Modelling of Water-Saturated Grounds under a Curved Section of an Oil and Gas Pipeline

Tatyana Maltseva[1,*] and *Tatyana* Saltanova[2]

[1]Industrial University of Tyumen, 625001 Volodarskogo str. 38, Tyumen, Russia
[2]Tyumen State University, 625000 Semakova str. 10, Tyumen, Russia

Abstract. The paper studies the influence of the oil and gas pipeline on the base of water-saturated soil in terms of external forces effect (from the pipeline). The developed mathematical model of stress-strain state of "the oil and gas pipeline and the grounds" system allows us to identify the maximum load resulting from the impact of a curved section of the pipeline on the base as well as the deformations occurring in the grounds.

1 Introduction

Currently, transportation of oil is carried out by one of the most profitable modes of transport – by pipeline.

Its main advantage is a low cost of the transfer. But it has its drawbacks, the main of which is a one-time large capital investment in the construction, as before starting to use the oil pipeline you need to build it from the starting point to its destination.

During the operation, a number of factors influences the pipeline: the pressure caused by curvilinear sections of the pipeline, internal pressure, temperature fields of soil and the pumped medium, the pressure of the overlying soil layer, the deformation of the base associated with the differential settlement of soil under its freezing and thawing. The paper considers a factor of pressure of the pipeline section and the deformation of the base. As shown by calculations, the stresses associated with a curved pipeline axis may exceed the stresses created by the weight of the tube and the pumped product [1].

The biggest part of the territory of oil and gas transportation is made up of weak clay soils. A big number of ways to increase the bearing capacity of foundation soil have been developed and implemented [2-6].

The calculation of water-saturated base is carried out in the framework of the mathematical theory of water-saturated soil, with an account of the effect of residual excess pore pressure in a stable condition when the time factor is absent [7].

The three-dimensional problem of the "oil and gas pipeline + base" system is divided into two sub-problems in the two-dimensional formulations: one problem is related to the horizontal plane   and is called the auxiliary one, the other problem deals with the vertical plane x_1Ox_2, is called the principal one.

When addressing the auxiliary two-dimensional problem in the horizontal plane parallel to the ground surface one should define the longitudinal force q3 (with respect to the pipe

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* Corresponding author: tgasu.maltseva.tatyana@mail.ru

axis) and transverse force q2. Then one solves the principal two-dimensional problem in the vertical plane, in which the horizontal force q2 is known as a result of the solution of the auxiliary problem. It is found with an account of the action of a longitudinal force q3. Thus, the solution of the three-dimensional problem (task) is replaced with the solution of two two-dimensional problems. Approximate taking account of the action of longitudinal force q3 is carried out through force q2.

2 Problem formulation

A mathematical model of vertical ultimate height (h) of a layer of water-saturated soil with an account of excessive residual pore pressure in a stable condition presents a system of differential equations with constant coefficients G, λ, b, c with respect to the displacement vector u=(u1, u2) of mineral soil particles [8]:

$$-\left(\left((G+\lambda)\frac{\partial \theta}{\partial x_i}+G\Delta u_i+b\frac{\partial^2 u_i}{\partial x_i^2}+c\frac{\partial u_i}{\partial x_i}\right)\right)=F_i, \; i=1,2, \tag{1}$$

$$G=\frac{E_S}{2(1+v)}, \quad \lambda=\frac{E_S v}{(1+v)(1-2v)}, \quad b=\frac{E_l}{\aleph^2}, \quad c=\frac{E_l}{\aleph h}, \quad \theta=div\, \boldsymbol{u}.$$

E_S - modulus of deformation and v - Poisson's ratio of mineral soil particles, E_l - mechanical constant of pore water, $\aleph$ – dimensionless quantity $(0<\aleph<1)$ determined from an experiment [9; 10], h – the height of the compressible strata.

The boundary conditions for part of the boundary S_1 – kinematic, for part S_2 - static:

$$\boldsymbol{u}\,|_{S_1}=0, \quad \boldsymbol{\sigma}|_{S_2}=\boldsymbol{q}(x_1,x_2), \tag{2}$$

$\boldsymbol{\sigma}$ - stress tensor in the mineral particle of the soil with the effect of pore water being taken account of only in diagonal elements.

Excessive residual pore pressures vanish on the day (x_1=0) surface of the body S_2 due to the draining coating and only the pressures in the mineral soil particles are left. Load $\boldsymbol{q}(x_1, x_2)$ - resultant of forces: q_1 - the weight per unit length of the pipeline with the product; horizontal force q_2 - the pressure of the curved section of the pipe on the ground. As already noted, it is defined with respect to the action of longitudinal force q_3.

Thus, the solutions of the two two-dimensional problems (the calculation of the curved section of the pipe and the calculation of the water-saturated soil base) have a common link, which is a transverse horizontal component q_2, arising as a result of the movement of oil products along the curved section of the pipeline. The dynamic smoothing method has been applied in order to assess the load rate q_2 [3, 4].

3 Research methods

Let us consider the auxiliary problem. According to the geodetic data the pipeline axis position is known. On the basis of elastic rod equilibrium equations [13; 14] and the use of the d'Alembert principle we obtained homogeneous nonlinear equations of motion of the rod in a Cartesian coordinate system:

$$m\ddot{x}_2 + \mu_{11}\dot{x}_2 + \mu_{12}\dot{x}_3 - \cos\varphi\frac{N' + \varphi'Q}{a} - \sin\varphi\frac{Q' - \varphi'N}{a} = 0,$$

$$m\ddot{x}_3 + \mu_{12}\dot{x}_2 + \mu_{22}\dot{x}_3 - \sin\varphi\frac{N' + \varphi'Q}{a} + \cos\varphi\frac{Q' - \varphi'N}{a} = 0.$$

$$(3)$$

In these equations the sign « ´ » denotes the derivative with respect to the Lagrangian coordinate of the point of the rod axis; the signs « ˙ and ¨ » stand for the first and the second time derivatives, m – mass per unit length of the pipeline with the product, x_2 and x_3 – Cartesian coordinates; φ – the angle between the pipe axis and the x_3 axis; N – the axial force, $N = D\varepsilon$, $\varepsilon = a - 1$ – axial deformation of the pipe; $D = E\cdot A$ – stiffness in tension, E – modulus of the pipe elasticity; A – cross sectional area of the pipe; $Q = H\cdot\varphi''$ – shear force, $H = E\cdot J$ – pipe bending stiffness, J – second area moment. Values a and φ are determined by known geodetic data – coordinates x_2', x_3' from the equations

$$x_2' = a\cos\varphi, \quad x_3' = a\sin\varphi.$$

Coefficients of resistance μ_{ij} are:
$\mu_{11} = \mu_\tau cos^2\varphi + \mu_n sin^2\varphi,$
$\mu_{22} = \mu_\tau sin^2\varphi + \mu_n cos^2\varphi,$
$\mu_{12} = (\mu_n - \mu_\tau) sin\varphi\ cos\varphi,$

where μ_τ and μ_n – known resistivity coefficients for movement along the tangent and the normal to the pipe axis respectively.

Let's introduce the boundary conditions: $x_2(0,t) = x_0$, $x_3(0,t) = y_0$, $x_2(l,t) = x_l$, $x_3(l,t) = y_l$, $\varphi(0,t) = \varphi_0$, $\varphi(l,t) = \varphi_l$.

The initial conditions: at time t = 0 $x_2(0) = x_0(0)$, $x_3(0) = y_0(0)$ - the initial form of a rod, $\dot{x}_2(0) = 0$, $\dot{x}_3(0) = 0$ - the initial velocities.

Transverse and longitudinal forces q_2 и q_3 are determined at any time by the formulas:

$$q_2 = \cos\varphi\frac{N' + \varphi'Q}{a} + \sin\varphi\frac{Q' - \varphi'N}{a}, \qquad q_3 = \sin\varphi\frac{N' + \varphi'Q}{a} - \cos\varphi\frac{Q' - \varphi'N}{a}.$$

The system of equations (3) is solved by the finite difference method with the use of the "cross" explicit difference scheme [15]. The derivatives of x_3 coordinate are approximated by finite-difference formulas of the 2nd order accuracy.

The system of equations (1) is solved with a modified finite element method. Triangular finite elements have been taken as finite elements [8; 16]. The movements inside the triangle elements are described by linear dependencies through the nodal displacements $\boldsymbol{u} = (u_1, u_2)$:

$$u_k = \frac{1}{2\Delta}\left[\left(p_i + d_i x_1 + n_i x_2\right)u_k^i + \left(p_j + d_j x_1 + n_j x_2\right)u_k^j + \left(p_m + d_m x_1 + n_m x_2\right)u_k^m\right],$$

$$p_i = x_1^j x_2^m - x_1^m x_2^j, \quad n_i = x_1^m - x_1^j, \quad d_i = x_2^j - x_2^m, \quad k = 1,2.$$

Nodal displacements u_k^i, u_k^j, u_k^m are the desired values.

When taking the integral over the area of the triangular element there appear definite integrals $\int_S x_1 dS$ and $\int_S x_2 dS$.

Consider taking one of the integrals on the basis of the well-known theorem on the static moment of area according to which the static moment equals the area of a triangle Δ times x_c distance to the center of gravity of the triangle, which coincides with the intersection of its medians:

$$\int_S x_1 dS = x_{1c} \cdot \Delta, \qquad x_{1c} = \frac{x_1^i + x_1^j + x_1^m}{3}, \qquad x_{2c} = \frac{x_2^i + x_2^j + x_2^m}{3}.$$

Here is a system of linear algebraic equations of the finite element method for finding the unknown nodal displacements $\{\delta\} = \left(u_1^i, u_2^i, u_1^j, u_2^j, u_1^m, u_2^m\right)$ [8]:

$$\left([k_1] + [k_2]\right)\{\delta\} = \{F\},$$

where:

$$[k_1] = \frac{1}{4\Delta} \begin{pmatrix} d_i & 0 & n_i \\ 0 & n_i & d_i \\ d_j & 0 & n_j \\ 0 & n_j & d_j \\ d_m & 0 & n_m \\ 0 & n_i & d_m \end{pmatrix} \cdot \begin{pmatrix} \lambda\frac{1-v}{v}+b_1 & 2G\frac{v}{1-2v} & 0 \\ 2G\frac{v}{1-2v} & \lambda\frac{1-v}{v}+b_2 & 0 \\ 0 & 0 & G \end{pmatrix} \cdot \begin{pmatrix} d_i & 0 & d_j & 0 & d_m & 0 \\ 0 & n_i & 0 & n_j & 0 & n_m \\ n_i & d_i & n_j & d_j & n_m & d_m \end{pmatrix}$$

$$[k_2] = \frac{1}{4\Delta} \begin{pmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ f_j & 0 & 0 \\ 0 & f_j & 0 \\ f_m & 0 & 0 \\ 0 & f_m & 0 \end{pmatrix} \cdot \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} d_i & 0 & d_j & 0 & d_m & 0 \\ 0 & n_i & 0 & n_j & 0 & n_m \\ n_i & d_i & n_j & d_j & n_m & d_m \end{pmatrix},$$

$$f_i = p_i + d_i x_{1c} + n_i x_{2c}, \quad f_j = p_j + d_j dx_{1c} + n_j x_{2c}, \quad f_m = p_m + d_m x_{1c} + n_m x_{2c},$$

$$\int_S dS = \Delta = \frac{1}{2} \begin{vmatrix} 1 & x_1^i & x_2^i \\ 1 & x_1^j & x_2^j \\ 1 & x_1^m & x_2^m \end{vmatrix}.$$

4 Numerical results

The system of equations (1) with the boundary conditions (2) was solved by a modified method of finite elements. The triangular finite elements were chosen as finite element.

The value of the intensity of the horizontal force q_2=0.005 MN/m

Acting at a distance of 10 m along x_3 axis ; own weight of the pipe q_1=0.0015 MN/m (Fig. 1)
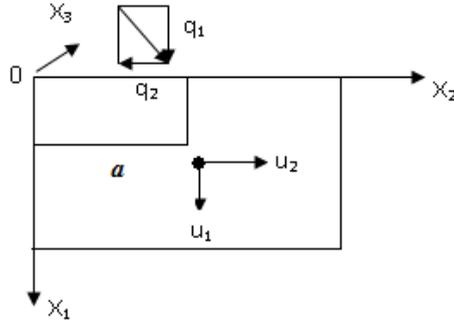
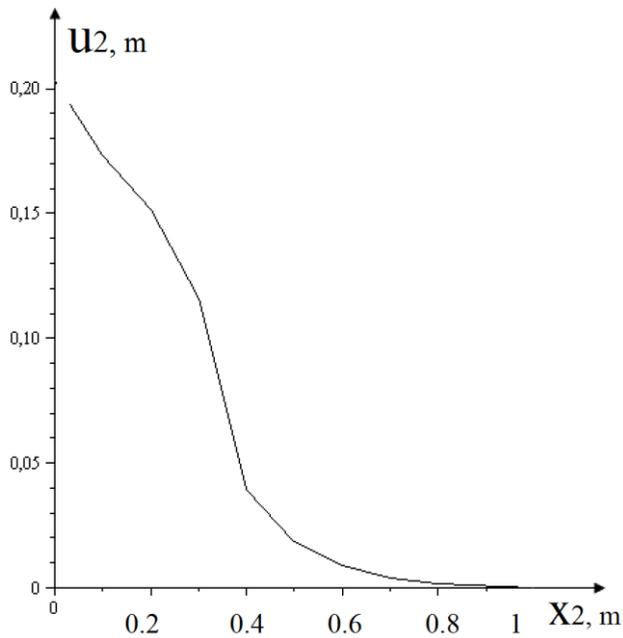**Fig. 1.** Forces acting on the water-saturated base.

The mechanical characteristics of the soil
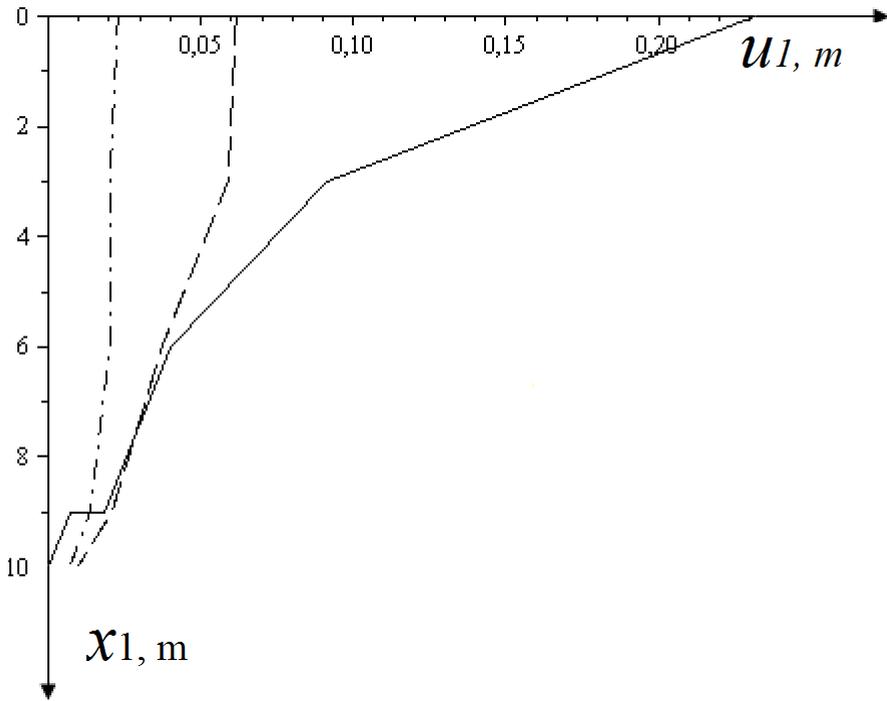
$$E_s = 5.2 \, MPa, \; E_l = 3.27 \, MPa,$$
$$v = 0.3, \; \aleph = 0.52,$$

the radius of the pipeline – 0.3 m.

The Fig. 2 shows the horizontal displacement of soil skeleton particles at $x_1 = 0$ м, $x_2 = 1$ m.

Fig.2. Horizontal displacement of soil skeleton particles

Fig. 3. Vertical displacement for different cross-sections.

Fig. 3 shows the graphs for vertical movement for $x_2 = 0$, at the distance

for $x_2=0$, at the distance $x_2=0.3$ m, $x_2= 0.9$ m from x_1 axis

The maximum horizontal displacements were 0.2 m, the minimum ones – 0.03 m.

The solution method solves the problem of determining the longitudinal and transverse forces of the interaction of the pipeline with the ground for sufficiently long pipeline sections. Transverse bends lead to the loss of stability of plane shape of the bending. The proposed model and solution algorithm allow us to take account of the possibility of the effect of thermal stresses in the pipeline on the strength of the interaction with the ground, but the article does not deal with this question. When solving the problem the maximum load of the pipeline on the ground was determined and it accounted for $q = 0.0166$MN/m

5 Conclusion

The obtained results can be used in the design and the calculation of sediment of engineering pipeline transportation facilities being built on weak grounds in determining the residual life of operated pipelines or in the processing of seismic results. The modified finite element method allows us to determine the stress and strain state of the base of the water-saturated soil under the action of external forces caused by the influence of the pipeline.

The solution to the problem of stress-strain state of a weak base will allow us to predict the maximum permissible values of the base strain and to consider and adopt measures to improve the safety of its construction and operation at the design stage.

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