Estimation Study of the Residual Stress Redistribution in Crack Propagation Process Based on Hull - Rimmer Theory

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Abstract. The redistribution of residual stress appears with the propagation of internal crack or damage, which also has an influence on the fatigue life of structure. A prediction model of residual stress redistribution is raised based on the Hull - Rimmer theory. And the quantitative expression between crack length, initiation crack location, initial distribution of residual stress and redistribution stress is achieved. Then the residual stress of crack tip and the redistribution stress of three instances are predicted, which also compared with the test results. It can be found from the comparison that the prediction results are basically same with the test results. Therefore, the prediction model of residual stress redistribution based on Hull-Rimmer theory can be well used for the precise prediction of residual stress redistribution in the crack propagation process.

Keywords: residual stress; redistribution; crack propagation; Hull - Rimmer theory

1 Introduction

As the common failure form of equipment [1], crack is becoming the research focus of industry. There are many researches focus on the crack propagation [2-3], while most of which ignores the influence of residual stress. The residual stress of crack tip and the distribution of residual stress always change with the crack propagation, and thus affect the fatigue life of structure. Fukuda [4] developed the analytic model to predict the residual stress in the crack propagation process. Tetada [5] established the equation of redistribution stress according to the test data. These two methods can be used for the prediction of stress redistribution. However, these methods are presented on the basis of empirical formula of specific material sample, and the precision of which is difficult to be guaranteed. Therefore, it is needed to establish a prediction model of residual stress redistribution according to the actual circumstance of crack propagation.

In this paper, a prediction model of residual stress redistribution is raised on the base of Hull - Rimmer theory. Then the residual stress of crack tip and the residual stress redistribution of three instances are predicted according to the prediction model, which also compared with the test results.

2 Prediction Model of Residual Stress Redistribution Based on Hull – Rimmer Theory

The theory of Hull - Rimmer describes the diffusion process from cavity to empty position and then grows up. The motion of diffusion process can be briefly described as 3 steps [6].

Step1 At the early stage of cavity growth, cavity nucleates at the interface. The changes of cavity number at this stage mostly due to the nucleation process.

Step2 At the middle stage of cavity growth, cavities significantly merge together. The changes of cavity number at this stage mostly due to the merge of cavity.

Step3 At the final stage of cavity growth, the space between big cavities significantly decrease and then extension together as the crack.

The chemical potential of vacancy source ($r = a/2$) and cavity surface ($r = a$) is:

$$(\mu_v)_{r=a/2} = -(\sigma_n - p)\Omega_A \tag{1}$$

$$(\mu_V)_{r=a} = -\frac{2\gamma_s \Omega_A}{R} \tag{2}$$

Where, μ_v is the chemical potential; R is the radius of cavity; a is the cavity space; σ_n is the stress; γ_s is the surface energy; p is the isostatic pressure; Ω_A is the volume of cavity.

The average gradient of chemical potential from vacancy source to cavity is:

$$\nabla\mu_v = -\frac{2\Omega_A}{a}(\sigma_n - p - \frac{2\gamma_s}{R}) \tag{3}$$

According to the first law of diffusion [7], the diffusion energy of vacancy can be expressed as:

$$J_V = \frac{2D_B}{k\lambda a}(\sigma_n - p - \frac{2\gamma_s}{R}) \tag{4}$$

Where, D_B is the diffusion coefficient; k is the boltzmann constant; λ is the spacing coefficient.

The diffusion area from vacancy into cavity can be expressed as:

$$A_{JV} = 2\pi R\delta_B \tag{5}$$

Where, δ_B is the thickness of grain boundary elements.

Then, the growing velocity of cavity volume dv/dt can be expressed as:

$$\frac{dv}{dt} = J_V \cdot A_{JV} \cdot \Omega_A = J_V(2\pi R\delta_B)\Omega_A = \frac{4\pi R(D_B\delta_B)\Omega_A}{k\lambda a}\sigma^* \tag{6}$$

Where, $\sigma^* = \sigma_n - p - \frac{2\gamma_s}{R}$.

Therefore, the propagation velocity of cavity in the direction of crack extension is:

$$V_i(\xi) = C\frac{D_B\delta_B\Omega_A}{k\lambda aR}\sigma_i^*(\xi) \tag{7}$$

Where, C is the constant; i is loading time; ξ is the distance between selected point and crack root.

At the same time, the stress of points on the crack propagation direction can be expressed as:

$$\sigma_i^*(\xi) = \frac{\sigma_{\max}[(n+1)R+\xi(i)]}{\sqrt{2(n+1)R\xi(i)+\xi^2(i)}} \tag{8}$$

Where, n is the number of cavity, which has the same radius R, $n = P\sigma_a^m N$; P is the nucleation factor of cavity; N is the loading time.

So the micro displacement can be obtained by substitute Eq. (8) into Eq. (7):

$$\varepsilon(\xi)=C\cdot\frac{D_B\delta_B\Omega_A\sigma_{\max}}{k\lambda aR}\cdot\int_{N_1}^{N_2}\frac{[(P\sigma_a^m N+1)R+\xi(i)]}{\sqrt{2(P\sigma_a^m N+1)R\xi(i)+\xi^2(i)}}dt \tag{9}$$

The redistribution of residual stress can be expressed as:

$$\sigma_{res}(x) = \sigma_{reso}(x) + \varepsilon(x)\cdot E \tag{10}$$

Where, $\sigma_{res}(x)$ is the redistribution of residual stresses; $\sigma_{reso}(x)$ is the distribution of residual stress before crack propagation; $\varepsilon(x)$ is the micro displacement caused by the crack propagation; E is the elasticity modulus.

Therefore, the redistribution result of residual stress can be expressed as:

$$\sigma_{res}(\xi)=\sigma_{reso}(\xi)+C\cdot\frac{D_B\delta_B\Omega_A\sigma_{\max}}{k\lambda aR}\cdot\int_{N_1}^{N_2}\frac{[(P\sigma_a^m N+1)R+\xi(i)]}{\sqrt{2(P\sigma_a^m N+1)R\xi(i)+\xi^2(i)}}dt \tag{11}$$

3 Instance Analysis and Verification

In order to validate the feasibility and accuracy of prediction model further, the following three instances as shown in Fig.1 are selected as the instance for analysis. It is shown in Fig.1 that the initial crack of instance 1 and instance 2 locate at the position 50 mm distance from the weld. The propagation direction is perpendicular to the weld,and the propagation direction of instance 1 is right and the propagation direction of instance 2 is left. Also the initial crack of instance 3 locates at the edge of plate. The initial crack length is 10mm. The length of plate perpendicular to the weld is 400mm, the length of weld is 300mm. The material of instance is Q345b.

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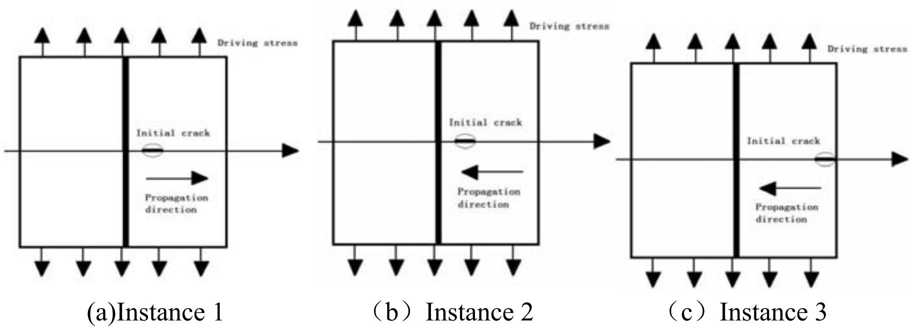


Fig.1 The instance of analysis

The initial distribution of residual stress is needed to be achieved before the calculation of residual stress redistribution. As shown in Fig. 2, the automatic submerged arc welding is used in the manufacturing process of instance first, and then the measurement position of instance is proper electrolytic polishing processed. Further, the X-ray residual stress measurement equipment is adopted, and the average initial distribution of residual stress as shown in Fig.3 can be obtained after the measuring.

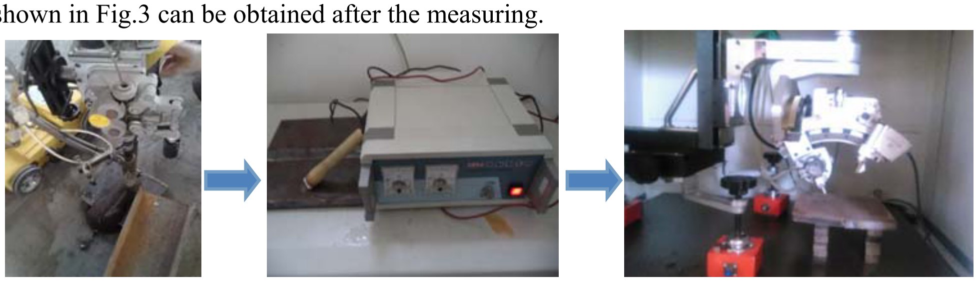


Fig. 2 The measurement of residual stress

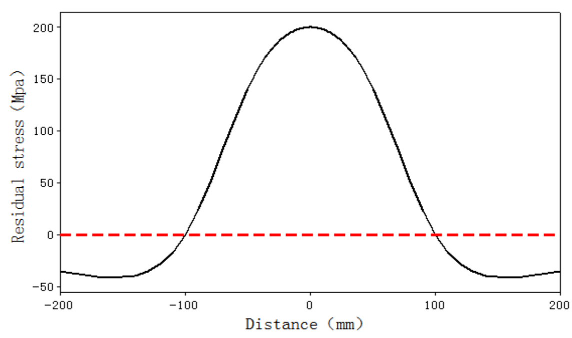
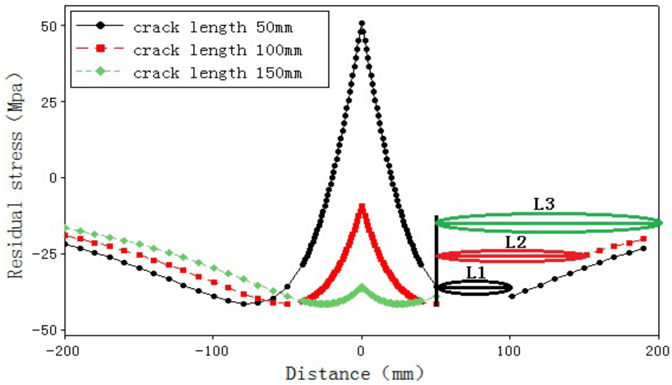
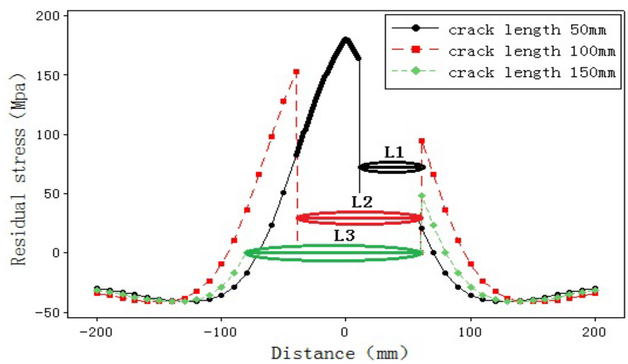


Fig. 3 The average initial residual stress distribution of instance

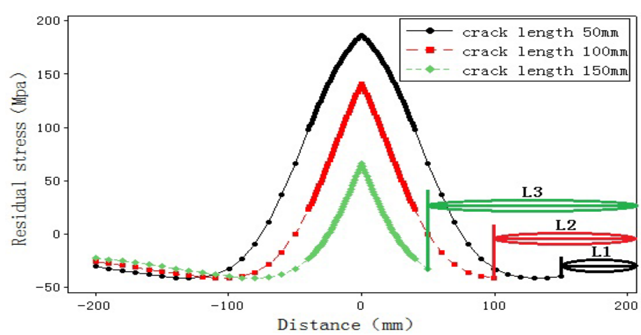
The prediction model based on Hull - Rimmer theory is used to analyze the instance after the crack propagation, contain the crack 50 mm, 100 mm and 150 mm in length. Then the results of residual stress redistribution as shown in Fig.4 can be obtained. It can be clearly found in Fig.4 that the peak of residual stress obviously decreases with the propagation of crack, and the distribution of residual stress shrink inside obviously with the propagation of crack.



(a) Instance 1



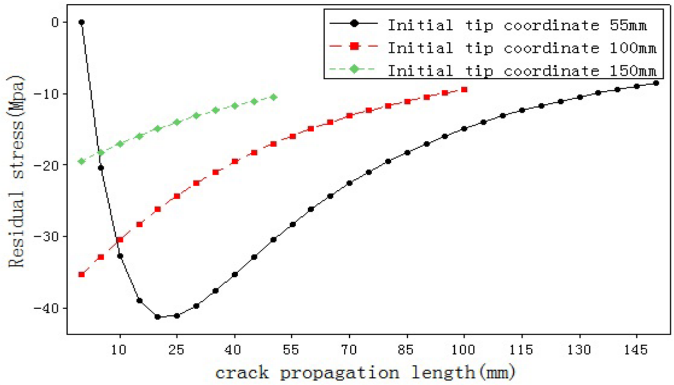
(b) Instance 2



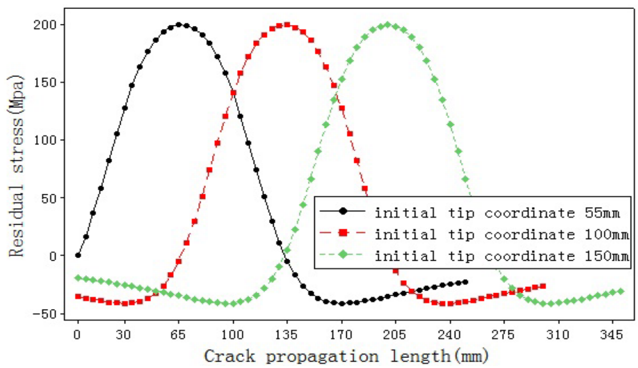
(c) Instance 3

Fig.4 The results of residual stress redistribution of instance

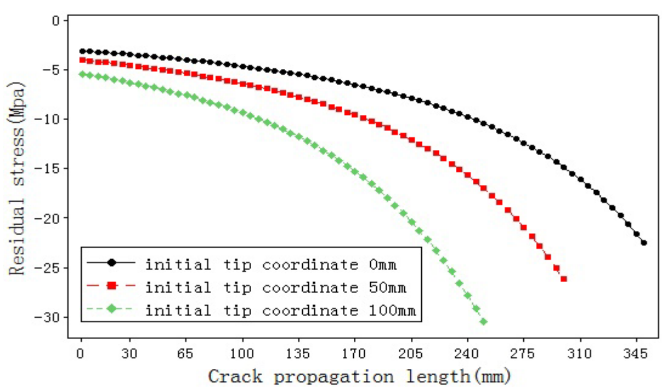
The residual stress of crack tip also changes accordingly with the propagation of crack and initial tip coordinate. The changing results of residual stress of crack tip can be obtained as shown in Fig.5. It can be clearly found in Fig.5 that the residual stress of crack tip is mainly related to the initial distribution of residual stress field and the initial position of crack.



(a) Instance 1



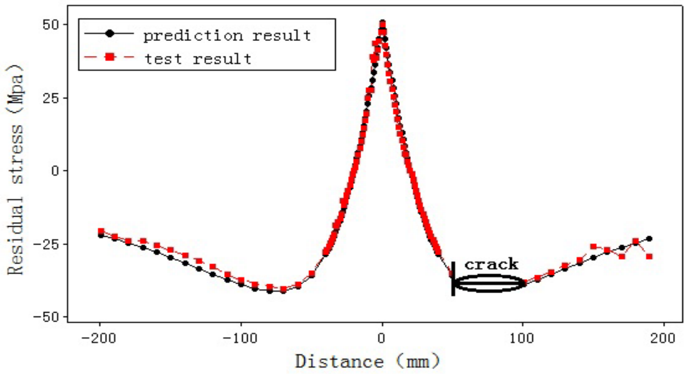
(b) Instance 2



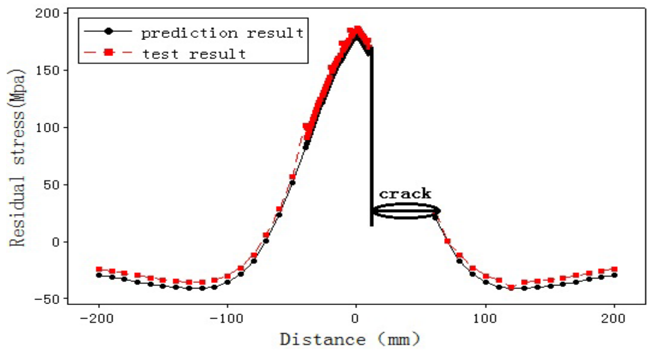
(c) Instance 3

Fig.5 The changing results of residual stress of crack tip

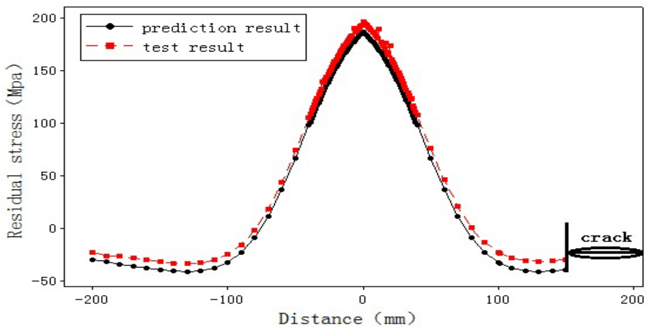
The X-ray residual stress measurement device is used to have a test, which contains the crack 50 mm in length caused by the tensile fatigue test. Then the comparison between prediction results and test results can be obtained as shown in Fig.6. It can be clearly found in Fig.6 that the results of prediction model based on Hull - Rimmer theory are basically same with the test results.



(a) Instance 1



(b) Instance 2



(c) Instance 3

Fig.6 The comparison between prediction result and test result

4 Conclusions

(1)A prediction model of residual stress redistribution is raised based on the theory of Hull - Rimmer. And then the quantitative expression between crack length, initiation location of crack, initial residual stress distribution and the redistribution stress is achieved.

(2)The residual stress of crack tip and the redistribution stress of three instances are predicted according to the prediction model, which also compared with the test results. It can be clearly found from the comparison that the results of prediction model based on Hull

- Rimmer theory are basically same with the test results.

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