Dynamic Model and Analysis of Asymmetric Telescopic Wing for Morphing Aircraft

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Abstract. Morphing aircraft has been the research hot topics of new concept aircrafts in aerospace engineering. Telescopic wing is an important morphing technology for morphing aircraft. This paper describes the dynamic equations and kinematic equations based on theorem of momentum and theorem of moment of momentum, which are available for all morphing aircrafts. Meanwhile,as simplified , dynamic equations for rectangular telescopic wing are presented. In order to avoid the complexity using aileron to generate rolling moment , an new idea that asymmetry of wings can generate roll moment is introduced. Finally, roll performance comparison of asymmetric wing and aileron deflection shows that asymmetric telescopic wing can provide the required roll control moment as aileron, and in some cases, telescopic wing has the superior roll performance.

1 Introduction

Morphing aircraft has been investigated extensively because of its superior flight performance[1, 2]. Morphing aircraft can adapt various complicated flight regimes.The Defense Advanced Research Projects Agency (DARPA) developed Morphing Aircraft Program in 1998. At the beginning of 21 century, NASA showed its morphing aicraft project. Ordinarily morphing aircraft can be classified three types:micro morphing、small morphing and large morphing. Micro morphing includes variable camber[3, 4] and twist[5], small morphing contains variable tip and variable horizontal tail, while variable sweep[6, 7],variable span[8] and folding wing belong to large morphing. This paper mainly concentrates on telescopic wing which is one of large morphing techniques.

Telescopic Wing is intended to change its wing span for various flight conditions to reduce drag. As aresult of increasing the wing span, the aspect ratio and wing area increase and the spanwise lift distribution decreases for the same lift[8].Telescopic wing has a long research history, many patents of invention have been declared[9], as shown in Fig.1, which is three patents about telescopic wing.

In this work, the dynamic and kinematic model are described to investigative performance of morphing aircraft. But, because the dynamic model involves some variable parameters with time, which leads to complexity of morphing aircraft. In order to obtain some basic characteristics, some hypotheses are put forward. Telescopic wing aircraft can change wing span and area

during flight to realize better aerodynamic and control performance. Although much work has been done, technique of telescopic wing is not well developed.Some fundamental reseach work of telescopic wing was done focusing on dynamic model and analysis in this paper .
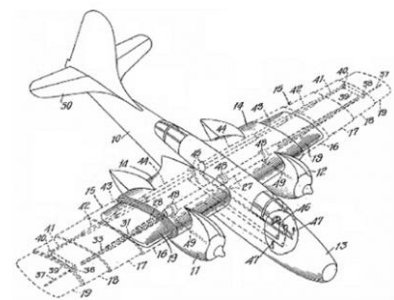
(a) Patent of variable erea by Gibson
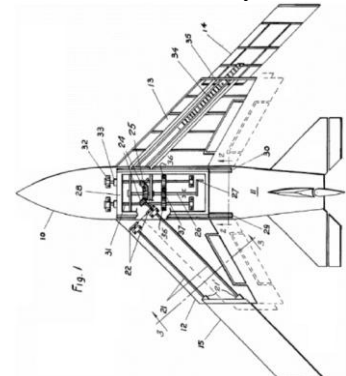
(b) Patent of telescopic wing by Kppenkin

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[a] Chen Lili: doublely@foxmail.com
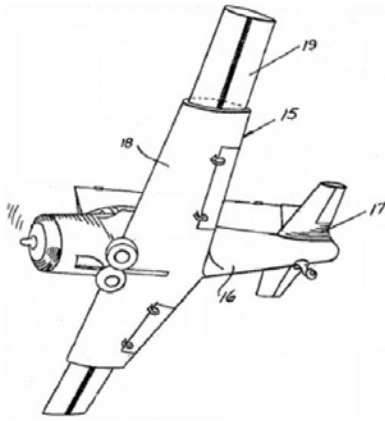
(c) Patent of variable erea by Murray

Figure.1 Patents of telescopic wings

2 Morphing aircraft model

2.1 Dynamic model

In this section the dynamic model of morphing aircraft is presented based on theorem of momentum and theorem of moment of momentum and also refers the symmetric wing dynamic model of ideal rigid-body, which is related to telescopic wing by Jin Dongping[10]. On the assumption that origin is located on symmetry position of fuselage, aircraft axes (or named as body axes)O-XYZ(as shown in Fig. 2) can be defined.In addition, inertial reference axes is defined as O_A-$X_A Y_A Z_A$ (as shown in Fig. 2) attached to ground (earch whirling ignored). The velocity and angular rate of morphing aircraft are V_A and ω_A relative to inertial reference axes,respectively. Usually,V_A and ω_A are projected on aircraft axes , marked as $V = (V_x, V_y, V_z), \omega = (\omega_x, \omega_y, \omega_z)$.

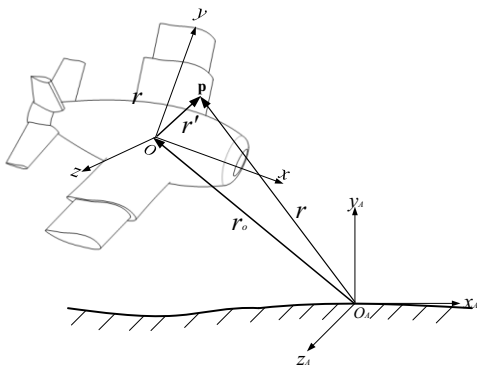**Figure 2** Chematic diagram of dynamic model

For any ***dm*** of body, its position vetor is ***r_i*** to inertial axes, the integral result can be expressed as:

$$p = \int_\Omega \overset{\square}{r_i} dm = \int_\Omega (\overset{\square}{r_o} + \overset{\square}{r'}) dm \quad (1)$$

According to theorem of moment of mass, the latter part can be noted as:

$$S = \int_\Omega r' dm \quad (2)$$

Derivative expression is

$$\overset{\square}{S} = \overset{\circ}{S_o} + \omega \times S_o \quad (3)$$

Where $\overset{\square}{S}$ and $\overset{\circ}{S_o}$ are the derivative of instantaneous mass moment in inertial reference axes and aircraft axes, respectively. "∘"represents the derivative to time in the aircraft axes.

Duo to $\overset{\square}{r_o} = V$,thus

$$p = mV + \overset{\circ}{S_o} + \omega \times S_o \quad (4)$$

Where m is mass of aicraft (including fuselage and wings ,also other parts),the moment of momentum relative to origin of aircraft axes can be expressed:

$$L_o = \int_\Omega r_i' \times v_i dm = \int_\Omega r_i' \times (V + \overset{\square}{r_i'}) dm$$
$$= S_o \times V + \int_\Omega r_i' \times \overset{\square}{r_i'} dm \quad (5)$$

Because aircaft axes has the rotation rate ω itself ,so

$$L_o = S_o \times V + \int_\Omega r_i' \times (\overset{\circ}{r_i'} + \omega \times r_i') dm \quad (6)$$

In the light of definition,

$$\int_\Omega r_i' \times (\omega \times r_i') dm = \int_\Omega [\omega(r_i' | r_i') - r_i'(\omega | r_i')] dm = I \cdot \omega \quad (7)$$

Where I is the aircraft instantaneous inertia tensor relative to origin O,the expression is as follows:

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \quad (8)$$

So $\quad L_o = S_o \times V + I \cdot \omega + \int_\Omega (r_i' \times \overset{\circ}{r_i'}) dm$

$$\quad (9)$$

Theorem of momentum： $\quad \overset{\square}{p} = F \quad (10)$

To make it simple,the relative theorem of moment of momentum of any point can be demonstrated in the translation axes as in Fig.3:
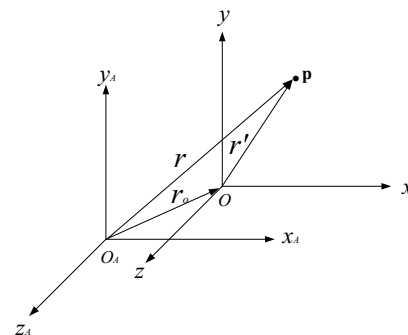

Figure 3 Inertial axes and translation axes

$$L_{Oi} = r_i' \times m_i v_i \quad (11)$$

Eq.(11) can be derivated ,applying Newton second law to the result：

$$\frac{dL_{Oi}}{dt} = \frac{dr_i'}{dt} \times m_i v_i + r_i' \times m_i \frac{dv_i}{dt} = \frac{dr_i'}{dt} \times m_i v_i + r_i' \times F_i$$

$$\quad (12)$$

Due to

$$\frac{dr_i'}{dt} = \frac{dr_i}{dt} - \frac{dr_o}{dt} = v_i - v_o \qquad (13)$$

$$r_i' \times F_i = M_{oi} \qquad (14)$$

Substituting Eq.(13) and Eq.(14) into Eq.(12) and integral yields the theorem of moment of momentum of whole aircraft

$$\frac{dL_o}{dt} = \int_\Omega v_i dm \times V + M_o = p \times V + M_o \qquad (15)$$

Substituting Eq.(4) and Eq.(9) into Eq.(15) and considering vetor cross product relationship:

$$\omega \times (S_o \times V) + V \times (\omega \times S_o) = S_o \times (\omega \times V) \quad (16)$$

Taking an inertial derivative of the vector with respect to time in whirling reference frame[11], morphing aircraft flight dynamic model afer simplification under body axes yields

$$\begin{cases} m(\overset{\square}{V} + \omega \times V) + \overset{\square\square}{S_o} + \overset{\square}{\omega} \times S_o + 2\omega \times \overset{\square}{S_o} \\ + \omega \times (\omega \times S_o) = F \\ \\ I \cdot \overset{\square}{\omega} + \overset{\square}{I} \cdot \omega + \omega \times (I \cdot \omega) + \overset{\square}{S_o} \times V + S_o \times (\omega \times V) \\ + \int_\Omega \{r_i' \times \overset{\square\square}{r_i'} + \omega \times (r_i' \times \overset{\square}{r_i'})\} dm = M_o \end{cases}$$

$$(17)$$

Where F is the force imposed on the aircraft under body axes, M_o is the force moment relative to the origin of body axes caused by external force. Eq.(17) shows that motion parameters of telescopic wing aircraft are tightly related with the time variable of moving wing,for instance, S_o、S_{oi}、I are functions of time. To rigid aircraft,wings maintain the constant relative position,then the equations reduce as:

$$\begin{cases} m(\overset{\square}{V} + \omega \times V) + \overset{\square}{\omega} \times S_o + \omega \times (\omega \times S_o) = F \\ I \cdot \overset{\square}{\omega} + \omega \times (I \cdot \omega) + \overset{\square}{S_o} \times V + S_o \times (\omega \times V) = M_o \end{cases}$$

$$(18)$$

If ,furthermore, center of mass is selected as the origin of body axes, Eq.(18) can be simplified as：

$$\begin{cases} m(\overset{\square}{V} + \omega \times V) = F \\ I \cdot \overset{\square}{\omega} + \omega \times (I \cdot \omega) = M_o \end{cases} \qquad (19)$$

This equation can be used as center of mass motion equation and around center of mass motion equation in noninertial system.

Generally, moving wings have a small part mass of the whole aircraft and that product of inertia is little relatively.So, it is proper that I_{xy}，I_{yz}，I_{zx} are equal to 0, scalar quantity format of dynamic equations in the body axes can obtain in the following：

$$\begin{cases} m(\overset{\square}{V}_x + \omega_y \cdot V_z - \omega_z \cdot V_y) + \overset{\square\square}{S}_{ox} + (\overset{\square}{\omega}_y \cdot S_{Oz} - \overset{\square}{\omega}_z \cdot S_{Oy}) \\ + 2(\omega_y \cdot \overset{\square}{S}_{Oz} - \omega_z \cdot \overset{\square}{S}_{Oy}) + \omega_y \cdot (S_{oy}\omega_x - S_{ox}\omega_y) \\ + \omega_z \cdot (S_{Oz}\omega_x - S_{ox}\omega_z) = F_x \\ \\ m(\overset{\square}{V}_y + \omega_z \cdot V_x - \omega_x \cdot V_z) + \overset{\square\square}{S}_{oy} + (\overset{\square}{\omega}_z \cdot S_{ox} - \overset{\square}{\omega}_x \cdot S_{oz}) \\ + 2(\omega_z \cdot \overset{\square}{S}_{ox} - \omega_x \cdot \overset{\square}{S}_{oz}) + \omega_z \cdot (S_{Oz}\omega_y - S_{oy}\omega_z) \\ - \omega_x \cdot (S_{oy}\omega_x - S_{ox}\omega_y) = F_y \\ \\ m(\overset{\square}{V}_z + \omega_x \cdot V_y - \omega_y \cdot V_x) + \overset{\square\square}{S}_{oz} + (\overset{\square}{\omega}_x \cdot S_{Oy} - \overset{\square}{\omega}_y \cdot S_{Ox}) \\ + 2(\omega_x \cdot \overset{\square}{S}_{Oy} - \omega_y \cdot \overset{\square}{S}_{Ox}) - \omega_x \cdot (S_{Oz}\omega_x - S_{ox}\omega_z) \\ - \omega_y \cdot (S_{Oz}\omega_y - S_{oy}\omega_z) = F_z \end{cases} \quad (20)$$

$$\begin{cases} I_x \cdot \overset{\square}{\omega}_x + \overset{\square}{I}_x \cdot \omega_x + (\omega_y(I_z \cdot \omega_z) - \omega_z(I_y \cdot \omega_y)) + (S_{Oy} \overset{\square}{V}_z \\ - S_{Oz} \overset{\square}{V}_y) - S_{Oy}(V_x \cdot \omega_y - V_y \cdot \omega_x) - S_{Oz}(V_x \cdot \omega_z - V_z \cdot \omega_x) \\ + \int_\Omega \{r_{iy}' \cdot \overset{\square\square}{r}_{iz}' - r_{iz}' \cdot \overset{\square\square}{r}_{iy}' + \omega_y(r_{iy}' \cdot r_{ix}' - r_{ix}' \cdot r_{iy}') \\ + \omega_z(r_{iz}' \cdot r_{ix}' - r_{ix}' \cdot r_{iz}')\} dm = M_{ox} \\ \\ I_y \cdot \overset{\square}{\omega}_y + \overset{\square}{I}_y \cdot \omega_y + (\omega_z(I_x \cdot \omega_x) - \omega_x(I_z \cdot \omega_z)) + (S_{Oz} \overset{\square}{V}_x \\ - S_{Ox} \overset{\square}{V}_z) + S_{Ox}(V_x \cdot \omega_y - V_y \cdot \omega_x) - S_{Oz}(V_y \cdot \omega_z - V_z \cdot \omega_y) \\ + \int_\Omega \{r_{iz}' \cdot \overset{\square\square}{r}_{ix}' - r_{ix}' \cdot \overset{\square\square}{r}_{iz}' + \omega_z(r_{iz}' \cdot r_{iy}' - r_{iy}' \cdot r_{iz}') \\ - \omega_x(r_{iy}' \cdot r_{ix}' - r_{ix}' \cdot r_{iy}')\} dm = M_{oy} \\ \\ I_z \cdot \overset{\square}{\omega}_z + \overset{\square}{I}_z \cdot \omega_z + (\omega_x(I_y \cdot \omega_y) - \omega_y(I_x \cdot \omega_x)) + (S_{Ox} \overset{\square}{V}_y \\ - S_{Oy} \overset{\square}{V}_x) + S_{Ox}(V_x \cdot \omega_z - V_z \cdot \omega_x) + S_{Oy}(V_y \cdot \omega_z - V_z \cdot \omega_y) \\ + \int_\Omega \{r_{ix}' \cdot \overset{\square\square}{r}_{iy}' - r_{iy}' \cdot \overset{\square\square}{r}_{ix}' - \omega_x(r_{iz}' \cdot r_{ix}' - r_{ix}' \cdot r_{iz}') \\ - \omega_y(r_{iz}' \cdot r_{iy}' - r_{iy}' \cdot r_{iz}')\} dm = M_{oz} \end{cases}$$

$$(21)$$

Obviously,the dynamic equation series is very complex and has 6-DOFs that couple with each other.It is difficult to solve. Whereas, the equation is available for all morphing aircrafts and can be applied to kinds of ridid or flexible morphing aircraft ,in the previous assumption.

2.2 Kinematic model

In order to obtain the movement trajectory of morphing aircraft in inertial axes,it is essential to bulid the relatinship of inertial axes and body axes.As usual,the relationship of the two axes is denoted by three Euler angles.Their definitions[12] (Fig.4) as follws:

Angle of pitch φ :the angle between the longitudinal axis of a flight vehicle ox and the horizontal surface $x_A o z_A$.

Angle of yaw ψ :the angle between the axis of ox_A and the longitudinal axis ox' projection on the horizontal surface $x_A o z_A$.

Angle of roll γ :the angle between the axis of oy

and vertical surface including the longitudinal axis ox of the fight vehicle.

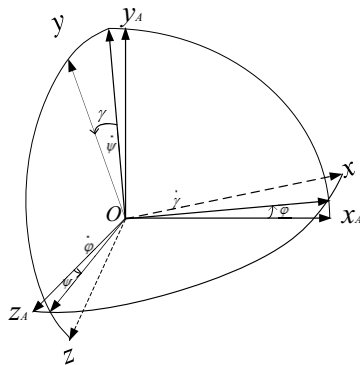Coordinate transformations[12] between inertial axes and body axes are shown as

Figure 4 Relationship of Euler angles

$$B_A = \begin{bmatrix} \cos\varphi\cos\psi \\ \cos\varphi\sin\psi\sin\gamma - \sin\varphi\cos\gamma \\ \cos\varphi\sin\psi\cos\gamma + \sin\varphi\sin\gamma \end{bmatrix}$$

$$\begin{matrix} \sin\varphi\cos\psi & -\sin\psi \\ \sin\varphi\sin\psi\sin\gamma + \cos\varphi\cos\gamma & \cos\psi\sin\gamma \\ \sin\varphi\sin\psi\cos\gamma - \cos\varphi\sin\gamma & \cos\psi\cos\gamma \end{matrix}$$

Then Kinematic equation in the inertial axes can be given by

$$\begin{bmatrix} V_{Ax}, V_{Ay}, V_{Az} \end{bmatrix}^{\mathrm{T}} = A_B \begin{bmatrix} V_x, V_y, V_z \end{bmatrix}^{\mathrm{T}}$$

Where $A_B = B_A^T$, the corresopnding scalar format is

$$\begin{cases} \dfrac{dx_A}{dt} = \cos\varphi\cos\psi V_x + (\cos\varphi\sin\psi\sin\gamma - \sin\varphi\cos\gamma)V_y \\ \quad + (\cos\varphi\sin\psi\cos\gamma + \sin\varphi\sin\gamma)V_z \\ \dfrac{dy_A}{dt} = \sin\varphi\cos\psi V_x + (\sin\varphi\sin\psi\sin\gamma + \cos\varphi\cos\gamma)V_y \\ \quad + (\sin\varphi\sin\psi\cos\gamma - \cos\varphi\sin\gamma)V_z \\ \dfrac{dz_A}{dt} = -\sin\psi V_x + (\cos\psi\sin\gamma)V_y + (\cos\psi\cos\gamma)V_z \end{cases}$$

(22)

The attitude of aircraft is described by three Euler angles φ、ψ and γ. The anglar equations of motion can be obtained throught finding the transformation relation of Euler angles[13].First, φ is formed by the angular rate $\dot\varphi$ around z_A axis, ψ is from the angular rate of new axis y', which is projected on body axes as $[0, \dot\psi\cos\gamma, -\dot\psi\sin\gamma]$, γ is the rotation angle around axis x in $\dot\gamma$.Thus

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = B_A \begin{bmatrix} 0 \\ 0 \\ \dot\varphi \end{bmatrix} + \begin{bmatrix} 0 \\ \dot\psi\cos\gamma \\ -\dot\psi\sin\gamma \end{bmatrix} + \begin{bmatrix} \dot\gamma \\ 0 \\ 0 \end{bmatrix} \quad (23)$$

The corresopnding scalar format is

$$\begin{cases} \omega_x = -\dot\varphi\sin\psi + \dot\gamma \\ \omega_y = \dot\varphi\cos\psi\sin\gamma + \dot\psi\cos\gamma \\ \omega_z = \dot\varphi\cos\psi\cos\gamma - \dot\psi\sin\gamma \end{cases} \quad (24)$$

Describing Eq.(24) in another form

$$\begin{cases} \dfrac{d\varphi}{dt} = \dfrac{1}{\cos\psi}(\omega_y\sin\gamma + \omega_z\cos\gamma) \\ \dfrac{d\psi}{dt} = \omega_y\cos\gamma - \omega_z\sin\gamma \\ \dfrac{d\gamma}{dt} = \omega_x + \tan\psi(\omega_y\sin\gamma + \omega_z\cos\gamma) \end{cases} \quad (25)$$

Eq.(25) is the angular equations of motion.

3 Analysis for an example

To make analysis simple, some hypothesis is necessary to define ahead,for example:

a) The aircraft is composed of three components: fuselage、left moving wing and right moving wing.

b) All wings have no aileron.

c) Fuselage(including fixed wing part) maintains relative fixed position.

d) All wings have no dihedral.

e) Stretching rate of telescopic wing is some contant value.

f) Mass of left and right wing is m_1 and m_2, supposing that $m_1=m_2$.

Based on the above hypothesis,a relative telescopic wing dynamic model would be established.While the wings are in the retractile status,there is one point O locating on symmetry surface of fuselage,where

$$S_{ox} = \int_\Omega r'_{ix}dm = 0, S_{oy} = \int_\Omega r'_{iy}dm = 0$$

Ω represents the integral of whole aicraft.As is shown in Fig.5,stretching rates of left and right wing are v_1, v_2, respectively. Sweep angle is χ.At the moment, only $S_{oy} = \int_\Omega r'_{iy}dm = 0$.

Then,

$$S_{ox} = m_1(d_2 - d_1)\sin(-\chi), S_{oz} = m_1(d_2 - d_1)\cos\chi \quad (26)$$
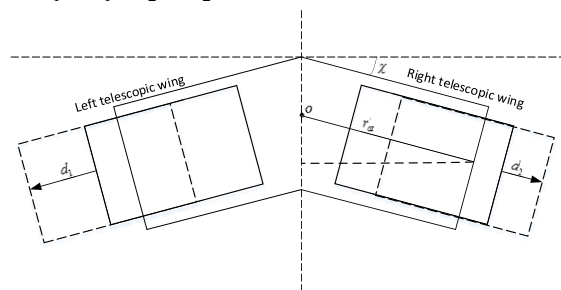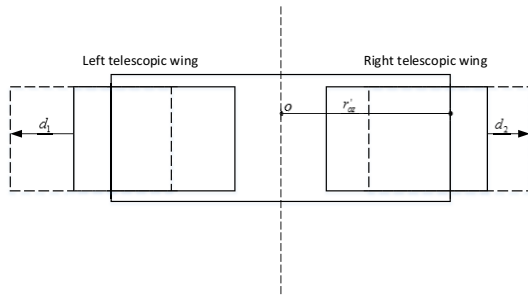
Where $d_1 = v_1 t, d_2 = v_2 t$.

Figure 5 Sweep telescopic wing

Figure 6 Rectangular telescopic wing

Futhrmore ,accoring to the feature of rectangular telescopic wing , Eq.(26) can be expressed as

$$S_{ox} = 0, S_{oz} = m_1(d_2 - d_1) = m_1(v_2 - v_1)t \qquad (27)$$

Dynamic and kinematic equations are simplified as

$$
\begin{cases}
m(\dot{V}_x + \omega_y \cdot V_z - \omega_z \cdot V_y) + \dot{\omega}_y \cdot S_{oz} + 2\omega_y \cdot \dot{S}_{oz} \\
+ \omega_x \omega_z S_{oz} = F_x \\
m(\dot{V}_y + \omega_z \cdot V_x - \omega_x \cdot V_z) + \dot{\omega}_x \cdot S_{oz} + 2\omega_x \cdot \dot{S}_{oz} \\
+ \omega_y \omega_z S_{oz} = F_y \\
m(\dot{V}_z + \omega_x \cdot V_y - \omega_y \cdot V_x) - \omega_x^2 S_{oz} - \omega_y^2 S_{oz} \\
= F_z
\end{cases}
\qquad (28)
$$

$$
\begin{cases}
I_x \cdot \dot{\omega}_x + \dot{I}_x \cdot \omega_x + (\omega_y(I_z \cdot \omega_z) - \omega_z(I_y \cdot \omega_y)) \\
+ S_{oz} \dot{V}_y - S_{oz}(V_x \cdot \omega_z - V_z \cdot \omega_x) = M_{ox} \\
I_y \cdot \dot{\omega}_y + \dot{I}_y \cdot \omega_y + (\omega_z(I_x \cdot \omega_x) - \omega_x(I_z \cdot \omega_z)) \\
+ S_{oz} \dot{V}_x - S_{oz}(V_y \cdot \omega_z - V_z \cdot \omega_y) = M_{oy} \\
I_z \cdot \dot{\omega}_z + \dot{I}_z \cdot \omega_z + (\omega_x(I_y \cdot \omega_y) - \omega_y(I_x \cdot \omega_x)) \\
= M_{oz}
\end{cases}
\qquad (29)
$$

4 Estimation of aerodynamic force

The theoretical value of thin airfoil lift line slope is 2π, the value of thick wing increases with relative thickness of airfoil increasing.Experiment value is about $0.9 \times 2\pi$, the correction value can approximately be expressed as

$$C_l^\alpha = 1.8\pi(1 + 0.8c) \qquad (30)$$

Where c is thickness of airfoil .Because of tip effect, the slope of wing lift line is lower than airfoil ,

$$C_L^\alpha = \frac{C_l^\alpha}{1 + C_l^\alpha / (\pi\lambda e)} \qquad (31)$$

Where C_L^α, C_l^α is the slope of wing and airfoil lift line, respectively, λ is aspect ratio, e is Oswald dfficiency factor, which is a function of aspect ratio and sweep angle. For rectangular wing,

$$e = 1.78(1 - 0.045\lambda^{0.68}) - 0.64 \qquad (32)$$

Drag coefficient of wing can be estimated that[14]

$$C_D = C_{D0} + C_{Di} \qquad (33)$$

$$C_{D0} = C_f(S_W / S), C_{Di} = KC_L^2 \qquad (34)$$

Where C_f is equivalent skin friction coefficient, S_w is wetted area of wing ,S is reference area of wing,K is damping factor by lift. At low velocity, equivalent skin friction coefficient of wing in turbulent flow can be approximately caculated as

$$C_f = \frac{0.455}{\left(\lg \text{Re}\right)^{2.58}} \qquad (35)$$

Usually , $$K = \frac{1}{\pi\lambda e} \qquad (36)$$

For some specific rectangular telescopic wing, lift and drag force can be obtained from above formulae , then it is necessary to project them on inertial axes, the following is transformation matrix.

$$
B_V = \begin{bmatrix}
\cos\alpha\cos\beta & \sin\alpha & -\cos\alpha\sin\beta \\
-\sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\
\sin\beta & 0 & \cos\beta
\end{bmatrix}
$$

Where α (defined as angle of attack) is the angle between the axis ox and velocity' projection on $x_A oz_A$ surface, β (defined as angle of sideslip) is the angle between velocity vetor and $x_A oz_A$ surface.

5 Analysis of roll performance

5.1 Rolling moment coefficient generated by aileron is caculated

Increment of rolling moment coefficient by aileron deflection

$$\mathrm{D}m_x = \frac{\mathrm{D}M_x}{qSb} = \frac{C_l qcydy}{qSb} = \frac{C_l cydy}{Sb} \qquad (37)$$

Where b is maximum wing span, S is maximum wing area, section lift coefficient of left wing can be given as

$$C_l = C_l^a \frac{da}{dd_a} d_a = C_l^a t_a d_a \qquad (38)$$

τ_a is aileron dfficiency factor, which represents equivalent angles of attack of wing section with aileron when aileron deflects1 degree, and can be estimated by $\tau_a = (c_a/c)^{0.5}$.

Where c_a is chord of aileron.For rectangular wing, chord of wing c is constant, consdering a couple of ailerons , through integral of Δm_x,

$$m_x = \frac{C_L^a c t_a}{Sb}(y_2^2 - y_1^2)d_a \qquad (39)$$

Where C_L^a and t_a have been corrected by considering effect of practical flow, y_1 and y_2 is the starting and ending position of aileron along span .

5.2 Rolling moment coefficient by asymmetric wing can be caculated

Rolling moment coefficient：

$$m_x = \frac{C_L^a a\, S_a\, \mathbf{D}L}{2Sb} \tag{40}$$

Where ΔL is difference value of left and right wing.Sa is the actual wing area. Fixed wing is 2m, moving wing span is 1m, chord is 1m.If aileron is installed in the 25% of chord and its length takes up the whole moving wing span. NACA LS 0417 is selected as airfoil, then C_l^a =2.0448π, S=6m^2, b=6m, τ_a=0.5.The results of roll moment coefficient are shown in Fig.7 and Fig.8.
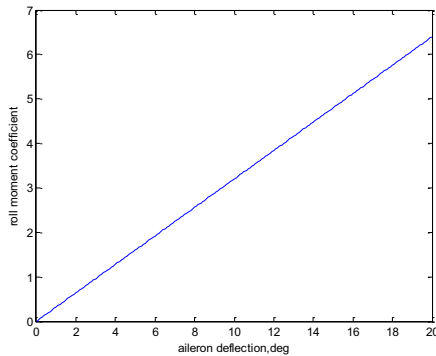

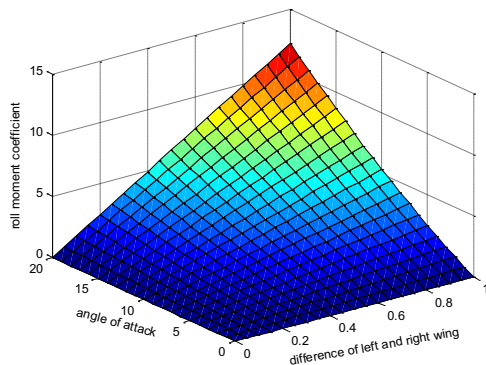**Figure 7** Rolling moment coefficient by aileron

Figure 8 Rolling moment coefficient by telescopic wing

It can be seen from Fig.7 and Fig.8 that rolling moment coefficient generated by aileron is linear change , while that by elescopic wing is non-linear. When rolling moment coefficient of aircraft in a large angle of attack is easier controlled than in a small angle of attack.Meanwhile, rolling moment coefficient increases with ΔL increasing.

6 Conclusions

In this paper, the dynamic equations and kinematic equations of asymmetric telescopic wing aircraft are developed.From the equations, it is very complex and difficult to solve such equation series , because several parameters change with time.Hower, the model is versatile for any telescopic morphing aircraft, in some simple assumptions. This paper also bulid the model of rectangular telescopic wing with no sweep and dihedral.Some estimation formulae are applied to caculate the aerodynamic force . Results show that at low velocity

high aspect ratio wing has better aerodynamic performance than low aspect ratio,while at high velocity low aspect ratio wing has superior performance. So, telescopic wing can enhance overall fight performance for a large velocity range.

The roll perfoamance also is evaluated by asymmetric wing span, aircraft can generate roll moment by change the stretching length of left and right wing. Results shows that roll moment coefficient varys evidently with difference value of telescopic wings' length and angle of attack. It is easy to controll the roll rate in this way. Moreover, roll performance of asymmetric wing is non-linear change. In the case of large angle of attack and Δ*L*, telescopic wings behave better roll performance.

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