MATEC Web of Conferences **71**, 05001 (2016) DOI: 10.1051/matecconf/20167105001
ICCPE 2015

Robust H_∞ Observer-Based Control of Uncertain Neutral Systems with Mixed Delays

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Abstract. In this paper, the robust In this paper, the robust H_∞ control problem of output dynamic observer-based control for a class of uncertain neutral systems with discrete and distributed time delays is considered. Linear matrix inequality (LMI) optimization approach is used to design the new H_∞ output dynamic controls. The minimal H_∞ -norm bound and the maximal perturbed bound are given. Based on the result of this paper, the constraint of matrix equality is not necessary for designing the H_∞ observer-based controls. Finally, a numerical example is given to illustrate the efficiency of the proposed approach.

1 Introduction

The existence of the time-delay phenomena in a dynamic system may cause instability or bad performances in dynamic systems [1-3]. In some practical systems, the system models can be described by functional differential equation of neutral type, which the models depend on the state delay but also depend on the state derivatives. Physical examples for neutral system have distributed networks, population ecology, process including steam, heat exchanges, lossless transmission line. Hence the stability and stabilization problems for neutral time-delay system have received some attenuation. By increasing in the equation number of summands and simultaneously decreasing the differences between neighbouring argument values, one naturally arrives at equations with distributed (or continuous) and mixed (both distributed and discrete) delay arguments [2]. In view of Fridman [2], the distributed delays, play an important role about the stability of system.

In the many real-world sysyems, state feedback control will fail to guarantee the stabilizability when some of system states are not measurable. In the observer-based control, output dynamic feedback control is provided and the system state can be estimated from the process. The observer-based controls are often be utilized to stabilize unstable systems or improve the system proformance. Hence, the observer-based control for systems has been an interesting topic in control theory. Lyapunov stability theory is used to design the nonlinear state observers for linear time varying systems [4]. In [5], the LMI approach was introduced to design the observer-based controls for uncertain systems.

On the other hand, the H_∞ control concept was proposed to reduce the effect of the disturbance input on the regulated output to within a prescribed level. The state feedback H_∞ controls for uncertain neutral time-delay systems had been considered in [6-7]. The output H_∞ filtering design for neutral system without uncertainties had been proposed in [8]. To the best of the knowledge of author, the robust H_∞ observer-based control for neutral systems with discrete and distributed time delays has never been considered in the past. In this paper, we will adopt this useful methodology to the design of the robust H_∞ observer-based controls for a class of uncertain neutral time-delay systems. The three classes of H_∞ observer-based controls with known and unknown (uncertain) time-delay values will be considered. Moreover, the minimal H_∞ -norm bound and the maximal perturbed bound for the observer-based controls are provided.

2 Problem formulations

Consider the following uncertain neutral system with discrete and distributed time delays:

$$\dot{x}(t)=\left(A_0+\Delta A_0(t)\right)x(t)+\left(A_1+\Delta A_1(t)\right)x(t-h)+\left(A_2+\Delta A_2(t)\right)\dot{x}(t-\tau)$$
$$+\left(A_3+\Delta A_3(t)\right)\int_{t-\eta}^{t}x(s)ds+\left(B+\Delta B(t)\right)u(t)+\left(B_w+\Delta B_w(t)\right)w(t), \quad (1a)$$

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$y(t) = C_1 x(t) + D_1 u(t)$, (1b)

$z(t) = C_2 x(t) + D_2 u(t) + D_3 w(t)$, (1c)

$x(t) = \phi(t),\ t \in [-H, 0]$, (1d)

where $x \in \Re^n$, x_t is the state at time t defined by $x_t(s) := x(t+s), \forall s \in [-H, 0]$, $H = \max\{h, \tau, \eta\} > 0$, $u \in \Re^m$ is the input, $w \in \Re^l$ is the disturbance input, $y \in \Re^p$ is the measured output, $z \in \Re^q$ is the regulated output, A_0, A_1, A_2, $A_3 \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $B_w \in \Re^{n\times l}$, $C_1 \in \Re^{p\times n}$, $D_1 \in \Re^{p\times m}$, $C_2 \in \Re^{q\times n}$, $D_2 \in \Re^{q\times m}$, and $D_3 \in \Re^{q\times l}$ are constant matrices, the pairs (A_0, B) and (A_0, C_1) are stabilizable and detectable, respectively. The matrices $\Delta A_0(t)$, $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta A_3(t)$, $\Delta B(t)$, and $\Delta B_w(t)$, are some perturbations with appropriate dimensions, and the initial vector ϕ is a continuous function. The following assumptions are made on system (1).

Assumption 1. Suppose that the matrix C_1 has full row rank (i.e., $rank(C_1) = p$).

For convenience of discussed, the singular value decomposition of C_1 as follows:

$$C_1 = U[S \quad 0]V^T,$$

where $S \in \Re^{p\times p}$ is a diagonal matrix with nonnegative diagonal elements in decreasing order, $0 \in \Re^{p\times(n-p)}$ is a zero matrix, and $U \in \Re^{p\times p}$ and $V \in \Re^{n\times n}$ are unitary matrices.

Assumption 2. The perturbed matrices $\Delta A_0(t)$, $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta A_3(t)$, $\Delta B(t)$, and $\Delta B_w(t)$ satisfy

$$\begin{aligned}&[\Delta A_0(t)\ \ \Delta A_1(t)\ \ \Delta A_2(t)\ \ \Delta A_3(t)\ \ \Delta B(t)\ \ \Delta B_w(t)]\\ &= M \cdot F(t) \cdot [N_0\ \ N_1\ \ N_2\ \ N_3\ \ N_4\ \ N_5]\end{aligned},$$

where matrices M, N_0, N_1, N_2, N_3, N_4, and N_5 are constant with appropriate dimensions and the uncertain parameter $F(t)$ satisfies $F^T(t)F(t) \le \nu \cdot I$ with $\nu > 0$.

Remark 1. The parameter perturbations $\Delta A_0(t)$, $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta A_3(t)$, $\Delta B(t)$, and $\Delta B_w(t)$, that appear in system (1) will represent the impossibility for exact mathematical model of a dynamic system due to the system complexity. The uncertainty has been widely used in many practical systems which can be either exactly modeled or overbounded by the condition $F^T(t)F(t) \le \nu \cdot I$. The ν is the bounding parameter on the uncertain perturbation $F(t)$, the matrix $F(t)$ contains the uncertain parameters, and constant matrices M and N_i, $i = 0,1,2,3,4,5$, specify how the uncertain parameter $F(t)$ affect the nominal matrix of system (1).

We wish to design the following modified observer-based control with the known time-delay values h, τ and η for system (1):

$$\begin{aligned}\dot{\hat{x}}(t) &= A_C \hat{x}(t) + A_D \hat{x}(t-h) + A_E \dot{\hat{x}}(t-\tau)\\ &+ A_F \int_{t-\eta}^{t} \hat{x}(s)ds + Bu(t) + L[y(t) - \hat{y}(t)]\end{aligned}, \quad (2a)$$

$\hat{y}(t) = C_1 \hat{x}(t) + D_1 u(t)$, (2b)

$u(t) = K\hat{x}(t)$, (2c)

where $\hat{x} \in \Re^n$ is the estimation of x, $\hat{y} \in \Re^p$ is the observer output, $K \in \Re^{m\times n}$ is the controller gain, $L \in \Re^{n\times p}$ is the observer gain, and A_C, A_D, A_E, $A_F \in \Re^{n\times n}$ are matrices to be determined.

Definition 1. Consider the system (1) with observer-based control (2) and the following conditions are satisfied:

(i) With $w(t) = 0$, the closed-loop system (1) with (2) is asymptotically stable.

(ii) With zero initial condition (i.e. $x_0 = \hat{x}_0 = 0$), the signals $w(t)$ and $z(t)$ are bounded by

$\int_0^\infty \|z(t)\|^2 dt \le \gamma^2 \cdot \int_0^\infty \|w(t)\|^2 dt$ (i.e., $\|z\|_2^2 \le \gamma^2 \cdot \|w\|_2^2$),

$\forall\, w \in L_2[0, \infty), w \ne 0$, for some $\gamma > 0$. In this condition, the system (1) is said to be stabilizable with disturbance attenuation γ and degree ν by observer-based control (2), and the control law (2) is said to be an H_∞ observer-based control for system (1). The parameter γ is said to be the H_∞ -norm bound for the H_∞ observer-based control, and the parameter ν is said to be the perturbed bound for the H_∞ observer-based control.

Lemma 1.[9] For a given $C_1 \in \Re^{p\times n}$ with $rank(C_1) = p$, assume that $X \in \Re^{n\times n}$ is a symmetric matrix, then there exists a matrix $\hat{X} \in \Re^{p\times p}$ such that $C_1 X = \hat{X} C_1$ if and only if

$$X = V \begin{bmatrix} \hat{X}_{11} & 0 \\ 0 & \hat{X}_{22} \end{bmatrix} V^T,$$

where $\hat{X}_{11} \in \Re^{p\times p}$ and $\hat{X}_{22} \in \Re^{(n-p)\times(n-p)}$.

Lemma 2. [10] Let $\Omega_0(x)$ and $\Omega_1(x)$ be two arbitrary quadratic forms over $\Re^n$, then $\Omega_0(x) < 0$ for all $x \in \Re^n - \{0\}$ satisfying $\Omega_1(x) \le 0$ if and only if there exist $\varepsilon \ge 0$ such that

$\Omega_0(x) - \varepsilon \cdot \Omega_1(x) < 0$, $\forall x \in \Re^n - \{0\}$.

3 Robust H_∞ control

By (2), system (1) and (2) can be rewritten as

$$\begin{aligned}\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{e}(t)\end{bmatrix} &= \begin{bmatrix} A_C + BK & LC_1 \\ A_0 - A_C & A_0 - LC_1\end{bmatrix}\begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix} + \begin{bmatrix} A_D & 0 \\ A_1 - A_D & A_1\end{bmatrix}\begin{bmatrix}\hat{x}(t-h)\\ e(t-h)\end{bmatrix}\\ &+ \begin{bmatrix} A_E & 0 \\ A_2 - A_E & A_2\end{bmatrix}\begin{bmatrix}\dot{\hat{x}}(t-\tau)\\ \dot{e}(t-\tau)\end{bmatrix} + \begin{bmatrix} A_F & 0 \\ A_3 - A_F & A_3\end{bmatrix}\int_{t-\eta}^{t}\begin{bmatrix}\hat{x}(s)\\ e(s)\end{bmatrix}ds\\ &+ \begin{bmatrix}0\\ B_w\end{bmatrix}w(t) + \begin{bmatrix}0\\ M\end{bmatrix}\tilde{\Delta}(t)\end{aligned}, \quad (3a)$$

where the signal $e(t) = x(t) - \hat{x}(t)$ is defined as the estimated error of system,
$\tilde{\Delta}(t) = F(t)((N_0 + N_4 K)\hat{x}(t) + N_0 e(t) + N_1 \hat{x}(t-h) + N_1 e(t-h)$

$+N_2\dot{\hat{x}}(t-\tau)+N_2\dot{e}(t-\tau)+N_3\int_{t-\eta}^{t}\hat{x}(s)ds+N_3\int_{t-\eta}^{t}e(s)ds+N_5w(t))$ and the uncertainty $\widetilde{\Delta}(t)$ satisfy the following quadratic inequality:

$$\widetilde{\Delta}^T(t)\cdot\widetilde{\Delta}(t)\le v\cdot\begin{bmatrix}\hat{x}(t)\\ e(t)\\ \hat{x}(t-h)\\ e(t-h)\\ \dot{\hat{x}}(t-\tau)\\ \dot{e}(t-\tau)\\ \int_{t-\eta}^{t}\hat{x}(s)ds\\ \int_{t-\eta}^{t}e(s)ds\\ w(t)\end{bmatrix}^T\begin{bmatrix}(N_0+N_4K)^T\\ N_0^T\\ N_1^T\\ N_1^T\\ N_2^T\\ N_2^T\\ N_3^T\\ N_3^T\\ N_5^T\end{bmatrix}\begin{bmatrix}(N_0+N_4K)^T\\ N_0^T\\ N_1^T\\ N_1^T\\ N_2^T\\ N_2^T\\ N_3^T\\ N_3^T\\ N_5^T\end{bmatrix}^T\begin{bmatrix}\hat{x}(t)\\ e(t)\\ \hat{x}(t-h)\\ e(t-h)\\ \dot{\hat{x}}(t-\tau)\\ \dot{e}(t-\tau)\\ \int_{t-\eta}^{t}\hat{x}(s)ds\\ \int_{t-\eta}^{t}e(s)ds\\ w(t)\end{bmatrix}. \quad (3b)$$

The minimal disturbance attenuation γ, the allowable maximal bound v, and the H_∞ observer-based control (2) could be solved simultanously from the following result.

Theorem 1. Consider the system (1) with the observer-based control (2). Suppose $\left\|\begin{bmatrix}A_E & 0\\ A_2-A_E & A_2\end{bmatrix}\right\|+\sqrt{2}\cdot\|M\|\cdot\|N_2\|<1$ and the following optimization problem:

$$\min_{\kappa,\rho,X_1,\hat{X}_{11},\hat{X}_{22},X_3,X_4,X_5,X_6,X_7,X_8,W_1,W_2,W_3,W_4,W_5,W_6}\kappa+\rho, \quad (4a)$$

subject to

$$\begin{bmatrix}
\Phi_{11} & \Phi_{12} & W_4 & 0 & W_5 & 0 & W_6 & 0 & 0 & 0 & X & 0 & \eta\cdot X & 0 & \Phi_{115} & \Phi_{116} & \Psi & \Phi_{118}\\
* & \Phi_{22} & \Phi_{23} & A_1X_4 & \Phi_{25} & A_2X_6 & \Phi_{27} & A_3X_8 & B_w & M & 0 & \Phi & 0 & \eta\cdot\Phi & \Phi_{215} & \Phi_{216} & \Phi C_2^T & \Phi_{218}\\
* & * & -X_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{315} & \Phi_{316} & 0 & \Phi_{318}\\
0 & * & 0 & -X_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{416} & 0 & \Phi_{418}\\
* & * & 0 & 0 & -X_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{515} & \Phi_{516} & 0 & \Phi_{518}\\
0 & * & 0 & 0 & 0 & -X_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{616} & 0 & \Phi_{618}\\
* & * & 0 & 0 & 0 & 0 & -X_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{715} & \Phi_{716} & 0 & \Phi_{718}\\
0 & * & 0 & 0 & 0 & 0 & 0 & -X_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{816} & 0 & \Phi_{818}\\
0 & * & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa\cdot I & 0 & 0 & 0 & 0 & 0 & 0 & B_w^T & D_3^T & N_5^T\\
0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & M^T & 0 & 0\\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_4 & 0 & 0 & 0 & 0 & 0 & 0\\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_7 & 0 & 0 & 0 & 0 & 0\\
0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_8 & 0 & 0 & 0 & 0\\
* & * & * & 0 & * & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_5 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & -X_6 & 0 & 0\\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0\\
* & * & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho\cdot I
\end{bmatrix}<0 \quad ,(4b)$$

has a solution $\kappa>0$, $\rho>0$, $X_1\in\Re^{n\times n}>0$, $\hat{X}_{11}\in\Re^{p\times p}>0$, $\hat{X}_{22}\in\Re^{(n-p)\times(n-p)}>0$, $X_3\in\Re^{n\times n}>0$, $X_4\in\Re^{n\times n}>0$, $X_5\in\Re^{n\times n}>0$, $X_6\in\Re^{n\times n}>0$, $X_7\in\Re^{n\times n}>0$, $X_8\in\Re^{n\times n}>0$, $W_1\in\Re^{n\times n}$, $W_2\in\Re^{m\times n}$, $W_3\in\Re^{n\times p}$, $W_4\in\Re^{n\times n}$, $W_5\in\Re^{n\times n}$, and $W_6\in\Re^{n\times n}$, where

$\Phi=V\begin{bmatrix}\hat{X}_{11} & 0\\ 0 & \hat{X}_{22}\end{bmatrix}V^T$, $\Psi=(C_2X_1+D_2W_2)^T$,

$\Phi_{11}=W_1+W_1^T+BW_2+W_2^TB^T$, $\Phi_{12}=W_3C_1+X_1A_0^T-W_1^T$,

$\Phi_{22}=A_0\Phi+\Phi A_0^T-W_3C_1-C_1^TW_3^T$, $\Phi_{23}=A_1X_3-W_4$,

$\Phi_{25}=A_2X_5-W_5$, $\Phi_{27}=A_3X_7-W_6$, $\Phi_{115}=(W_1+BW_2)^T$,

$\Phi_{215}=C_1^TW_3^T$, $\Phi_{315}=W_4^T$, $\Phi_{515}=W_5^T$, $\Phi_{715}=W_6^T$,

$\Phi_{116}=X_1A_0^T-W_1^T$, $\Phi_{216}=\Phi A_0^T-C_1^TW_3^T$,

$\Phi_{316}=X_3A_1^T-W_4^T$, $\Phi_{416}=X_4A_1^T$, $\Phi_{516}=X_5A_2^T-W_5^T$,

$\Phi_{616}=X_6A_2^T$, $\Phi_{118}=(N_0X_1+N_3W_2)^T$, $\Phi_{218}=\Phi N_0^T$,

$\Phi_{314}=X_3N_1^T$, $\Phi_{418}=X_4N_1^T$, $\Phi_{518}=X_5N_2^T$, and

$\Phi_{618}=X_6N_2^T$. Then the system (1) is robustly stabilizable with disturbance attenuation $\gamma=\sqrt{\kappa}$ and degree $\upsilon=\rho^{-1}$ by the H_∞ observer-based control (2). The matrices $A_C=W_1X_1^{-1}$, $A_D=W_4X_3^{-1}$, $A_E=W_5X_5^{-1}$, $A_F=W_6X_7^{-1}$, $K=W_2X_1^{-1}$, and $L=W_3US\hat{X}_{11}^{-1}S^{-1}U^T$ are obtained, respectively.

Proof: Due to the limitation of page, it is omitted.

4 Example

Consider the neutral system (1) with the parameters:

$$A_0=\begin{bmatrix}1 & 0.5 & 1\\ -0.5 & -2 & 1\\ 1 & -1 & -1.1\end{bmatrix}\qquad A_1=\begin{bmatrix}-0.7 & 0.6 & -0.6\\ -0.3 & -0.2 & 0.7\\ 0.2 & 0.5 & -0.3\end{bmatrix},$$

$$A_2=\begin{bmatrix}-0.1 & 0.1 & -0.2\\ -0.3 & -0.2 & 0.1\\ 0 & 0 & -0.3\end{bmatrix},\qquad A_3=\begin{bmatrix}0.1 & 0 & 0.2\\ 0.1 & 0.2 & 0.1\\ 0.1 & 0 & 0\end{bmatrix},$$

$$B=\begin{bmatrix}-1 & 0\\ 1 & -1\\ 0 & -1\end{bmatrix},\quad B_w=\begin{bmatrix}0.1 & 0\\ 0 & 0\\ 0.1 & 0\end{bmatrix},\quad C_1=\begin{bmatrix}0.1 & 0.2 & 0.1\\ 0.1 & 0.5 & 0\end{bmatrix},$$

$$D=\begin{bmatrix}0.1 & 0.1 & 0\end{bmatrix},\quad D_2=\begin{bmatrix}0.1 & 0\end{bmatrix},\quad D_3=\begin{bmatrix}0 & 0.1\end{bmatrix},\quad M=\begin{bmatrix}0.2\\ 0.3\\ 0.4\end{bmatrix},$$

$$N_0=\begin{bmatrix}0 & 0.5 & 0\end{bmatrix},\quad N_1=\begin{bmatrix}0.2 & 0 & 0\end{bmatrix},\quad N_2=\begin{bmatrix}0.1 & 0.1 & 0\end{bmatrix},$$

$$N_3=\begin{bmatrix}0.1 & 0 & 0.1\end{bmatrix},\ N_4=\begin{bmatrix}0.2 & 0.1\end{bmatrix},\ N_5=\begin{bmatrix}0.1 & 0.1\end{bmatrix},\ \eta=0.1. \quad (4)$$

By Theorem 1 with (A1)-(A2), and the optimization problem (4), the dynamic feedback gains in (2) are given by

$$A_C=\begin{bmatrix}0.4654 & 0.2771 & 0.8436\\ -2.541 & -2.8088 & 0.4225\\ 2.8507 & -0.2738 & -0.5808\end{bmatrix},\ A_D=\begin{bmatrix}-0.8729 & 0.6307 & -0.6077\\ -0.445 & -0.1854 & 0.6961\\ 0.0142 & 0.5494 & -0.3123\end{bmatrix}$$

$$A_E=\begin{bmatrix}-0.1792 & 0.0368 & -0.21\\ -0.3692 & -0.2617 & 0.0952\\ -0.0813 & -0.0554 & -0.3161\end{bmatrix},\ A_F=\begin{bmatrix}2.9477 & -0.1598 & 2.9731\\ -0.1598 & 0.114 & -0.0912\\ 2.9731 & -0.0912 & 3.0539\end{bmatrix},$$

$$K=\begin{bmatrix}-15.9336 & -9.5613 & -5.2457\\ 89.6026 & 37.1001 & 26.8625\end{bmatrix},\ L=\begin{bmatrix}510.453 & -240.4732\\ 130.0944 & -53.6957\\ 524.9408 & -255.4172\end{bmatrix}.$$

The system (1) is stabilizable by the H_∞ observer-based control (2) with (18), disturbance attenuation $\gamma=\sqrt{\kappa}=0.5453$, and degree $\nu=\rho^{-1}=0.9206$.

In this situation, the states of the system (1) with (4) could not be measurable or have no any practical sense. Hence, the state feedback control schemes in [7] cannot be applied to design the H_∞ control of system. Obviously, the design scheme derived in this paper is more efficient and flexible.

5 Conclusions

In this paper, the problem of H_∞ control design for a class of uncertain systems has been considered. LMI optimization approach has been developed to construct the output H_∞ dynamic observer-based feedback control. A numerical example has been given to demonstrate the merits of the proposed results. Finally, we have noticed that the useful results on the robust observer-based control problem for linear systems with perturbations [11]. Based on the LMI approach, the robust observer-based control has been derived. However, the conditions including the constraint of matrix equality are not the classic LMI feasible form. Their results can not use the efficient software MATLAB to solve the LMI problem with equality constraint. Our obtained results can be performed directly by LMI toolbox of MATLAB.

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