z)}{\partial z} \Big|_{z=d-\delta} + \\
 &+ \frac{\alpha_2}{\delta} (t_u(\tau) - t_2(\tau, x, y)) = c_a \frac{\partial t_2(\tau, x, y)}{\partial \tau}, \\
 &-a < x < a, -b < y < b,
 \end{aligned}
 \tag{6}$$

where t_2 is external panel skin temperature, t_u is the average air temperature outdoors, α_2 is heat exchange coefficient on an external panel skin.

For the insulation layer the heat flows only in the direction Z according to the accepted assumptions. $Q_x = Q_y = 0$ is considered. Let us consider the balance of heat flows for an insulation element $dx \times dy \times dz$.

The inward flow

$$Q_z(\tau, x, y, z) = -\lambda_s \cdot dx \cdot dy \cdot \frac{\partial t_s(\tau, x, y, z)}{\partial z} \cdot d\tau \quad (7)$$

where t_s is temperature insulation, λ_s is thermal conductivity of the insulation material. The outward flow

$$Q_z(\tau, x, y, z + \Delta z) = -\lambda_s \cdot dx \cdot dy \cdot \frac{\partial t_s(\tau, x, y, z + \Delta z)}{\partial z} \cdot d\tau. \quad (8)$$

The equation of thermal balance is:

$$\lambda_s \cdot dx \cdot dy \cdot d\tau \left(\frac{\partial t_s(\tau, x, y, z + \Delta z)}{\partial z} - \frac{\partial t_s(\tau, x, y, z)}{\partial z} \right) = c_s \cdot \frac{\partial t_s(\tau, x, y, z)}{\partial \tau} \cdot d\tau \cdot dV, \quad (9)$$

where c_s is heat capacity of insulation material unit volume, dV is the insulation unit volume.

After the rearrangement (9) will take a form:

$$\frac{\lambda_s}{c_s} \frac{\partial^2 t_s(\tau, x, y, z)}{\partial z^2} = \frac{\partial t_s(\tau, x, y, z)}{\partial \tau}, \quad (10)$$

$-a < x < a, -b < y < b, \delta \leq z \leq d - \delta.$

Boundary conditions are set similarly [1]:

$x = -a$:

$$\frac{\partial t_1(\tau, x, y)}{\partial x} = -\frac{\partial t_2(\tau, x, y)}{\partial x} = \frac{(t_1(\tau, x, y) - t_2(\tau, x, y))}{l_2} + \frac{MC_p(t(\tau) - t_{cw}(\tau, x, y))}{\delta \lambda_a}. \quad (11)$$

$x = +a$:

$$-\frac{\partial t_1(\tau, x, y)}{\partial x} = \frac{\partial t_2(\tau, x, y)}{\partial x} = \frac{(t_1(\tau, x, y) - t_2(\tau, x, y))}{l_1} + \frac{MC_p(t(\tau) - t_{cw}(\tau, x, y))}{\delta \lambda_a}. \quad (12)$$

$$\begin{aligned}
 & y = -b: \\
 & \frac{\partial t_1(\tau, x, y)}{\partial y} = -\frac{\partial t_2(\tau, x, y)}{\partial y} = \\
 & \frac{(t_1(\tau, x, y) - t_2(\tau, x, y))}{l_4} + \frac{MC_p(t(\tau) - t_{cw}(\tau, x, y))}{\delta\lambda_a}.
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 & y = +b: \\
 & -\frac{\partial t_1(\tau, x, y)}{\partial y} = \frac{\partial t_2(\tau, x, y)}{\partial y} = \\
 & \frac{(t_1(\tau, x, y) - t_2(\tau, x, y))}{l_3} + \frac{MC_p(t(\tau) - t_{cw}(\tau, x, y))}{\delta\lambda_a}.
 \end{aligned} \tag{14}$$

where l_1, l_2, l_3, l_4 are length dimension parameters characterizing heat-conducting inclusions, M is the discharge of infiltrating air per unit length per second, t_{cw} is temperature of the cold air coming inside through a joint; C_p is specific air heat capacity.

The left part of boundary conditions characterizes a heat flow due to heat conductivity of a panel skin, the right one is proportional to a temperatures difference of external and internal part of a joint.

It is advisable to accept

$$t_s(\tau, x, y, \delta) = t_1(\tau, x, y), \quad t_s(\tau, x, y, d - \delta) = t_2(\tau, x, y). \tag{15}$$

Initial conditions:

$$\begin{aligned}
 & t_1(0, x, y) = t_w(0), \\
 & t_2(0, x, y) = t_u(0), \\
 & t_s(0, x, y, z) = t_w(0) - z \cdot \frac{t_w(0) - t_u(0)}{d}, \\
 & -a \leq x \leq a, \quad -b \leq y \leq b, \quad \delta \leq z \leq d - \delta.
 \end{aligned} \tag{16}$$

The simultaneous solution of (5), (6), (10), (11) - (16) are used to construct the temperature field of the panel.

Numerical solution of the boundary value problem of non-steady heat conduction

For the numerical solution of the problem (5), (6), (10), (11) - (16) we use the implicit method of alternating directions. We introduce the grid

$$\omega_{h\tau} = \omega_h \times \omega_\tau = \left\{ \begin{aligned} & (i_1 h_1, i_2 h_2, i_3 h_3, j \Delta \tau), \\ & i_1 = 0, 1, \dots, N_1, i_2 = 0, 1, \dots, N_2, \\ & i_3 = 0, 1, \dots, N_3, j = 0, 1, \dots, N_\tau \end{aligned} \right\}$$

With grid steps $h_1 = \frac{2a}{N_1}$, $h_2 = \frac{2b}{N_2}$, $h_3 = \frac{d}{N_3}$, $\Delta\tau = \frac{T}{N_\tau}$.

For the equation of an internal panel skin we receive two boundary problems. The index i_3 is omitted, since we get $i_3 = 0$ for points of the inner panel skin.

The first problem is

$$\begin{aligned}
 &0,5\beta_1 t_{i_1-1,i_2}^{j+1/2} - k_1 t_{i_1,i_2}^{j+1/2} + 0,5\beta_2 t_{i_1+1,i_2}^{j+1/2} = -F_{i_1,i_2}^{j+1/2}, \quad i_1 = 1, 2, \dots, N_1 - 1 \quad (0 < i_2 < N_2), \\
 &\beta_1 = \frac{\lambda_a \Delta\tau}{c_a (h_1)^2}, \quad \beta_2 = \frac{\lambda_a \Delta\tau}{c_a (h_2)^2}, \quad k_1 = 1 + \beta_1 - \frac{\Delta\tau}{2\delta c_a} \left(\alpha_1 + \frac{\lambda_s}{h_3} \right), \\
 &F_{i_1,i_2}^{j+1/2} = 0,2\beta_2 \left(t_{i_1,i_2-1}^j + t_{i_1,i_2+1}^j \right) + (1 - \beta_2) t_{i_1,i_2}^j - \frac{\lambda_a \Delta\tau}{2c_a} f^{j+1/2}, \\
 &f^{j+1/2} = \left(\frac{\lambda_s}{\lambda_a \delta h_3} t_{s,i_1,i_2}^j + \frac{\alpha_1}{\lambda_a \delta} t_w(\Delta\tau(j+0,5)) \right).
 \end{aligned} \tag{17}$$

The boundary conditions are:

$$\begin{aligned}
 t_{10,i_2}^{j+1/2} &= t_{11,i_2}^{j+1/2} + h_1 \left(\frac{(t_{10,i_2}^j - t_{20,i_2}^j)}{l_2} + L \right), \\
 t_{1N_1,i_2}^{j+1/2} &= t_{1N_1-1,i_2}^{j+1/2} + h_1 \left(\frac{(t_{2N_1,i_2}^j - t_{1N_1,i_2}^j)}{l_1} + L \right),
 \end{aligned} \tag{18}$$

where $L = \frac{MC_p (t_u^j - t_{cw}^j)}{\delta\lambda_a}$.

The second problem is:

$$\begin{aligned}
 &0,5\beta_2 t_{i_1,i_2-1}^{j+1} - k_2 t_{i_1,i_2}^{j+1} + 0,5\beta_1 t_{i_1,i_2+1}^{j+1} = -F_{i_1,i_2}^{j+1}, \quad i_2 = 1, 2, \dots, N_2 - 1 \quad (0 < i_1 < N_1), \\
 &\beta_1 = \frac{\lambda_a \Delta\tau}{C_a (h_1)^2}, \quad \beta_2 = \frac{\lambda_a \Delta\tau}{C_a (h_2)^2}, \quad k_2 = 1 + \beta_2 - \frac{\Delta\tau}{2\delta c_a} \left(\alpha_1 + \frac{\lambda_{u3}}{h_3} \right), \\
 &F_{i_1,i_2}^{j+1} = 0,5\beta_1 \left(t_{i_1-1,i_2}^{j+1/2} + t_{i_1+1,i_2}^{j+1/2} \right) + (1 - \beta_1) t_{i_1,i_2}^{j+1/2} - \frac{\lambda_a \Delta\tau}{2c_a} f^{j+1/2}, \\
 &f^{j+1/2} = \left(\frac{\lambda_{u3}}{\lambda_a \delta h_3} t_{u3,i_1,i_2}^j + \frac{\alpha_1}{\lambda_a \delta} t_w(\Delta\tau(j+0,5)) \right).
 \end{aligned} \tag{19}$$

The boundary conditions are:

$$\begin{aligned}
 t_{1i,0}^{j+1} &= t_{1i,0}^{j+1} + h_2 \left(\frac{(t_{1i,0}^j - t_{2i,0}^j)}{l_4} + L \right), \\
 t_{1i,N_2}^{j+1} &= t_{1i,N_2-1}^{j+1} + h_2 \left(\frac{(t_{2i,N_2}^j - t_{1i,N_2}^j)}{l_3} + L \right);
 \end{aligned}
 \tag{20}$$

The scheme of alternating directions for the equation of an internal panel skin is derived similarly.

For the insulation layer the differential analog of the equation (10) will have a form:

$$\frac{\lambda_s \Delta \tau}{c_s (h_3)^2} t_{s i_1, i_2, i_3-1}^{j+1} - \frac{2\lambda_s \Delta \tau}{c_s (h_3)^2} t_{s i_1, i_2, i_3}^{j+1} + \frac{\lambda_s \Delta \tau}{c_s (h_3)^2} t_{s i_1, i_2, i_3+1}^{j+1} = -t_{s i_1, i_2, i_3}^{j+1}, i_3 = 1, 2, \dots, N_3 - 1. \tag{21}$$

The boundary conditions are

$$t_{s i_1, i_2, 0}^{j+1} = t_{i_1, i_2}^j, t_{s i_1, i_2, N_3}^{j+1} = t_{2 i_1, i_2}^j. \tag{22}$$

The received systems allows to use the tridiagonal matrix algorithm to solve them.

The constructed scheme of alternating directions is stable and converges at a speed of

$$O\left(\sum_{i=1}^3 h_i^2 + (\Delta \tau)^2\right).$$

To calculate the temperature of infiltrated air, the methodology described in [1] has been used. When computer modelling the temperature field the symmetry was taken into account. All information was stored for a quarter of the fragment and restored due to the symmetry for other sections of the construction. Besides, variable step toward the panel edges was used, for the temperature field at the fragment edges has very strong heterogeneity. On each time step heat rejection ratio of a vertical and horizontal joints surface, the cold air temperature arriving through joints, and calculation of temperature at an internal and external panel skin surface as well as in the insulation layer were defined. To check the calculations accuracy by the proposed method additional calculations were made, the results of which were compared with the calculations results by stationary model, for which there is a comparison with in - situ measurements [1]. The comparison showed almost total agreement between the calculated temperatures. Figure 1 shows the results of the calculated steady temperature fields of the inner and outer surfaces according to [1].

The temperature field calculation by the above method allows to calculate the heat flow through the panels and through the joints and assess their contribution to the overall heat loss at any time.

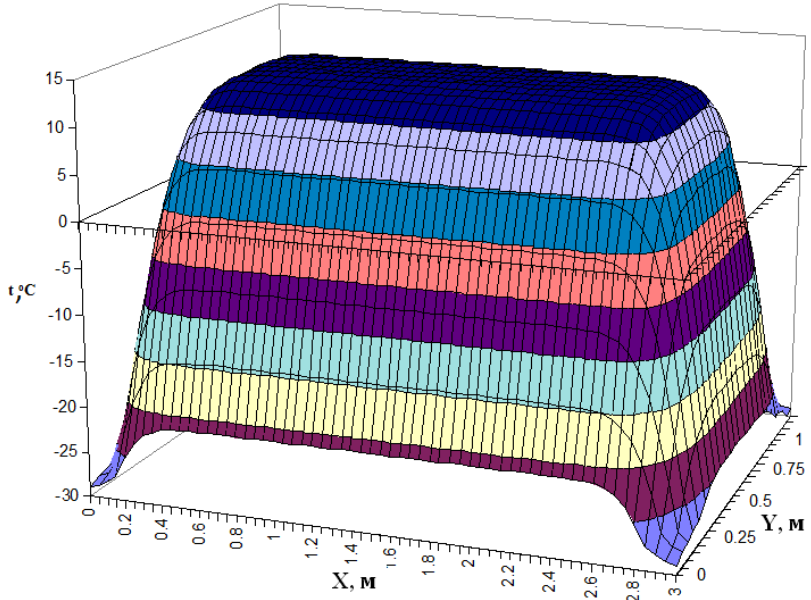
Heat flow through panels

$$Q(\tau) = \int_{-(a-\delta)}^{a-\delta} dx \int_{-(b-\delta)}^{b-\delta} \alpha_1(t_w(\tau) - t_1(\tau, x, y)) dy.$$

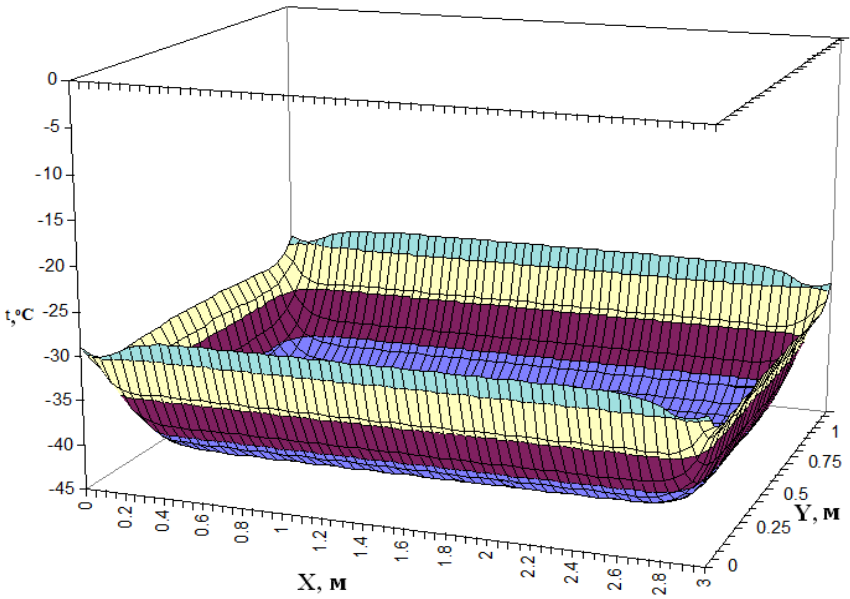
Heat flow through a panel joint $x = a$:

$$Q(\tau)|_{x=a} = -\lambda_a \cdot \delta \cdot \int_{-b}^b \left[\frac{\partial t_1(\tau, x, y)}{\partial x} \Big|_{x=a} \right] dy,$$

similarly $x = -a, y = \pm b$.



a)



b)

Fig. 1. Temperature field of the panel skin; a) is an internal surface; b) is an external surface.

3 Results

The physical and mathematical model of heat exchange through the enclosure structure is developed. The model considers non-stationarity of the process and heat exchange connected with heat-conducting inclusions and infiltration of air in panel joints.

This model can be rather effective to design a multilayered heat insulation panels as the above calculations have shown that at non-stationary heat exchange process heat loss depends on sequence of layers along cross section.

4 Conclusion

When using the enclosure structure it takes from 15 to 20 hours to establish the stationary condition. Thus, the stationary condition practically does not occur. Accurate thermo - physical calculation can be made only with the help of the non-stationary model.

In lightweight enclosure structure up to 75% of heat loss falls on the joints between panels. And about 30% is lost through infiltration and 45% through the joints formed in "thermal bridge". To reduce heat loss through the enclosure structure it is more efficient to seal joints and destroy "thermal bridge" but not increase the panel thickness or use a better thermal insulator.

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