Function of One Regular Separable Relation Set Decided for the Minimal Covering in Multiple Valued Logic

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ABSTRACT: Multiple-valued logic is an important branch of the computer science and technology. Multiple-valued logic studies the theory, multiple-valued circuit & multiple-valued system, and the applications of multiple-valued logic included.In the theory of multiple-valued logic, one primary and important problem is the completeness of function sets, which can be solved depending on the decision for all the precomplete sets(also called maximal closed sets)of K-valued function sets noted by P_K^*, and another is the decision for Sheffer function, which can be totally solved by picking out all of the minimal covering of the precomplete sets.In the function structure theory of multi-logic, decision on Sheffer function is an important role. It contains structure and decision of full multi-logic and partial multi-logic. Its decision is closely related to decision of completeness of function which can be done by deciding the minimal covering of full multi-logic and partial-logic. By theory of completeness of partial multi-logic, we prove that function of one regular separable relation is not minimal covering of P_K^* under the condition of $m = 2, \sigma = e$.

KEYWORD: Regular Separable Relation; Sheffer Function; Minimal Covering; Maximal Closed Set;

1 INTRODUCTION

Let $E_K = \{0,1,2,\Lambda, K-1\}, K \geq 2$ .Function $f(x_1, x_2, \Lambda, x_n,)$ is from E_K to E_K. Function $f(x_1, x_2, \Lambda, x_n,)$ is called complete K-valued function when if for all of $(\alpha_1, \alpha_2, \Lambda, \alpha_n)$ $f(x_1, x_2, \Lambda, x_n,)$ is a function. Otherwise $f(x_1, x_2, \Lambda, x_n,)$ is called non-complete K-valued function. All of the complete and non-complete K-valued function is called partial K-valued function. The set of all of the complete K-valued function is signed as P_K. The set of all of the non-complete K-valued function is signed as P_K^* [1-6].

Let $A \subseteq P_K^*, f(t_1, t_2, \Lambda, t_m) \in A, g_i(x_1, x_2, \Lambda, x_n) \in A$ or $g_i(x_1, x_2, \Lambda, x_n)$ is one of x_1, x_2, Λ, x_n ,here $i = 1,2,\Lambda, m$. $f(g_1(x_1, \Lambda, x_n), \Lambda, g_m(x_1, \Lambda, x_n))$ is called one function which is composite from the function of A. The set of all of functions which is composite from the function of A, signed as A'. A is called one closed set if $A = A'$.

Let $A \subseteq P_K^*$, $GEN(A) = A \cup A' \cup A'' \cup \Lambda \cup A^{(i)} \cup \Lambda$, if $GEN(A) = P_K^*$, A is called a complete set.

For non-complete function $f(x_1, x_2, \Lambda, x_n,)$, if $f(\alpha_1, \alpha_2, \Lambda, \alpha_n)$ is not belong to E_K , we let $f(\alpha_1, \alpha_2, \Lambda, \alpha_n) = *$.

The sequence which includes m numbers of E_K called one m- sequence. m- sequence is signed as

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$<\alpha_1,\alpha_2,\Lambda\ ,\alpha_n>$. The set of m- sequence is signed as G_m.

$G_m=\overline{G}_m\cup\overline{G}_m^{\sigma_1}\cup\overline{G}_m^{\sigma_2}\cup\Lambda\ \cup\overline{G}_m^{\sigma_h}$ is called regular separable if it have these conditions below: existing one division of E_K:

$$D_i:E_K=A_1^i+A_2^i+\Lambda\ +A_m^i,i=1,2,\Lambda\ ,n.$$

For all of the $<\alpha_1,\alpha_2,\Lambda\ ,\alpha_m>\in\overline{G}_m$ there exists one division

$$D_i(1\le h\le n):\alpha_i\in A_i^{(h)},i=1,2,\Lambda\ ,m,$$

and also for all of $D_j,(1\le j\le h-1)$ there exiting $A_{j_1}^{(j)},\Lambda\ ,A_{j_n}^{(j)},(1\le j_1,\Lambda\ ,j_n\le m)$ have the condition of $\alpha_1,\alpha_2,\Lambda\ ,\alpha_{r_1}\in A_{j_1}^{(j)},\Lambda\ ,\alpha_{r_{n-1}+1},\alpha_{r_{n-1}+2},\Lambda\ ,\alpha_{r_n}\in A_{n_1}^{(j)}$. The set of all of regular separable function is called regular separable function set which is signed $S_{R,m}$.

One function $f(x_1,x_2,\Lambda\ ,x_n)\in P_K^*$ is called Sheffer function if $GEN\{f(x_1,x_2,\Lambda\ ,x_n)\}=P_K^*$. That is to say, $f(x_1,x_2,\Lambda\ ,x_n)$ can produce all functions of P_K^*.

Let

$\sum=\{A_i\mid A_i\subset P_K^*\text{ and }A_i\text{ is maximal closed set}\},\sum'\subseteq\sum$

$S_{\sum'}=\{f\in P_K^*\mid\text{if there exists one }A'_j,f\in A'_j\}$.

$\sum'(\subseteq\sum)$ is called the covering of $\sum$, if $S_{\sum'}=S_{\sum}$. For one set A(here A is maximal closed set), If there exists one function $f\in A$ and f is not belong to any maximal closed set except A, A is one of member of the minimal closed set.

Function $f(x_1,x_2,\Lambda\ ,x_n)\in P_K^*$ is called protecting of G, if for any function

$g_i(x)\in G,i=1,2,\Lambda\ ,n$,

$f(g_1(x),g_2(x),\Lambda\ ,g_n(x))\in P_K^*$.

All of $f(x_1,x_2,\Lambda\ ,x_n)\in P_K^*$ which is called protecting of G is signed as $T(G)$.

The relation of regular separable

$$G_2=G_2(\{1,2\})\cup\overline{G}_2$$

can be described by relation of $<a,b>$. It can be represented as one graph which has K vertices[1-6].

2 MAIN CONCLUSION

Theorem Under the condition of $\sigma=e$, if the basic graph of regular separable set

$$G_2=G_2(\{1,2\})\cup\overline{G}_2$$

is only one loop, then it is not the minimal closed set of P_K^*.

Proof Because of the Characteristics of loop, without loss of generality, we let vertices $0,1,2,\Lambda\ ,k-1$ is in turn connected to each other. We will proof this loop can't be one strongly connected graph.

Let

$<0,1>,<1,2>,\Lambda\ ,<k-2,k-1>,<k-1,0>\in G_2$,

According to the definition of regular separable set, let division $D_i:E_K=A_1^{(i)}+A_2^{(i)},i=1,2,\Lambda\ ,n$, for any $<a_1,a_2>\in\overline{G}_2$, there existing one division $D_h(1\le h\le n):a_i\in A_i^{(h)},i=1,2$ and also for any $D_j(1\le j\le h-1)$, there exists

$$A_{j_j}^{(j)},1\le j_1\le 2$$

which satisfies $a_1,a_2\in A_i^{(h)}$.

Let $D_i:E_K=A_1^{(i)}+A_2^{(i)},i=1,2,\Lambda\ ,n$ satisfy the definition of regular separable set. For the sequence of binary regular separable relation $<0,1>\in G_2$, without loss of generality, let exist one division D_1 which satisfies $0\in A_1^{(1)},1\in A_2^{(2)}$.

Next we consider $<1,2>\in G_2$, it is clear that D_1 can't satisfy $1\in A_1^{(1)}$.

In the same way, we consider $<2,3>\in G_2$, it is clear that D_2 can't satisfy $1\in A_1^{(1)}$.

Now we consider $<k-1,0>\in G_2$, it is clear that D_{k-1} can't satisfy $k-1\in A_1^{(1)}$, So there also exists one division D_k which satisfies

$$k-1\in A_1^{(1)},0\in A_2^{(2)}.$$

But according to the definition of division, this gets contradiction: $0\in A_1^{(1)}\text{and }0\in A_2^{(2)}$.

So we prove that this loop is not one strongly connected graph.

The following we will prove this theorem.

Because the basic graph of this relation graph is only one loop, this basic graph is connected. For any $f\in T(G_2)$, without loss of generality, let $f(0,0,\Lambda\ ,0)=i,0\le i\le K-1$. There must exist one path L from 0 to *i*, let $|L|$ be the length of L. we have possibilities as below.

1. if $|L|=0,f\in T(G_0)$,
2. if $|L|=1$, that is $f(0,0,\Lambda\ ,0)=1$, that is to say $<0,1>\in G_2$. Next we will consider

$$<1,2>\in G_2,or<2,1>\in G_2.$$

Because the basic graph is only one loop, we have possibilities as below.

(1) $f(1,1,\Lambda\ ,1)=1$,

(2) $f(1,1,\Lambda\ ,1)=*$,

(3) $f(1,1,\Lambda,1)=2$,

For (1)(2),we can get $f\in T_{\{1\}}$, and $<2,1>\in G_2$. We get $f\in T(G_2)$ isn't the member of minimal closed set.

For (3), we can get $<1,2>\in G_2$. Next we consider $<2,3>\in G_2, or <3,2>\in G_2$. We also have possibilities as below.

(a) $f(2,2,\Lambda,2)=2$,

(b) $f(2,2,\Lambda,2)=*$,

(c) $f(2,2,\Lambda,2)=3$,

For (a)(b),we can get $f\in T_{\{2\}}$, and $<3,2>\in G_2$. We get $f\in T(G_2)$ isn't the member of minimal closed set.

For (c), we can get $<2,3>\in G_2$. Next we consider $<3,4>\in G_2, or <4,3>\in G_2$. We can do like above. We also have possibilities as below.

(e) $f(k-2,k-2,\Lambda,k-2)=k-2$,

(f) $f(k-2,k-2,\Lambda,k-2)=*$,

(g) $f(k-2,k-2,\Lambda,k-2)=k-1$,

For (e)(f),we can get $f\in T_{\{k-2\}}$, and $<k-1,k-2>\in G_2$. We get $f\in T(G_2)$ isn't the member of minimal closed set.

For (g), we can get $<k-2,k-1>\in G_2$. Because this basic graph is not strongly connected. We have $<0,k-1>\in G_2$. We have possibilities as below.

(h) $f(k-1,k-1,\Lambda,k-1)=k-1$,

(i) $f(k-1,k-1,\Lambda,k-1)=*$,

For (h)(i), we can get $f\in T_{\{k-1\}}$ .We get $f\in T(G_2)$ isn't the member of minimal closed set.

3. if $|L|>1$, without loss of generality, let $f(0,0,\Lambda,0)=i$, here $2\le i\le i-1$. Let $L_1=L$, we consider $f(1,1,\Lambda,1)$. We have possibilities as below.

(a_1) If $f(1,1,\Lambda,1)=i$, we consider the path L_2: from vertices 1 to vertices *i*. Here we get $|L_2|=|L_1|-1$,

(a_2) If $f(1,1,\Lambda,1)=i-1$, we consider the path L_2: from vertices 1 to vertices *i*-1. Here we get $|L_2|=|L_1|-2$,

(a_3) If $f(1,1,\Lambda,1)=i+1$, we consider the path L_2: from vertices 1 to vertices *i*-1. Here we get $|L_2|=|L_1|$,

(a_4) If $f(1,1,\Lambda,1)=*$, we have $f\in T_{\{1\}}$,

Now we consider path L_2 and we can get L_2 which have the same features like L_1. So we use the technology like above, we can get one of $f\in T_{\{1\}}$ or $f\in T_{\{2\}}$, or we can get L_3. If we do like above, we can have possibilities as below.

(b_1) $|L_{n+1}|=|L_n|-1$,

(b_2) $|L_{n+1}|=|L_n|-2$,

(b_3) This directional path is connected from 0 to $k-1$,

For (b_1) (b_2), because the path becomes smaller and smaller, $f\in T_E$.

For (b_3), because this loop can't be strongly connected graph, $<k-1,0>\notin G_2$, that is to say $<0,k-1>\in G_2$. And this path is one directional connected path from 0 to k-2. We can get $<k-2,k-1>\in G_2$. We can have possibilities as below.

(c_1) $f(k-1,k-1,\Lambda,k-1)=k-1$,

(c_2) $f(k-1,k-1,\Lambda,k-1)=*$,

For (c_1) (c_2) we can get the result $f\in T_E$.

According to the proof of above, we prove that if under the condition of $\sigma=e$, the basic graph of regular separable set $G_2=G_2(\{1,2\})\cup\overline{G}_2$ is only one loop, then it is not the minimal closed set of P_K^*.

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