Research on Optimization of Bearing Span of Main Reducer Gear System Based on the Transfer Matrix

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Abstract. The research object is the bearing span of the main reducer gear of the crossover vehicle for improving the performance of the Noise-Vibration-Harshness (NVH) on the rear axle. It could get the optimal bearing span by modal analysis of the driving gear shaft and the vibration test of the main reducer, based on the transfer matrix. It is verified that the vibration and transmission stability are greatly improved with this optimal bearing span by the dynamic simulations of different bearing spans in ADAMS. And it can provide reference for the structural optimization design of the main reducer and the research on the vibration and noise reduction.
Keywords. bearing span, transfer matrix, ADAMS, NVH

1 Introduction

The main reducer is one of the most important parts of the automobile transmission system, and its vibration and noise can directly affect the NVH performance of the car[1]. The bearing support which connects the gear shaft and the main reducer shell can affect the coupling vibration effect of the components in the main reducer. And its support stiffness can affect the meshing quality of the bevel gear pairs, thereby affecting the vibration response of the main reducer[2-3]. When the bearing preload is certain, bearing support span has a direct effect on support stiffness[4]. The bearing span is taken as the research object for achieving the purpose of improving the vibration response of the main reducer and the NVH performance of the crossover vehicle. The optimal bearing span under certain bearing preload is obtained by the modal analysis of driving gear shaft and the vibration test of main reducer, based on the transfer matrix. And the result is verified by the dynamic simulations in ADAMS.

2 The optimal bearing span based on the transfer matrix

Transfer matrix can describe the relationship between input and output of the linearized MIMO system (Multi Input Multi Output system). The input and the output are the end state vectors of each unit, including displacement, angular displacement, bending moment and shear stress. The basic idea of the transfer matrix method is to discretize the system into several feature units, and then the overall multi-body dynamics model is gotten by combining the each feature unit. The main reducer gear system is discretized into several feature units, and the transfer matrix of each unit is respectively established. Then the total transfer matrix of the system is obtained by combining the sub transfer matrices. The characteristic equation can be obtained with the free end as constraint conditions, then the system natural frequency under the different bearing span is gotten by calculating with relevant parameters[5-6]. The resonance frequency range which should be avoided can be determined by the modal analysis of the driving gear shaft and the vibration test of the main reducer, and the optimal bearing span is gotten.

2.1 System transfer matrix

The main reducer gear system is discretized in its section as shown in figure 1a. It contains 5 parts: driving gear, shaft-I, shaft-II, shaft-III and shaft-IV. The bearing is mounted on the shaft-II. The main reducer gear system is continued to be discretized into several feature units for establishing the transfer matrix model easily, as shown in figure 1b. Shaft-IV and shaft-III are set as a general shaft. The general shaft is composed of a non-mass elastic shaft and two centralized mass stations. Two bearing support are set as a non-mass elastic support station, and the acting point is in the center of the two support bearing. The shaft-II and bearings are simplified as a whole mass unit, and then it is divided into two parts: shaft-II-2 and shaft-II-1. The length of shaft-II-1 is l/2, and l is the bearing span. As the diameter of shaft-II-1 is larger than its length, shaft-II-2 and shaft-II-1 are set as the wheel stations. The shaft-I and the driving gear are integrally set as a shaft with an elastic support, and it can be equivalent to a non-mass elastic shaft and a rigid mass unit with an elastic support. The simplified mechanical models of the

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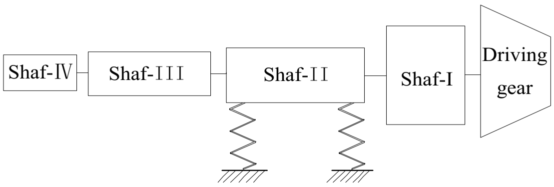
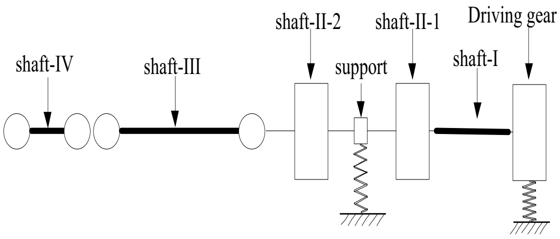
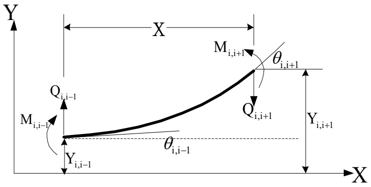
feature units are shown in figure 2. The transfer matrix of each sub-unit can be obtained by the dynamic equations.



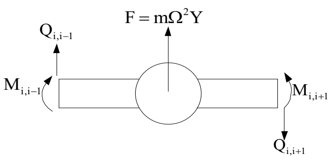
a. The initial discrete model



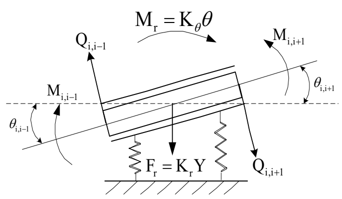
b. The final discrete model
Figure 1.The discrete model of the main reducer gear system



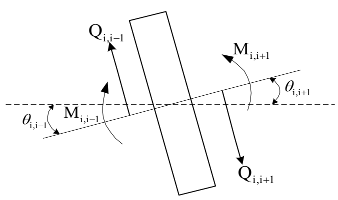
a. Non-mass elastic shaft



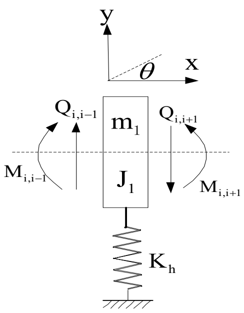
b. Centralized mass station



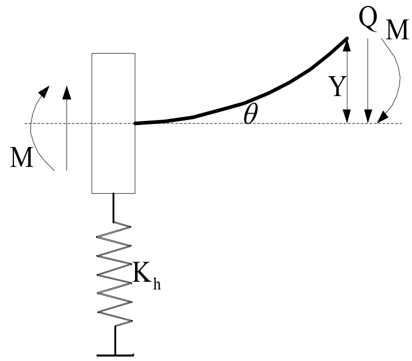
c. Non-mass elastic support station



d. Wheel station



e. Rigid mass unit with an elastic support



f. Shaft with an elastic support
Figure 2. The simplified mechanical models of feature units

2.1.1 General shaft

The general shaft is composed of a non-mass elastic shaft and two centralized mass stations. Figure 2a shows the definition of coordinate and the state vectors of the non-mass elastic shaft. The sections of shaft from $X_{i,i-1}$ to $X_{i,i+1}$ are called as i-1 section and i section. The end state vectors include: displacement (*Y*), angular displacement (*θ*), bending moment (*M*) and shear stress (*Q*). *I*is the section inertia, and *E* is the elastic modulus. The dynamic equations of the non-mass elastic shaft can be obtained by the mechanical model.

$$\begin{cases} Y_{i,i+1} = Y_{i,i-1} + x \cdot \theta_{i,i-1} + \frac{x^2}{2EI} M_{i,i-1} + \frac{x^3}{6EI} Q_{i,i-1} \\ \theta_{i,i+1} = \theta_{i,i-1} + \frac{x}{EI} M_{i,i-1} + \frac{x^2}{2EI} Q_{i,i-1} \\ M_{i,i+1} = M_{i,i-1} + l \cdot Q_{i,i-1} \\ Q_{i,i+1} = Q_{i,i-1} \end{cases} \quad (1)$$

Transforming the equation (1) into the matrix form, the coefficient matrix of the state vectors is the transfer matrix of the non-mass elastic shaft, denoted as T_{sh}.

$$T_{sh} = \begin{bmatrix} 1 & x & \frac{x^2}{2EI} & \frac{x^3}{6EI} \\ 0 & 1 & \frac{x}{EI} & \frac{x^2}{2EI} \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The dynamic equations of the centralized mass station can be obtained by the mechanical model shown in figure 2b.

$$\begin{cases} Y_{i,i+1} = Y_{i,i-1} \\ \theta_{i,i+1} = \theta_{i,i-1} \\ M_{i,i+1} = M_{i,i-1} \\ Q_{i,i+1} = Q_{i,i-1} + m\mathbf{\Omega}^2 Y_{i,i-1} \end{cases} \quad (3)$$

Transforming the equation (3) into the matrix form, the coefficient matrix of the state vectors is the transfer matrix of the centralized mass station, denoted as T_m. *T* is the transfer matrix of the general shaft.

$$T_m = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m\mathbf{\Omega}^2 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$T = T_m T_{sh} T_m = \begin{bmatrix} \frac{m\mathbf{\Omega}^2 x^3}{6EI}+1 & x & \frac{x^2}{2EI} & \frac{x^3}{6EI} \\ \frac{m\mathbf{\Omega}^2 x^2}{2EI} & 1 & \frac{x}{EI} & \frac{x^2}{2EI} \\ \frac{m\mathbf{\Omega}^2 x}{EI} & 0 & 1 & \frac{x}{EI} \\ m\mathbf{\Omega}^2 + m\mathbf{\Omega}^2\left(\frac{m\mathbf{\Omega}^2 x^3}{6EI}+1\right) & m\mathbf{\Omega}^2 x & \frac{m\mathbf{\Omega}^2 x^2}{2EI} & \frac{m\mathbf{\Omega}^2 x^3}{6EI}+1 \end{bmatrix} \quad (5)$$

The transfer matrix of the shaft-IV and the shaft-III can be gotten by putting their physical parameters into the equation (5). T_4 is the transfer matrix of the shaft-IV. T_3 is the transfer matrix of the shaft-III.

2.1.2 Non-mass elastic support station

The two bearing supports are simplified as an elastic supporting station, as shown in figure 2c. And it can be equated to a spring with radial stiffness and angular stiffness. K_r is the equivalent radial stiffness of two supporting bearings, K_θ is the equivalent angular stiffness, and l is the bearing span. The effect of rotational speed and load on the bearing stiffness can be not considered, thus the bearing stiffness is regarded as a constant. In the center of the two bearings, there will be a reaction force F_r and a reaction moment M_r.

$$\begin{cases} K_\theta = K_r \cdot l^2/4 \\ F_\mathrm{r} = K_r \cdot Y \\ M_\mathrm{r} = K_\theta \cdot \theta \end{cases} \quad (6)$$

The effect of damping of the rolling bearing will not bring large error to the calculation results, thus it can be ignored. The dynamic equations of the non-mass elastic support station can be obtained by the mechanical model shown in figure 2c.

$$\begin{cases} Y_{i,i+1} = Y_{i,i-1} \\ \theta_{i,i+1} = \theta_{i,i-1} \\ M_{i,i+1} = M_{i,i-1} + K_\theta \theta_{i,i-1} \\ Q_{i,i+1} = Q_{i,i-1} - K_r Y_{i,i-1} \end{cases} \quad (7)$$

Transforming the equation (7) into the matrix form, then its coefficient matrix of the state vectors is the transfer matrix of the non-mass elastic support station, denoted as T_b.

$$T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & K_\theta & 1 & 0 \\ -K_r & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{K_r l^2}{4} & 1 & 0 \\ -K_r & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

2.1.3Wheel station

As shown in figure 2d, the wheel station is of mass and moment of inertia, and there will be the moment when it rotates with the angular velocity $\mathbf{\Omega}$. m is the mass of the wheel station. J is the moment of inertia of the wheel station. The dynamic equations of the wheel station can be obtained by its mechanical model.

$$\begin{cases} Y_{i,i+1} = Y_{i,i-1} \\ \theta_{i,i+1} = \theta_{i,i-1} \\ M_{i,i+1} = M_{i,i-1} + J\mathbf{\Omega}^2 \theta_{i,i-1} \\ Q_{i,i+1} = Q_{i,i-1} + m\mathbf{\Omega}^2 Y_{i,i-1} \end{cases} \quad (9)$$

Transforming the equation (9) into the matrix form, then its coefficient matrix of the state vectors is the transfer matrix of the wheel station, and the transfer matrix of the shaft-II-2 and shaft-II-1 are $T_{2\text{-}2}$ and $T_{2\text{-}1}$.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & J\mathbf{\Omega}^2 & 1 & 0 \\ m\mathbf{\Omega}^2 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

$$T_{2-2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & J_{2-2}\mathbf{\Omega}^2 & 1 & 0 \\ m_{2-2}\mathbf{\Omega}^2 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$T_{2-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & J_{2-1}\mathbf{\Omega}^2 & 1 & 0 \\ m_{2-1}\mathbf{\Omega}^2 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

2.1.4Shaft with an elastic support

The difference value between the diameter of shaft-I and the diameter of driving gear is small. The shaft-I and the driving gear can be integrally set as a shaft with an elastic support. And it can be equated to a non-mass elastic shaft and a rigid mass unit with an elastic support, as shown in figure 2f. The mechanical model of a rigid mass unit with an elastic support is shown in figure 2e. K_h is the meshing stiffness of the driving gear, m_1 is the overall mass of the shaft-I and the driving gear, and J_1 is the overall moment of inertia of the shaft-I and the driving gear. When the shaft-I and the driving gear rotate with the angular velocity $\mathbf{\Omega}$, there will be an inertia force and an inertia moment: $m_1\mathbf{\Omega}^2 Y$ and $J_1\mathbf{\Omega}^2\theta$. The dynamic equations of the rigid mass unit with an elastic support can be obtained by the mechanical model.

$$\begin{cases} Y_{i,i+1} = Y_{i,i-1} \\ \theta_{i,i+1} = \theta_{i,i-1} \\ M_{i,i+1} = M_{i,i-1} + J_1\mathbf{\Omega}^2 \theta_{i,i-1} \\ Q_{i,i+1} = Q_{i,i-1} + m_1\mathbf{\Omega}^2 Y_{i,i-1} - K_h Y_{i,i-1} \end{cases} \quad (13)$$

Transforming the equation (13) into the matrix form, then its coefficient matrix of the state vectors is the transfer matrix of the rigid mass unit with an elastic support, denoted as T_k.

$$T_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & J_1\mathbf{\Omega}^2 & 1 & 0 \\ m_1\mathbf{\Omega}^2 - K_\mathrm{h} & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

The transfer matrix of the non-mass elastic shaft has been derived, denoted as T_{sh}. x_1 is the overall length of the shaft-I and the driving gear. $I_1$is the section inertia of the shaft-I and the driving gear. T_g is the transfer matrix of the shaft with an elastic support.

$$T_g = T_{sh}T_k = \begin{bmatrix} (m_1\boldsymbol{\Omega}^2 - K_h)\frac{x_1{}^3}{6EI_1}+1 & J_1\boldsymbol{\Omega}^2\frac{x_1{}^2}{2EI_1}+x_1 & \frac{x_1{}^2}{2EI_1} & \frac{x_1{}^3}{6EI_1} \\ (m_1\boldsymbol{\Omega}^2 - K_h)\frac{x_1{}^2}{2EI_1} & J_1\boldsymbol{\Omega}^2\frac{x_1}{EI_1}+1 & \frac{x_1}{EI_1} & \frac{x_1{}^2}{2EI_1} \\ (m_1\boldsymbol{\Omega}^2 - K_h)\frac{x_1}{EI_1} & J_1\boldsymbol{\Omega}^2 & 1 & x_1 \\ m_1\boldsymbol{\Omega}^2 - K_h & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

2.1.5Transfer matrix of the system

According to the discrete relationship, all feature sub units are connected in series. It could get the total transfer matrix of the main reducer gear system by putting each transfer matrix together. T_N is the transfer matrix of the system, and it is a 4x4 matrix.

$$T_N = T_4T_3T_{2-2}T_bT_{2-1}T_g = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \quad (16)$$

2.2 Calculation of natural frequency

According to the discrete model of the main reducer gear system, the initial cross section and the final section both are the free end constraints, thus M_I=0, Q_I=0, M_N=0 and Q_N=0. The relationship of the state vectors between the initial cross section and the final section of the main reducer gear system can be obtained.

$$\begin{bmatrix} Y \\ \theta \\ M \\ Q \end{bmatrix}_N = \begin{bmatrix} Y \\ \theta \\ 0 \\ 0 \end{bmatrix}_N = T_N \begin{bmatrix} Y \\ \theta \\ 0 \\ 0 \end{bmatrix}_1 \quad (17)$$

$$\begin{bmatrix} Y \\ \theta \\ 0 \\ 0 \end{bmatrix}_N = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \\ u_{41} & u_{42} \end{bmatrix} \begin{bmatrix} Y \\ \theta \end{bmatrix}_1 \quad (18)$$

Y_I and θ_I cannot both be 0 at the same time when the main reducer gear system is vibrating. Therefore, the equation (19) will exist based on the equation (18).

$$\Delta(\boldsymbol{\Omega}^2) = \begin{vmatrix} u_{31} & u_{32} \\ u_{41} & u_{42} \end{vmatrix} = 0 \quad (19)$$

In this main reducer gear system, K_r=1023.65N/um, K_h=2.0436N/um, and the material of the gear shaft is 20CrMnTi. The other related parameters are shown in table 1. The relationship between the critical speed and bearing span can be obtained after calculating by the equation (19) with related parameters. The curve of the natural frequencies and the bearing span can be gotten because of f=$\boldsymbol{\Omega}$/60, as shown in figure 3.

The calculation result shows that the first-order natural frequency of the main reducer gear system is mainly in the range of 2700~3500Hz. And the first-order natural frequency is under 3000Hz when the bearing span is in the range of 53~58mm.

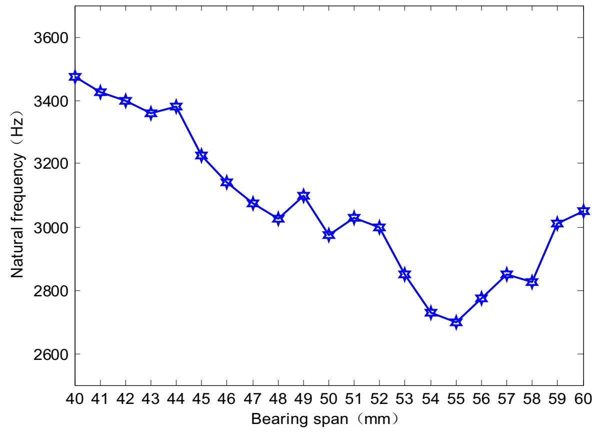


Figure 3. Natural frequencies - bearing span

2.3The optimal bearing span

The 3D model of the driving gear shaft was established in UG, based on the engineering drawing. The end faces of the bearings were constrained after the model imported into ANSYS. The modal analysis was performed by Block Lanczos method. The analysis result shows that the first-order natural frequency of the driving gear shaft is 3247.3Hz, and the two-order natural frequency is 3262.0Hz.

The natural frequency of the main reducer can be obtained by analyzing its vibration signal in the frequency domain [7]. The vibration test of the main reducer was completed by "the rear axle main reducer vibration performance testing system" which developed by the research group. The testing results of the three testing samples are shown in figure 4.

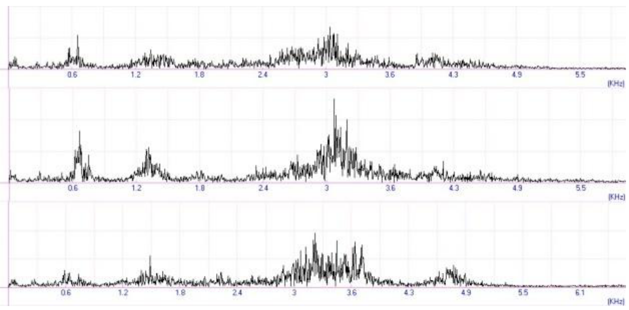


Figure 4. Vibration test results of the main reducers

The testing results show that the vibration of the main reducer is more obvious when the frequency in the range of 3000~3500Hz. That is the resonance frequency range of the main reducer. Thus the natural frequency in the range of 3000~3500Hz, especially around 3200Hz, should be avoided when designing the parts of the main reducer. And it could effectively reduce the vibration of the main reducer. It can be inferred that the better bearing span is in the range of 53~58mm by the curve shown in figure 3 which reflects the relationship between the natural frequencies and the bearing span. The first-order natural frequency is the most far away from 3000Hz when the bearing span is 55mm. Thus it can avoid resonance of the main reducer as far as possible when the bearing span is 55mm.

Table 1. The related parameters

	x $(10^{-3}$m)	$m(10^{-3}$kg)	$I(10^{-8}$m$^4)$	$J(10^{-6}$kg·m$^2)$
Shaft-IV	25	49.6	0.5153	
Shaft-III	69	264.2	1.9175	
Shaft-II-2	57-l/2	254.562-2.233l		23196.96-203.48l
Shaft-II-1	l/2	2.233l		203.48l
Shaft-I & Driving gear	65	358.4	3.9761	4.032

3Dynamic simulations in ADAMS

The physical parameters and the constraints and loads were defined based on the actual situation after the 3D model of the main reducer gear system imported into ADAMS. The dynamic model of the main reducer gear system is shown in figure 5.

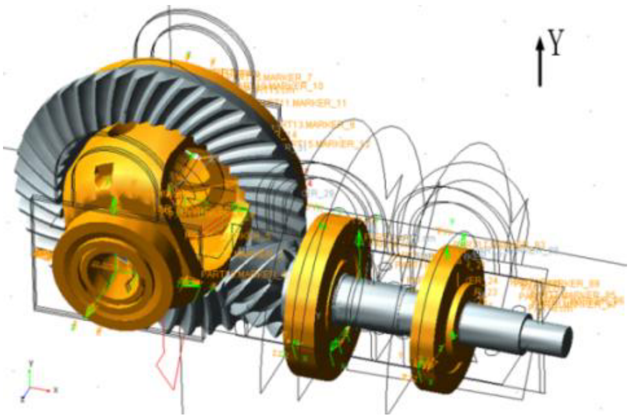
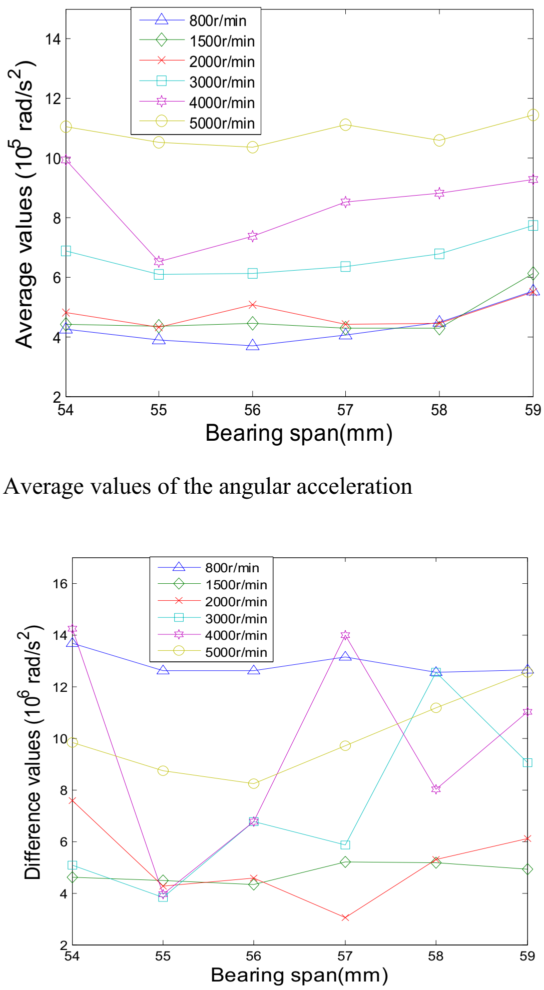


Figure 5. Dynamic model of the main reducer gear system

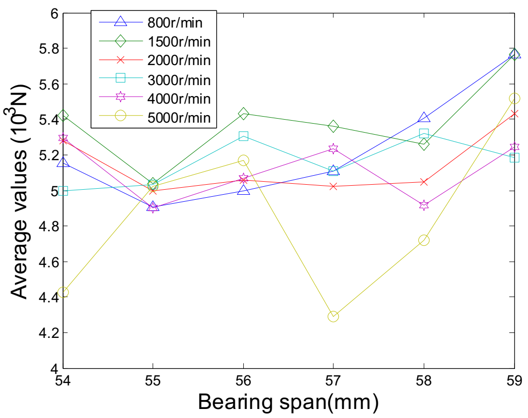
6 sets of bearing span models are prepared for the comparative simulation analysis, including 54mm, 55mm, 56mm, 57mm, 58mm and 59mm. At the same time, 6 sets of driving speed are set for the simulation, including 800r/min, 1500r/min, 2000r/min, 3000r/min, 4000r/min and 5000r/min. The angular acceleration and the contact force in the Y direction (vertical direction) of the driven gear are obtained by the simulation. The driven gear contact force in the Y direction can directly affect the vertical vibration when bevel gear pair meshing, thereby affecting the ride comfort of the car. The system is in the acceleration phase in the first 0.2 seconds. The simulation data of the acceleration phase should be excluded. The average values and difference values of the angular acceleration and the contact force in the Y direction at different driving speed are shown in figure 6 and figure 7.



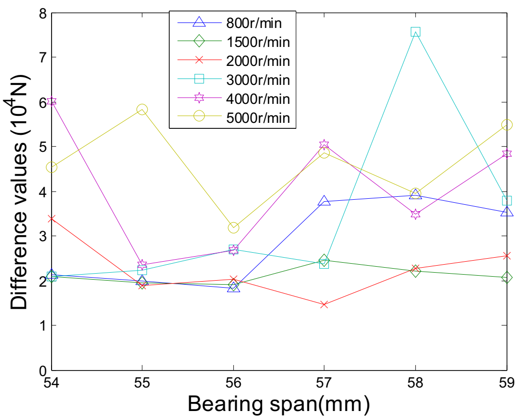
a. Average values of the angular acceleration

b. Difference values of the angular acceleration
Figure 6. Angular acceleration of the driven gear

The figure 6a shows that the average value of the angular acceleration of the driven gear closes to the minimum value when the bearing span is 55mm. The figure 6b shows that the difference values of the angular acceleration of the driven gear are fluctuating in different simulation driving speed, and the difference values is basically close to or in the minimum value when the bearing span is 55mm. From the above analysis, the vibration of the main reducer gear system is relatively stable when the bearing span is 55mm. The vibration and noise in the transmission process can be effectively reduced while the vibration fluctuation of the main reducer gear system is small.



a. Average values of the contact force (Y direction)



b. Difference values of the contact force (Y direction)
Figure 7. Contact force (Y direction) of the driven gear

The figure 7a shows that the average values of the contact force (Y direction) are relatively concentrated, and the average values are close to or in the minimum value when the bearing span is 55mm. The figure 7b shows that the difference values of the contact force (Y direction) are relatively small if the driving speed is not higher than 4000r/min when the bearing span is 55mm. It indicates that the impact of the transmission process is small, and the vibration is relatively stable.

Comprehensive analysis shows that the transmission of the main reducer gear system is relatively stable with small vibration fluctuation and the better vibration performance when the bearing span is 55mm.

4 Conclusions

The resonance of the main reducer can be avoided as far as possible when the bearing span is 55mm by the theoretical analysis and calculation, combined with the modal analysis of the driving gear shaft and the vibration test of the main reducer. The results of dynamic simulation shows that the angular acceleration and the contact force in Y direction of the driven gear are smaller when the bearing span is 55mm. This verifies the theoretical result. With this optimal bearing span, it can reduce the vibration of the main reducer effectively and improve the NVH performance of the crossover vehicle.

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