VERIFICATION OF MODEL OF CALCULATION OF INTRA-CHAMBER PARAMETERS IN HYBRID SOLID-PROPELLANT ROCKET ENGINES

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Abstract. On the basis of obtained analytical estimate of characteristics of hybrid solid-propellant rocket engine verification of earlier developed physical and mathematical model of processes in a hybrid solid-propellant rocket engine for quasi-steady-state flow regime was performed. Comparative analysis of calculated and analytical data indicated satisfactory comparability of simulation results.

In order to verify mathematical model for calculation of intra-chamber parameters in a hybrid solid-propellant rocket engine [1, 2] adequacy assessment of calculation methods was performed using analytical estimation of engine's characteristics for quasi-steady-state flow regime, constant flow section area *S* and perimeter P of cylindrical grain channel with the radius r. Other sufficiently accurate solutions of this problem were not obtained.

For abovementioned assumptions the change of oxidizer flow along the grain channel of a hybrid engine can be described by the following equation:

$$\frac{d}{dx}\left(\varrho u S\right) = -\mathrm{P}\alpha_{ox}\varrho_f a\left(\varrho u\right)^{\nu}, \tag{1}$$

where ϱ,u – oxidizer density and flow rate;
a,ν – burning rate law constants; ϱ_f – fuel density; $\alpha_{ox}=m_{ox}/m_f$ – ratio of "consumed" oxidizer mass flow m_{ox} to the mass of fuel m_f in the process of combustion.
Solution of equation (1) regarding oxidizer flow at any channel point (section) is as follows

$$\left.\left(\varrho u\right)\right|_x = \left[\left(\varrho u\right)_0^{1-\nu} - \varphi x\right]^{1/(1-\nu)}, \ \varphi = \frac{2}{r}\alpha_{ox}\varrho_f\left(1-\nu\right)a \tag{2}$$

where $\left(\varrho u\right)_0 = \left(m_{ox}^{in}\right)/S$ – mass flow rate of oxidizer injection (m_{ox}^{in} – total oxidizer consumption).

As it is implied from equation (2) oxidizer mass consumer for combustion m_{ox} can be determined from the following relation

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$$m_{ox}=\begin{cases} m_{ox}^{in}-S\cdot\left[\left(\varrho u\right)_0^{1-\nu}-\varphi L\right]^{1/(1-\nu)}, & \textit{если}\ u\left(\varrho u\right)_0^{1-\nu}>\varphi L,\\ m_{ox}^{in}, \ \textit{если}\ u\left(\varrho u\right)_0^{1-\nu}\le\varphi L. \end{cases}$$

Mass of fuel m_f consumed in the process of combustion is equal to $m_f = m_{ox}/\alpha_{ox}$ and the total mass of combustion products and "unused" oxidizer is equal to $m_\Sigma = m_{ox}^{in} + m_f$.

Thermodynamic characteristics of the mixture (gas constant R and isobaric heat capacity C_p) are determined additively using mass individual characteristics of oxidizer gas $\left(R_{ox}, C_{p_{ox}}\right)$ and combustion products $\left(R_f, C_{p_f}\right)$

$$R = c_{ox}R_{ox} + c_f R_f,\ C_p = c_{ox}C_{p_{ox}} + c_f C_{p_f},$$

where mass fractions of "unused" oxidizer c_{ox} and combustion products c_f are determined by the following relations

$$c_{ox} = \frac{m_{ox}^{in} - m_{ox}}{m_\Sigma},\ c_f = \frac{m_f + m_{ox}}{m_\Sigma} = 1 - c_{ox}.$$

The temperature of mixture of combustion products and "unused" oxidizer can be determined from the law of conservation of energy:

$$\frac{\gamma}{\gamma-1}RTm_\Sigma = \left(m_f + m_{ox}\right)\left(\frac{\gamma}{\gamma-1}RT\right)_f - m_{ox}^{in}\left(\frac{\gamma}{\gamma-1}R\right)_{ox}\left(\tilde{T} - T_{ox}\right). \qquad (3)$$

The first member in the right part of equation (3) describes energy input due to fuel combustion products with the temperature T_f entering the flow. The last member of the right part of this equation represents energy losses due to heating of oxidizer gas from its initial temperature T_{ox} to some average value $\tilde{T}$. At a first approximation it can be taken as $\tilde{T} = \left(T_f + T_{ox}\right)/2$.

Then from the balance of mass of gas supplied to combustion chamber m_{ox}^{in} and combustion products m_Σ taking into account the fact that they are equal to flow rate G through the nozzle block with throat area S_* in case of critical flow, pressure p in combustion chamber can be determined

$$p = \frac{m_\Sigma}{S_* G_*(\gamma)}\sqrt{RT},\ \ G = S_* G_*(\gamma)\cdot\frac{p}{\sqrt{RT}} = m_\Sigma,\ \ G_*(\gamma) = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}.$$

Comparison of numerical calculation results and analytical estimation data obtained using the abovementioned relationships was performed for various levels of oxidizer supply to combustion chamber of hybrid solid-propellant rocket engine with the following parameters.

Initial data for calculation

Engine geometry parameters	Oxidizer parameters
Channel radius r = 1 cm;	Gas temperature T_{ox} = 300 K;
Throat area S_* = 0.79 cm^2;	Adiabatic index γ_{ox} = 1.4;
Fuel grain length L = 50 cm.	Gas constant R_{ox} = 296 J/kg/K.

Fuel parameters

Burning rate law $a = 0.07$ mm/s, $\nu = 0.65$;
Fuel density $\varrho_f = 1000$ kg/m3;
Gas constant of combustion products $R_f = 320$ J/kg/K;
Adiabatic index of combustion products $\gamma_f = 1.22$;
Fuel-oxidizer ratio $\alpha_{ox} = 2.7$;
Burning temperature $T_f = 3000$ K.

Fig.1 illustrates pressure distribution along the relative channel length for m_{ox}^{in}=0.2 kg/s. Relatively smooth pressure profile (power-law type) typical for initial conditions of engine operation is observed. It should be noted that for initial data as well as for other values of oxidizer flow rate the profile is transformed smoothly due to non-uniform fuel burnout and as the front part of fuel grain burns completely a pressure peak is formed and moves together with fuel burnout boundary.

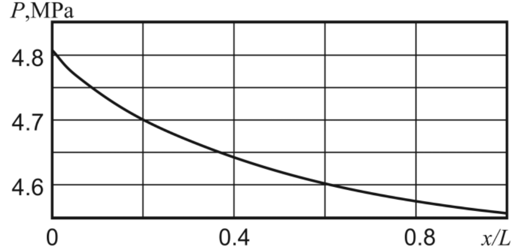

Fig. 1. Pressure distribution along the channel length.

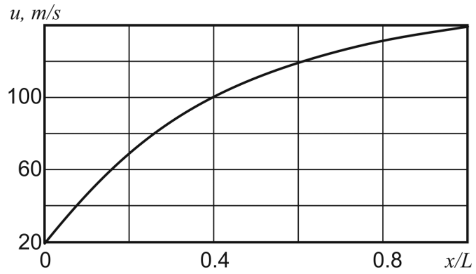

Fig. 2. Gas flow rate distribution along the channel length.

Table 1. – Results of comparison of numerical and analytical data.

m_{ox}^{in}, kg/s	p, MPa		$c_{ox}\|_{x=L}$		$T\|_{x=L}$, K		G, kg/s	
	num	theor	num	theor	num	theor	num	theor
0.10	2.33	2.40	0.002	0.002	2385	2523	0.137	0.137
0.20	4.59	4.71	0.029	0.026	2361	2480	0.271	0.271
0.35	7.84	7.97	0.070	0.068	2326	2404	0.467	0.468
0.80	16.87	16.91	0.163	0.159	2233	2235	1.034	1.035
1.00	20.73	20.63	0.189	0.187	2206	2180	1.281	1.281
2.00	38.88	37.95	0.283	0.281	2095	1992	2.481	2.482

Δ, %	1.70	3.01	3.36	0.06

Δ – mean difference between parameter values.

The results of comparison of numerical and analytical data (“num” – numerical calculation results; “theor” – analytical estimation data) for various levels of oxidizer supply to combustion chamber of hybrid solid-propellant rocket engine are given in Table 1.

Mean difference between values of parameters determining the process (Table 1) does not exceed 3.5% for selected configuration.

Thus, an analytical estimate of engine's characteristics for verification of developed physical and mathematical model of processes in a hybrid solid-propellant rocket engine for quasi-steady-state flow regime, constant flow section area and constant perimeter of working surface of cylindrical grain channel is given in this paper. Performed parametric comparative analysis of calculated and analytical data indicated more than satisfactory comparability of results of numerical calculation and analytical estimation – mean difference between values of parameters determining the process does not exceed 3.5% for selected configuration.

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