Heat transfer optimization of \( \text{SCO}_2 \) porous flow based on Brinkman model

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Abstract. The purpose of this study is to obtain the optimal operating condition in order to find the maximum supercritical \( \text{CO}_2 \) heat extraction in the enhanced geothermal system (EGS). In this study, the heat transfer model conjugated with the Brinkman model is used to evaluate the thermal behavior in the reservoir of the EGS. This numerical model is validated by experiment. Optimization is processed based on the Nelder-Mead approach. The optimal operating conditions are proposed with different pressure, porosity. This study will build the optimal platform of heat source of geothermal power plant.

1 Introduction

With the progress of science and technology rapidly, the energy growing is emphasized gradually. Among the growing energy, the renewable energy such as the geothermal energy continues to be upgraded. For this reason, many countries begin to support the research of geothermal energy and hope to obtain the better low-cost power generation.

Nowadays, the deep geothermal power generation technology common with enhanced geothermal system and closed-loop heat collection system (CEEG). The principle of EGS is to inject the fluid with high pressure for generating the multiple artificial fractures under heat reservoir. Then, the working fluid flows through the reservoir to absorb the heat and flows out to ground through production well. Water is first applied on the EGS as the working fluid for its high latent heat, specific enthalpy and cheap cost. However, a number of disadvantages have to overcome, the main ones are viscosity coefficient and density. These will bring the pipeline fouling.

\( \text{CO}_2 \)-EGS is first proposed by Brown in 2000 [1], which leads the research of EGS proliferated recently. It brings many benefits, such as low-temperature and high-density of carbon dioxide injection to production occurred phenomena of thermal siphon. It results in the lower pumping power.

Fouillac et al. [2], Ueda et al. [3] and Wan et al. [4], have also proposed \( \text{CO}_2 \)-EGS for generating electricity, further and even more importantly. It will result in the \( \text{CO}_2 \) storage and prevent the pipeline fouling. There is a lot of previous research to study the possibility and potential of \( \text{CO}_2 \), such as Pruess et al. [5] and Zhang et al. [6]. In addition, Zhang et al. [6] compare carbon dioxide and water to evaluate the efficiency. Li et al. assess the renewable energy in the world [7], that will confirm the importance of geothermal energy. Xu et al. conduct a series of studies on the interaction of fluid and rock [8], and Jiang et al. discuss the influence of buoyancy of supercritical carbon dioxide in a porous media tube [9], and Liao et al. find the buoyancy effect significant influence flow direction [10]. Even a lot of researches attend the \( \text{CO}_2 \)-EGS but few experiment of reservoir based on the optimal heat extraction. The purpose of this study proposes an integrated method which combined experiment, optimal scheme to obtain the maximum heat extraction of \( \text{CO}_2 \)-EGS in the reservoir.

2 Experimental Section

The schematic diagram of the experimental system is shown in figure 1. The experimental system consisted of a \( \text{CO}_2 \) cylinder, high pressure pump, preheated water bath, cooling water bath, heating unit, test section, computer, and data logger. The test section is composed of stainless tube, and ten calibrated T-type thermocouples and pressure transducers are inserted into test section to record experimental data. The test section had inside and outside diameters of 29.9 and 55.0mm, and 235.0mm of length. The test section is filled with silica-base particles. To compare effects of particles, the empty of test section is also examined.

The particle mean mass diameters 1.54 and 2.03mm, respectively. The porosity of porous tube was approximately fixed at 0.2 and 0.39. To simulate reservoir...
temperature, the test section wall is heated as 200°C. The heated power is about 53W.

The system pressure is set as 7.5, 8.5, 9.5, 10.5, 11.5 and 12.5MPa; the flow rate is set as 20, 40, 60, and 80 ml/min. The overall uncertainty of the heat transfer coefficient of the supercritical CO2 is evaluated using the standard approach of Kline and McClintock [11]. The analysis shows that the measured uncertainties of temperature, flow rate, pressure, extraction power and heat transfer coefficient are estimated to be less than ±0.1 °C, ±0.5 %, ±2.3 %, ±2.37 % and ±17.6%, respectively.

Figure 1. The schematic diagram of experimental system.

3 Simulation

In this model, a 3D free and porous media flow model combined with heat transfer model is established by finite element method – COMSOL multiphysics package. The material of tube is stainless steel. The properties of supercritical carbon dioxide are interpolated function based on the National Institute of Standards and Technology (NIST) standard reference database 69. The schematic diagram of this model is shown in figure 2.

![Figure 2. The schematic diagram of test tube.](image)

The non-porous medium in the Navier-Stokes equation is described as below:

**Continuity equation is:**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]  

(1)

**Momentum equation is:**

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla : \mathbf{I})] + \mathbf{F} \]  

(2)

where \( \mu \) is viscosity; \( \mathbf{u} \) is velocity; \( \rho \) is fluid density; \( p \) is pressure and \( \mathbf{F} \) is forced term.

The porous medium of the Brinkman equation is described as below:

**Continuity equation is:**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = Q_{br} \]  

(3)

**Momentum equation is:**

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot \left[ \frac{1}{\varepsilon} \left( \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \mu (\nabla : \mathbf{I}) \right) \right] - \left( \frac{\mu}{\varepsilon} + \frac{Q_{br}}{\varepsilon^3} \right) \mathbf{u} + \mathbf{F} \]  

(4)

where \( \varepsilon \) is porosity; \( \kappa \) is permeability and \( Q_{br} \) is mass force.

The heat transfer of porous medium of the Fourier's law is described as below:

**Energy balance equation is:**

\[ \frac{\partial \rho}{\partial t} \left( 1 - \varepsilon \right) \rho C_p \left( \rho \mathbf{u} \right) = \nabla \cdot (k_p \nabla T_p) \]  

(5)

**Energy balance equation is:**

\[ \frac{\partial \rho}{\partial t} \left( 1 - \varepsilon \right) C_p \ität( \rho \mathbf{u} \right) = \nabla \cdot (k \nabla T) - \varepsilon \nabla \cdot (k \nabla T) \]  

(6)

where \( T \) is temperature; \( C_p \) is specific heat; \( k \) is thermal conductivity and \( D \) is Darcy flow; subscript \( f \) means fluid and \( p \) is media.

Figure 3 shows the comparison between the heat extraction of experiment and simulation. Through figure 3, the simulation is validated by the experiment. The error of simulation is limited within 2.3%. Therefore, we will process the optimization under this validated model.

![Figure 3. The comparison between experiment and simulation (porosity is 0.2).](image)

The Nelder-Mead non-gradient method is used as the optimal method [11].

here, the constraints of design variables (pressure and flow rates) are shown as:
7.5 ≤ P ≤ 12.5 ; 20 ≤ U ≤ 80 \tag{7}

where \( P \) = pressure (MPa); \( U \) = flow rate (ml/min).

4 Results and Discussion

This study processes the optimization of heat extraction under the varied pressure and flow rate. Based on the validated model, three kinds of test tube is optimized.

Case 1: empty tube, porosity is 0.
Case 2: particle size 1.54mm, porosity is 0.2.
Case 3: particle size 2.03mm, porosity is 0.39.

First, we can find the best heat extraction is 26.86W occurred at 9.5MPa and 20ml/min as shown figure 4. After optimization, we can see more clearly the changes around. The optimal results show that the result is happened at 9.4MPa and 28.007ml/min, the corresponding heat extraction is 27.966W as shown figure 5.

Figure 4. The initial simulation results of case 1.

Figure 5. The optimal results of case 1.

In the porous medium as case 2 and case 3, the similar results are obtained. As shown in figures 6 and 7, the optimal results of case 2 is obtained. We can find the best heat extraction is 28.166W, occurred at 9.4MPa and 27.418ml/min.

In addition, the optimal heat extraction of case 3 is 28.168W, occurred at 9.4MPa and 27.409ml/min. The results are shown in figures 8 and 9.

Figure 6. The initial simulation results of case 2.

Figure 7. The optimal results of case 2.

Figure 8. The initial simulation results of case 3.
Figure 9. The optimal results of case 3.

The detailed results are listed in Table 1.

Table 1. Optimal results of the objective function

<table>
<thead>
<tr>
<th>Case</th>
<th>q(W)</th>
<th>P(MPa)</th>
<th>U(ml/min)</th>
</tr>
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<tbody>
<tr>
<td>Case 1-Empty</td>
<td>Initial</td>
<td>26.860</td>
<td>9.5</td>
</tr>
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<td></td>
<td>optimum</td>
<td>27.966</td>
<td>9.4</td>
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<tr>
<td>Case 2-1.54mm</td>
<td>Initial</td>
<td>27.138</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>optimum</td>
<td>28.166</td>
<td>9.4</td>
</tr>
<tr>
<td>Case 3-2.03mm</td>
<td>Initial</td>
<td>27.142</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>optimum</td>
<td>28.168</td>
<td>9.4</td>
</tr>
</tbody>
</table>

5 Conclusions

This study is to find the optimal operating condition of the heat extraction of supercritical CO₂ in the geothermal reservoir. The results show that the optimal heat extraction can be obtained from the optimal process. Through the optimization, it shows the suitable flow rate is necessary to obtain the highest capability of heat extraction. It can be observed that the best heat extraction is close 9.4MPa. It results from the characteristics of supercritical CO₂. In addition, while as can be seen from the result, the influence of porosity on the heat extraction is minor. This study can improving the heat extraction and reduces the cost of actual test of enhanced geothermal system.

Acknowledgments

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References and Note

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