# ANALYTICAL STUDY OF MICROWAVE HEATING OF THE COAL LAYER WITH MIXED BOUNDARY CONDITIONS OF I AND III TYPES

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**Abstract**. This work is dedicated to the search for the exact analytical dependences of microwave heating due to absorption of a plane electromagnetic wave by coal layer with asymmetric and non-uniform heat dissipation conditions I and III kind. Some of simplifications have been made, such as one-dimensional problem, uniformity and isotropic coal material, and the constancy of the electrical properties of thermal coal during heating of microwave radiation. This has led to the fact that the Maxwell's task is solved separately from the Fourier's task, and a heat source generated in the carbon layer is subject to Bouguer law. For the system of equations of heat transfer has been found a new dependent variable, which is to simplify the search for a final solution. All this has given the possibility of finding rigorous analytical solution of the problem of microwave heating of the coal layer in the presence of asymmetric and inhomogeneous boundary conditions I and III kind.

## 1 Features of microwave heating of materials

In traditional conditions the heating of materials in the heat treatment process is performed by heat transfer from the surface to the inside by heat conduction. Absolutely other heating mechanism is connected with the microwave absorption in volume dielectric material, including coal. In this mode, the material forming the internal heat sources, which provide its heating. Energy conversion efficiency of the electric field to the heat increases in direct proportion to the oscillation frequency and square of the electric field. It should be noted the easy feeding of microwave energy for almost any of the heated area of the body. An important advantage of microwave heating is thermal absence of inertia i.e. the ability to almost instantly turn on and off the thermal influence on the material being processed. Hence, high precision of adjustment and reproducibility of the heating process are observed. The advantage of microwave heating is also fundamentally high conversion efficiency of the microwave to heat energies that is allocated in the amount of heated bodies. The theoretical value of this efficiency is close to 100%. An important advantage of

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microwave heating is the possibility of the implementation and practical application of new and unusual his species, for example, selective, uniform, ultra-pure, self-regulating.

*Selective heating* is based on dielectric loss depending on the wavelength, i.e. depending of the dielectric loss tangent $\delta$ as a function of wavelength $\lambda$. In the multicomponent mixture of dielectrics it will be heated only those portions where high tg $\delta$. So, in the coal array it will be harder to heat the organic part, rather than a mineral.

*Uniform heating*. Typically, the heat transfer is by convection and radiation. Hence it is inevitable the temperature gradient from the surface to the depth of the material, and the larger, the smaller the thermal conductivity. Reduce or eliminate the temperature gradient is almost possible by increasing the processing time. In many cases only by slow heating it is possible to avoid overheating of the surface layers. With microwave energy it is possible not only to uniformly heating over its volume, but also to receive any desired predetermined temperature distribution. Therefore, in the microwave heating there are a lot of opportunities for acceleration of a number of technological processes.

*Ultrapure heating*. In compare with heating by a gas flame and by arc torches which generally pollute materials, the microwave energy can be supplied to the material through the protective shell of solid dielectrics. The heat transfer medium may be absent at all. As a result the surface contamination from the product is almost completely eliminated. In addition, placing the heated material in a controlled volume environment or an inert gas, it is possible to eliminate surface oxidation. Pollution of the dielectric through which the supplied microwave energy is very small, because in the case of small losses, even with the passage of a large microwave power it remains practically cold.

*Self-regulating heating*. Upon heating for drying purposes the quality of the material is greatly improved due to the fact that the heating is stopped automatically as places dried. This is explained by the fact that the dielectric loss tangent of such materials, such as coal, is directly proportional to moisture. Therefore, with decreasing moisture the loss of the microwave energy during drying is reduced and heating is continued only on those portions of the processed material, which has remained high humidity.

The search for optimal microwave heating modes, designing microwave setups, the calculation of maximum temperature and heat flux and others parameters can be most effectively done using the theoretical approaches. Most interesting are primarily analytically rigorous solutions of microwave heating tasks. With their help it is readily accomplished to make a parametric analysis of microwave heating, to conduct a rapid calculation of thermal processes, identifies the fundamental laws. The purpose of this study is related to the search for such solutions, in particular, are often encountered in practice cases of microwave exposure to carbon layer with the mixed conditions of heat removal from its external surfaces of the first and the third kind.

## 2 The mathematical formulation of problems and build their solutions

We will solve the two tasks of this class.

### *Task 1*

Diagram of the task is shown in figure 1.

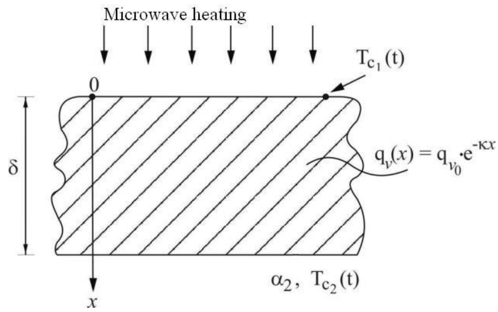


**Fig. 1.** Scheme of microwave heating task of the coal's flat layer with a mixed asymmetric boundary conditions I and III kind.

The microwave heating of the coal layer system of equations with the adopted simplifications is as follows:

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{q_{v_0}}{c\rho}e^{-kx} \tag{1.1}$$

$$T(x,0) = f(x) \tag{1.2}$$

$$T(0,t) = T_{c_1}(t) \tag{1.3}$$

$$\lambda\frac{\partial T(\delta,t)}{\partial x} = \alpha_2\left[T(\delta,t) - T_{c_2}(t)\right] \tag{1.4}$$

$T(x,t)$ – current temperature,K; $x$ – transverse coordinate, m; $a$ – thermal diffusivity, $m^2/s$; c – specific heat, J/kg·K; $\rho$ – dencity, $kg/m^3$; t – time, s; $q_{v_0}$ – the maximum value of the internal heat source, $W/m^3$; k – bsorption coefficient in the Bouguer law,1/m; λ-coal thermal conductivity, $W/(m \cdot K)$ ; $T_{c_1}(t)$ – time-variable temperature of the outer surface, K; $\alpha_2$ – heat transfer coefficient on the lower surface, $W/(m^2 \cdot K)$.

This system (1.1-1.4) by introduction of a new dependent variable:

$$u(x,t) = T(x,t) - T_{c_1}(t) - \frac{T_{c_1}(t) - T_{c_2}(t)}{\frac{\lambda}{\alpha_2} - \delta}x \tag{1.5}$$

It is reduced to a simpler system of equations with homogeneous boundary conditions:

$$\frac{\partial u(x,t)}{\partial t} = a\frac{\partial^2 u(x,t)}{\partial x^2} + q_v(x,t), \tag{2.1}$$

$$u(x,0) = F(x), \tag{2.2}$$

$$u(0,t) = 0, \tag{2.3}$$

$$\lambda\frac{\partial u(\delta,t)}{\partial x} = \alpha_2 u(\delta,t), \tag{2.4}$$

$q_v(x,t) = \frac{q_{v_0}}{c\rho}e^{-kx} - \frac{\partial T_{c_1}(t)}{\partial t} - \frac{\partial(T_{c_1}(t) - T_{c_2}(t))}{\partial t\,(\frac{\lambda}{\alpha_2} - \delta)}x$ – a new distribution of heat sources, F(x) = f(x) - $T_{c_1}(0) - \frac{T_{c_1}(0)-T_{c_2}(0)}{\frac{\lambda}{\alpha_2}-\delta}x$ – new initial temperature distribution

The transformed system (2.1-2.4) with an inhomogeneous equation (2.1) and inhomogeneous initial condition (2.2) is more convenient to solve, divided into two [1]. We construct the general solution in the form of a superposition of two solutions $u(x,t)=u_1(x,t)+u_2(x,t)$, for each part of which should be solved the following boundary value problems:

for $u_1$

$$\frac{\partial u_1(x,t)}{\partial t} = a\frac{\partial^2 u_1(x,t)}{\partial x^2} + q_v(x,t), \quad (3.1)$$

$$u_1(x,0) = 0, \quad (3.2)$$

for $u_2$

$$\frac{\partial u_2(x,t)}{\partial t} = a\frac{\partial^2 u_2(x,t)}{\partial x^2}, \quad (4.1)$$

$$u_2(x,0) = F(x)\ , \quad (4.2)$$

under the previous boundary conditions (2.3), (2.4)

To solve each system of equations is necessary to find the eigenvalues and eigenfunctions of the homogeneous problem (system 4.1 and 4.2). The unknown function $u_i$ - can be represented as the product of two independent functions:

$$u_i=\psi_i(x)\varphi_i(t)$$

Then, find the eigenvalues $\gamma_k$ and eigenfunctions $\Psi_k(x)$ of homogeneous problem. This leads to two equations:

$$\varphi_i{}'(t) + (a\gamma)^2\varphi_i(t) = 0 \quad (5.1)$$

$$\psi_i{}''(x) + \gamma^2\psi_i(x) = 0 \quad (5.2)$$

Solutions for $\psi_i(x)$ generally following $\psi_i(x) = c_1\sin(\gamma x) + c_2\cos(\gamma x)$
Substituting in equations of the boundary conditions, we find:
1) $c_2=0$
2) $\mathrm{tg}(\gamma\delta) = \frac{\lambda}{\alpha_2}\gamma$. From this equation, we find the eigenvalues $\gamma_k$
Then for the functions $\Psi_n(x)$ and $\varphi_n(x)$ we have the following expression

$$\Psi_n(x) = \sin(\gamma_n x),$$

$$\varphi_n(x) = a_n e^{-(\sqrt{a}\gamma_n)^2 t}$$

$a_n$ – integrating constant.
This is enough to find private solutions for the boundary value task u2:

$$u_{2n} = a_n e^{-(\sqrt{a}\gamma_n)^2 t}\sin(\gamma_n x)$$

Then we draw up a series of:

$$u_2 = \sum_{n=1}^{\infty} a_n e^{-(\sqrt{a}\gamma_n)^2 t}\sin(\gamma_n x)$$

Coefficients $a_n$ can be found method of orthogonality,
Using the initial distribution $u_2(x,0)$:

$$F(x) = \sum_{n=1}^{\infty} a_n \sin(\gamma_n x)$$

This is a decomposition of a given function in a trigonometric Fourier series of sines.
The expansion coefficients are found by the formula:

$$a_n = \frac{1}{\|\varphi_n\|^2} \int_a^b r(\xi) F(\xi) \varphi_n(\xi) d\xi \ ,$$

$\varphi_n\ (\xi)$ – eigenfunction ($\sin(\gamma_n x)$), r($\xi$) – weighting function (in this case equal to 1)

a=0, b=$\delta$, $\|\varphi_n\|^2 = \int_0^\delta \sin(\gamma_n x)^2\, dx = \left(\frac{\delta}{2} - \frac{\sin(2\gamma_n \delta)}{4\gamma_n}\right)$

Next, we find the desired function:

$$u_2(x,t) = \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n \delta)}{4\gamma_n}} e^{-(\sqrt{a}\gamma_n)^2 t} \sin(\gamma_n x) \int_0^\delta F(\xi) \sin(\gamma_n \xi) d\xi$$

To find the solution of (3.1-3.2) $u_1$ use applicable in this case, the theorem of Steklov VI [1], according to which $u_1(x,t)$ can be represented as a Fourier series in eigenfunctions of the homogeneous problem (4.1-4.2):

$$u_1(x,t) = \sum_{n=1}^{\infty} b_n\,(t) \Psi_n(x)$$

To find the coefficients $b_n(t)$ expand a given function q (x, t) in a Fourier series in eigenfunctions $\Psi_n(x)$:

$$q_v(x,t) = \sum_{n=1}^{\infty} q_n\,(t) \Psi_n(x),$$

$$q_n(t) = \frac{1}{||\Psi_n(x)||^2} \int_0^\delta q_v(x,t)\, \Psi_n(x) dx$$

Next, we substitute the expression for $q(x,t)$ и $u_1(x,t)$ в (3.1):

$$\sum_{n=1}^{\infty} b_n{}'\,(t) \Psi_n(x) = \sum_{n=1}^{\infty} a\, b_n\,(t) \Delta\Psi_n(x) + \sum_{n=1}^{\infty} q_n\,(t) \Psi_n(x)$$

Then, using (5.2) and comparing the coefficients of $\Psi_n(x)$, we find for $b_n(t)$:

$$b_n(t)' + \left(\sqrt{a}\gamma_n\right)^2 b_n(t) = q_n(t)$$

with the initial condition (3.2), which implies: $b_n(0) = 0$
Solution of linear inhomogeneous differential equation has the form:

$$b_n(t) = \int_0^t e^{-\left(\sqrt{a}\gamma_n\right)^2 (t-\tau)} q_n(\tau) d\tau$$

As a result for $u_1(x,t)$:

$$u_1(x,t) = \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n\delta)}{4\gamma_n}} \sin(\gamma_n x) \int_0^t \int_0^\delta e^{-(\sqrt{a}\gamma_n)^2(t-\tau)} q(\xi,\tau) \sin(\gamma_n \xi)\, d\tau d\xi, \quad (5.3)$$

The interim solution u(x,t)= $u_1(x,t) + u_2(x,t)$ of system $(2.1 - 2.4)$:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n\delta)}{4\gamma_n}} \sin(\gamma_n x) \int_0^t \int_0^\delta e^{-(\sqrt{a}\gamma_n)^2(t-\tau)} q(\xi,\tau) \sin(\gamma_n \xi) d\tau d\xi + \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n\delta)}{4\gamma_n}} e^{-(\sqrt{a}\gamma_n)^2 t} \sin(\gamma_n x) \int_0^\delta F(\xi) \sin(\gamma_n \xi) d\xi \quad (6.1)$$

After inverse substitutions we get the final solution of the system (1.1-1.4)

$$T(x,t) = T_{c_1}(t) + \frac{T_{c_1}(t) - T_{c_2}(t)}{\frac{\lambda}{\alpha_2} - \delta} x + \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n\delta)}{4\gamma_n}} \sin(\gamma_n x) \int_0^t \int_0^\delta e^{-(\sqrt{a}\gamma_n)^2(t-\tau)} \left\{ \frac{q_{v_0}}{c\rho} e^{-k\xi} - \frac{\partial T_{c_1}(\tau)}{\partial \tau} - \frac{\partial (T_{c_1}(\tau) - T_{c_2}(\tau))}{\partial \tau \left(\frac{\lambda}{\alpha_2} - \delta\right)} \xi \right\} \sin(\gamma_n \xi) d\tau d\xi + \sum_{n=1}^{\infty} \frac{1}{\frac{\delta}{2} - \frac{\sin(2\gamma_n\delta)}{4\gamma_n}} e^{-(\sqrt{a}\gamma_n)^2 t} \sin(\gamma_n x) \int_0^\delta \left\{ \mathrm{f}(\xi) - T_{c_1}(0) - \frac{T_{c_1}(0) - T_{c_2}(0)}{\frac{\lambda}{\alpha_2} - \delta} \xi \right\} \sin(\gamma_n \xi) d\xi \quad (6.2)$$

## *Task 2.*

Now consider the case when the upper boundary of the layer satisfies the III kind of condition, the bottom - I kind.

The scheme of this problem is shown in figure 2.

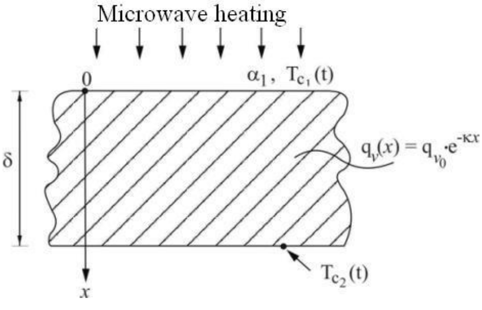


**Fig. 2.** Scheme of task of microwave heating of the coal's flat layer with mixed asymmetric boundary conditions I and III kind.

The microwave heating system's equations to the coal layer of this formulation of the problem is of the form:

$$\frac{\partial T(x,t)}{\partial t} = a \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{q_{v_0}}{c\rho} e^{-kx}, \quad (7.1)$$

$$T(x,0) = f(x), \quad (7.2)$$

$$-\lambda \frac{\partial T(0,t)}{\partial x} = \alpha_1\left[T(0,t) - T_{c_1}(t)\right], \tag{7.3}$$

$$T(\delta, t) = T_{c_2}(t), \tag{7.4}$$

In this case, a new variable, which simplifies the system (7.1-7.4), is as follows:

$$u(x,t) = T(x,t) - \frac{\delta^2 - x^2}{\delta^2}\left[T_{c_1}(t) - T_{c_2}(t)\right] - T_{c_2}(t) \tag{7.5}$$

Now a simple system with homogeneous boundary conditions as follows:

$$\frac{\partial u(x,t)}{\partial t} = a\frac{\partial^2 u(x,t)}{\partial x^2} + q_v(x,t), \tag{8.1}$$

$$u(x,0) = F(x), \tag{8.2}$$

$$-\lambda \frac{\partial u(0,t)}{\partial x} = \alpha_1 u(0,t), \tag{8.3}$$

$$u(\delta, t) = 0, \tag{8.4}$$

$q_v(x,t) = \frac{q_{v_0}}{c\rho}e^{-kx} - (\frac{\delta^2 - x^2}{\delta^2})\frac{\partial(T_{c_1}(t) - T_{c_2}(t))}{\partial t} - \frac{\partial T_{c_2}(t)}{\partial t}$ -2 $a[T_{c_1}(t) - T_{c_2}(t)]$ – a new distribution of heat sources,

F(x) = f(x) $-\left(\frac{\delta^2 - x^2}{\delta^2}\right)[T_{c_1}(0) - T_{c_2}(0)] - T_{c_2}(0)$ – new initial temperature distribution.

Further, by performing actions similar to the solution of problem 1, we divide the task into two subtasks with two solutions $u_1$ и $u_2$ . Then we find a new own functions:

$$\Psi_n(x) = \cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x),$$

$$\varphi_n(x) = a_n e^{-(\sqrt{a}\gamma_n)^2 t},$$

The eigenvalues are found from a transcendental relations:

$$\text{tg}(\gamma_n\delta) = \frac{\lambda}{\alpha_1}\gamma_n,$$

$a_n$ – integrating constant.

This is enough to find a private solution of the second sub-tasks (u2):

$$u_{2n} = a_n e^{-(\sqrt{a}\gamma_n)^2 t}(\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x))$$

Then we draw up a series of

$$u_2 = \sum_{n=1}^{\infty} a_n e^{-(\sqrt{a}\gamma_n)^2 t}(\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x))$$

Coefficients $a_n$ can be found from the initial condition for the distribution of $u_2$ (8.2):

$$F(x) = \sum_{n=1}^{\infty} a_n(\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x))$$

This expansion is a given function by a trigonometric series of eigenfunctions.

The coefficients of decomposition are found $a_n$ from:

$a_n = \frac{1}{\|\varphi_n\|^2}\int_a^b r(\xi)F(\xi)\varphi_n(\xi)d\xi,$

$\varphi_n(\xi)$ - eigenfunction - $(\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma x))$, r($\xi$) – weighting function (in this case equal to 1)

We introduce a new value $c_n$ =- $\frac{\alpha_1}{\lambda\gamma_n}$, simplifies subsequent calculations record:

a=0, b= $\delta, \|\varphi_n\|^2 = \int_0^\delta(\cos(\gamma_n x) + c_n\sin(\gamma_n x))^2\,dx = \frac{\delta}{2}(1 + c_n{}^2) + \frac{1}{4\gamma_n}[(1 - c_n{}^2)\sin(2\gamma_n\delta) - 2c_n(\cos(2\gamma_n\delta) - 1)],$

Next, we find the desired function:

$$u_2(x,t) = \\ = \sum_{n=1}^{\infty}\frac{1}{\frac{\delta}{2}(1 + c_n{}^2) + \frac{1}{4\gamma_n}[(1 - c_n{}^2)\sin(2\gamma_n\delta) - 2c_n(\cos(2\gamma_n\delta) - 1)]}e^{-(\sqrt{\alpha}\gamma_n)^2 t}\cdot$$

$$[\cos(\gamma_n x) + c_n\sin(\gamma_n x)]$$

$$\int_0^\delta F(\xi)\,[\cos(\gamma_n\xi) + c_n\sin(\gamma_n\xi)]\,d\xi,$$

Searching solutions for another subtask $u_1$ performed similarly. So:

$$u_1(x,t) = \\ = \sum_{n=1}^{\infty}\frac{1}{\frac{\delta}{2}(1 + c_n{}^2) + \frac{1}{4\gamma_n}[(1 - c_n{}^2)\sin(2\gamma_n\delta) - 2c_n(\cos(2\gamma_n\delta) - 1)]}[\cos(\gamma_n x) + c_n\sin(\gamma_n x)]$$

$$\int_0^t\int_0^\delta e^{-(\sqrt{\alpha}\gamma_n)^2(t-\tau)}q_v(\xi,\tau)\,[\cos(\gamma_n\xi) + c_n\sin(\gamma_n\xi)]\,d\tau d\xi$$

The total solution u(x,t)= $u_1(x,t) + u_2(x,t)$ of $(8.1 - 8.4)$ is:

$$u(x,t) = \\ \sum_{n=1}^{\infty}\frac{1}{\|\varphi_n\|^2}[\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x)]\int_0^t\int_0^\delta e^{-(\sqrt{\alpha}\gamma_n)^2(t-\tau)}q_v(\xi,\tau)\,[\cos(\gamma_n\xi) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n\xi)]\,d\tau d\xi + \\ \sum_{n=1}^{\infty}\frac{1}{\|\varphi_n\|^2}e^{-(\sqrt{\alpha}\gamma_n)^2 t}\,[\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x)]\int_0^\delta F(\xi)\,[\cos(\gamma_n\xi) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n\xi)]\,d\xi, (9.1)$$

After inverse substitutions final solution of the system (7.1-7.4) following:

$$T(x,t) = \\ \frac{\delta^2-x^2}{\delta^2}\left[T_{c_1}(t) - T_{c_2}(t)\right] + T_{c_2}(t) + \\ \sum_{n=1}^{\infty}\frac{1}{\|\varphi_n\|^2}[\cos(\gamma_n x) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x)]\int_0^t\int_0^\delta e^{-(\sqrt{\alpha}\gamma_n)^2(t-\tau)}(\frac{q_{v_0}}{c\rho}e^{-k\xi} - \\ (\frac{\delta^2-\xi^2}{\delta^2})\frac{\partial(T_{c_1}(\tau) - T_{c_2}(\tau))}{\partial\tau} - \frac{\partial T_{c_2}(\tau)}{\partial\tau} - 2a[T_{c_1}(\tau) - T_{c_2}(\tau)])\,[\cos(\gamma_n\xi) - \frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n\xi)]\,d\tau d\xi +$$

$\sum_{n=1}^{\infty}\frac{1}{\|\varphi_n\|^2}e^{-(\sqrt{a}\gamma_n)^2 t}\left[\cos(\gamma_n x)-\frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n x)\right]\int_0^{\delta}\left(\mathrm{f}(\xi)\ -\left(\frac{\delta^2-\xi^2}{\delta^2}\right)\left[T_{c_1}(0)-T_{c_2}(0)\right]-T_{c_2}(0)\right)\left[\cos(\gamma_n \xi)-\frac{\alpha_1}{\lambda\gamma_n}\sin(\gamma_n \xi)\right]d\xi,$ (9.2)

The derived analytically rigorous solution of the problem of microwave heating of the coal layer with mixed boundary conditions I and III type of (6.2), (9.2) in general allow for a wide range of private simplistic solutions with the possibility of a detailed parametric analysis. The information on the distribution of temperature fields is the base for the evaluation of microwave-ignition of coal fuel parameters determining the thermo-destructive forces, the search control of automated microwave heating effects, the implementation of optimal parameters of microwave technology coal arrays and other.

From these solutions can extract important information about the parameters of microwave heating of the coal massif:

1) To investigate the extremum of temperature function (6.2), (9.2), it is possible to find the position and the value of the maximum temperature;
2) In case of equality the maximum temperature of the coal layer and ignition temperature it is possible to find moment of ignition;
3) These solutions can also be used to find the maximum temperature difference between the coal layer, which directly affects the level of hazardous thermo-destructive forces.
4) Other process parameters for optimization, automation, etc.

## 3 Conclusion

Rigorous analytical solutions of two problems of microwave heating of the coal layer with mixed and fairly arbitrary boundary conditions I and III kind were built. Getting these solutions due to the adoption of the constancy of the electrical and thermal characteristics of the coal. With the help of two new substitutions the initial system of equations is reduced to a transformed with homogeneous boundary conditions that simplify the search for the final decision. The final solution was found by the method of making a superposition of two less complex subtasks. Internal heat source of the absorption of electromagnetic waves modeled by Bouguer law. With these solutions can be obtained vital information to a wide range of special cases: the temperature distribution on the layer with the passage of time, the maximum temperature, the ignition timing, the greatest temperature difference allotted heat and other important parameters of the coal heating layer of microwave radiation. The derived analytical solutions allow to obtain a plurality of individual results, which demand by engineering practice, which, of course, is the foundation for the creation of energy efficient and environmentally friendly thermal coal treatment microwave technology.

## References

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