

# Threshold Characteristics Analysis of Dual-end-pumped Nd<sup>3+</sup>-doped Gain-guided and Index-antiguided Fiber Lasers

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**Abstract.** An analytical model for dual-end-pumped Nd<sup>3+</sup>-doped gain-guided and index-antiguided (GG-IAG) fiber laser is established. The corresponding analytical expression of the threshold pump power is obtained by solving the improved rate equations. Pump light propagation and threshold characteristics are both researched. Simulation results show that there are optimum values of  $N$ ,  $a$  and  $L$  for the lowest  $P_{th}$ . When  $a=50\mu\text{m}$ ,  $L=9\text{cm}$ ,  $N=1.8 \times 10^{20}\text{cm}^{-3}$ ,  $R_{1s}=1$ ,  $R_{2s}=0.75$ , the lowest threshold pump power  $P_{th}$  of the GG-IAG fiber laser is about 11.95W, the results of the work may be helpful for the later experiments.

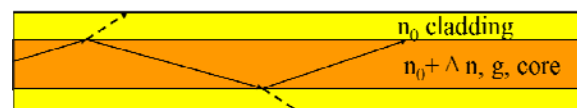
## 1 Introduction

High power fiber lasers have many important applications in industry, defense and commercial fields. However, the major limitations to high power lasers in conventional single mode fibers are undesired optical nonlinear effects [1-4]. The most direct method to solve these problems is increasing the mode area to reduce the optical power density [5], thus large mode area single-mode fibers have recently attracted a lot of attentions, such as photonic crystal fibers [6], chirally coupled core fibers [7-8] and so on. However, the maximal obtainable mode field diameter (MFD) is still less than  $100\mu\text{m}$  [3]. In 2003, A.E. Siegman proposed a new type of gain-guided and index-antiguided (GG-IAG) fiber with large mode area [9-10], and the maximal MFD reported is about  $400\mu\text{m}$  [1]. In this paper, we report an analytical model for a dual-end-pumped Nd<sup>3+</sup>-doped GG-IAG fiber laser, and an exact analytical expression of the threshold pump power for this fiber laser has been obtained by solving the improved rate equations (REs) with the additional leakage losses of the

signal light. The effects of each parameters of the fiber on the threshold pump power are also discussed.

## 2 Theoretical analyses

The structure of the GG-IAG fiber is showed in fig.1. The refractive index of the fiber cladding  $n_2 = n_0$ , the refractive index difference of the core and the cladding is  $\Delta n$  ( $\Delta n < 0$ ), the power gain factor of the signal light is  $g$ .

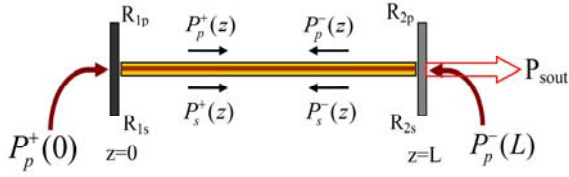


**Fig.1** The schematic diagram of GG-IAG fiber

The GG-IAG fiber laser is as shown in Fig.2. Dual-end-pumped model means that pump light  $P_p^+(0)$  and  $P_p^-(L)$  are injected into the fiber simultaneously from both ends with  $R_{1p} = R_{2p} = 0$ .  $R_{1s}$  and  $R_{2s}$  are the reflectivity of the signal light. Forward and backward

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pump light power are  $P_p^+(z)$  and  $P_p^-(z)$ , respectively. Forward and backward signal light power are  $P_s^+(z)$  and  $P_s^-(z)$ , respectively.



**Fig.2** Schematic diagram of GG-IAG fiber laser

There are leakage losses of signal light and pump light, then according to the literature [11], a set of improved REs. of  $\text{Nd}^{3+}$  doped dual-end-pumped GG-IAG fiber laser can be expressed as formulas (1), (2) and (3):

$$\frac{N_2(z)}{N} = \frac{\frac{[P_p^+(z) + P_p^-(z)] \cdot \sigma_{ap} \Gamma_p}{h\nu_p A} + \frac{[P_s^+(z) + P_s^-(z)] \cdot \sigma_{as} \Gamma_s}{h\nu_s A}}{\frac{[P_p^+(z) + P_p^-(z)] \cdot \sigma_p \Gamma_p}{h\nu_p A} + \frac{1}{\tau} + \frac{[P_s^+(z) + P_s^-(z)] \cdot \sigma_s \Gamma_s}{h\nu_s A}} \quad (1)$$

$$\pm \frac{dP_p^\pm(z)}{dz} = \Gamma_p [\sigma_p N_2(z) - \sigma_{ap} N] P_p^\pm(z) - \alpha_p P_p^\pm(z) \quad (2)$$

$$\pm \frac{dP_s^\pm(z)}{dz} = \Gamma_s [\sigma_s N_2(z) - \sigma_{as} N] P_s^\pm(z) - \alpha_s P_s^\pm(z) \quad (3)$$

Where,  $N_2(z)$  is the upper lasing level population density,  $N$  is the concentration of dopant  $\text{Nd}^{3+}$ .  $\Gamma_p$  and  $\Gamma_s$  are the pump and signal field overlaps, respectively.  $\nu_p$  and  $\nu_s$  are the pump and signal light frequencies, respectively.  $A$  is the cross-sectional area of the core.  $\tau$  is the spontaneous lifetime of the upper-level atoms and the parameter  $h$  is Planck's constant. The parameter  $c$  is the speed of light in vacuum.

$\sigma_{tp} = \sigma_{ap} + \sigma_{ep}$ ,  $\sigma_{ap}$  and  $\sigma_{ep}$  are absorption and emission cross sections of pump light, respectively.  $\sigma_{ts} = \sigma_{as} + \sigma_{es}$ ,  $\sigma_{as}$  and  $\sigma_{es}$  are absorption and emission cross sections of signal light respectively.  $\alpha_{tp} = \alpha_p + l_p$ ,  $\alpha_{ts} = \alpha_s + l_s$ , where  $\alpha_p$  and  $\alpha_s$  represent the fundamental material absorption coefficients of the pump and signal light respectively,  $l_p$  and  $l_s$  represent the additional leakage

losses of pump and signal light respectively, and they are [12]:

$$l_p = g_{01}^{th} = [\lambda_p^2 / (2\pi)^2 a^3] \sqrt{133.8 / (-2n_0^3 \Delta n)} \quad (4)$$

$$l_s = g_{01}^{th} = [\lambda_s^2 / (2\pi)^2 a^3] \sqrt{133.8 / (-2n_0^3 \Delta n)} \quad (5)$$

Eq. (3) can be derived as:

$$\frac{1}{L} \int_0^L N_2(z) dz = \frac{-\frac{\ln(R_{1s} R_{2s})}{2L} + \Gamma_s \sigma_{as} N + \alpha_{ts}}{\Gamma_s \sigma_{ts}} \quad (6)$$

From Eq. (2), Eqs. (7) and (8) can be obtained:

$$P_p^+(z) = P_p^+(0) g \exp(\beta g_{th} - \gamma) z \quad (7)$$

$$P_p^-(z) = P_p^-(L) g \exp(-\beta g_{th} + \gamma)(z - L) \quad (8)$$

Where:  $\beta = \Gamma_p \sigma_{tp} / \Gamma_s \sigma_{ts}$ ,  $\gamma = \Gamma_p \sigma_{ap} N + \alpha_{tp}$

Let:  $P_{s,sat} = \frac{h\nu_s A}{\Gamma_s (\sigma_{es} + \sigma_{as})}$ ,  $P_{p,sat} = \frac{h\nu_p A}{\Gamma_p (\sigma_{ep} + \sigma_{ap})}$ ,

$$K = \frac{1}{L} \int_0^L N_2(z) dz \cdot$$

Then, Eq. (1) can be expressed as:

$$\begin{aligned} & \frac{P_s^+(z) + P_s^-(z)}{P_{s,sat}} g \frac{N_2(z) (\sigma_{as} + \sigma_{es}) - N \sigma_{as}}{N (\sigma_{as} + \sigma_{es})} \\ &= -\frac{K}{N} + \frac{P_p^+(z) + P_p^-(z)}{P_{p,sat}} g \frac{N \sigma_{ap} - N_2(z) (\sigma_{ap} + \sigma_{ep})}{N (\sigma_{ap} + \sigma_{ep})} \end{aligned} \quad (9)$$

Combining Eqs. (2) and (3), the following equation can be derived:

$$\begin{aligned} & \frac{\tau}{h\nu_s A} \left\{ \frac{dP_s^+(z)}{dz} - \frac{dP_s^-(z)}{dz} + \alpha_{ts} [P_s^+(z) + P_s^-(z)] \right\} + \\ & \frac{\tau}{h\nu_p A} \left\{ \frac{dP_p^+(z)}{dz} - \frac{dP_p^-(z)}{dz} + \alpha_{tp} [P_p^+(z) + P_p^-(z)] \right\} + K = 0 \end{aligned} \quad (10)$$

From Eq. (3), the following conclusion can be obtained:

$$P_s^+(z) g P_s^-(z) = P_s^+(0) g P_s^-(0) = P_s^+(L) g P_s^-(L) = \text{Constant}, \quad (11)$$

The boundary conditions of dual-end-pumped fiber laser are as followings:

$$P_s^+(0) = R_{1s} P_s^-(L) \quad (12a)$$

$$P_s^-(L) = R_{2s} P_s^+(0) \quad (12b)$$

Combining Eqs. (11) and (12a-b), after Eq. (10) is integrated, the threshold pump power can be derived as following:

$$P_{in} = P_p^+(0) + P_p^-(L) = P_{s,sat} \frac{(N\Gamma_s\sigma_{as} + \alpha_s)L + \ln \frac{1}{\sqrt{R_{1s}R_{2s}}}}{\frac{v_s}{v_p} g(1 - \frac{\alpha_p}{\alpha}) (g - \exp(-\alpha \cdot L))} \quad (13)$$

Where,  $\alpha = N\Gamma_p\sigma_{ap} - \Gamma_p(\sigma_{ap} + \sigma_{ep})KL + \alpha_p$ .

### 3 Results and discussions

All the parameters used in the calculations are given in table1.

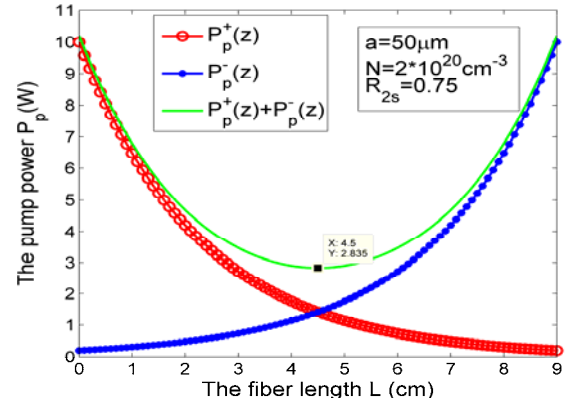
**Table 1.** The parameters of the Nd3+-doped GG-IAG fiber laser.

PRMs	value	PRMs	value
$\Gamma_p$	0.25	$\tau$	$1.9 \times 10^{-4}$ s
$\Gamma_s$	0.6	$\sigma_{ap}$	$0.7 \times 10^{-20}$ cm <sup>2</sup>
$\lambda_s$	1054nm	$\sigma_{ep}$	$3.76 \times 10^{-22}$
$\lambda_p$	803nm	$\sigma_{as}$	$7.82 \times 10^{-22}$ c
$L$	0-15cm	$\sigma_{es}$	$4.4 \times 10^{-20}$
$a$	40-100μm	$\alpha_p$	$0.016$ cm <sup>-1</sup>
$n_0$	1.5734	$\alpha_s$	$0.016$ cm <sup>-1</sup>
$\Delta n$	-0.0045	$R_{1s}$	1
$N$	$1 \times 10^{20} - 1 \times 10^{21}$ cm <sup>-3</sup>	$R_{2s}$	0.75
$h$	$6.6260775 \times 10^{-34}$ J·s	$R_{1p}, R_{2p}$	0

#### 3.1 The pump light propagation characteristics

The forward and backward pump light can be expressed as  $P_p^+(z)$  and  $P_p^-(z)$  respectively, and the total pump power in the fiber core is  $P_p^+(z) + P_p^-(z)$ . Supposing  $P_p^+(0) = P_p^-(L) = 10W$ , then the pump light distribution in the fiber core is shown in fig.3.

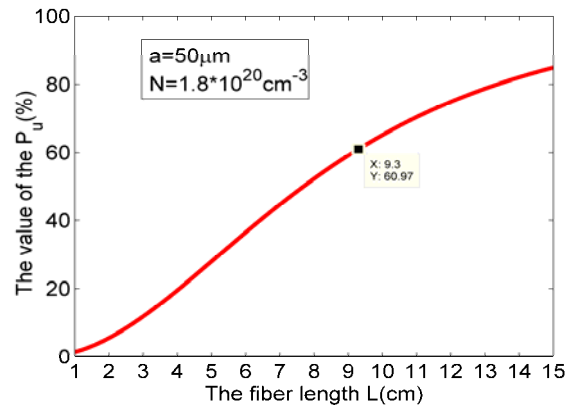
$P_p^+(L)$  and  $P_p^-(0)$  are very low when fiber length  $L$  is 9cm, and at the middle of the fiber the total pump power is 2.835W.



**Fig.3**  $P_p$  distribution along the fiber length

Supposing that the maximal (minimal) pump power along the whole fiber is  $P_{pmax}$  ( $P_{pmin}$ ), respectively. The allowed fractional variation in the pump light distribution in the fiber core should be below 61% [4] for GG-IAG fiber to maintain single mode oscillation, and which is called uniformity of pump light distribution here and can be defined as:  $P_u = (P_{pmax} - P_{pmin}) / P_{pmax} \times 100\%$  [4].  $P_u$  changes process with the fiber length  $L$  is shown in Fig.4.  $P_u$  increases with the increasing of  $L$ .

When  $P_u$  is 61%, the allowed maximal fiber length  $L$  is about 9.3 cm, which is shown in Fig.4.



**Fig.4**  $P_u$  changes with the fiber length

### 3.2 Threshold pump power characteristics

Supposing the threshold pump power is  $P_{th}$ , which changes with  $N$  is shown in Fig.5 and Fig.6. It can be found that in a certain concentration range, the threshold pump power is lower and keeps nearly invariable, but when the concentration is beyond this range, the threshold increases higher. The reason is when  $N$  is lower, it is difficult to supply enough gain to overcome the losses, and when  $N$  is too high, the pump power is absorbed fast at the two ends of the fiber, so both make the threshold power increase. In this paper, when  $L=9\text{cm}$ ,  $a=50\mu\text{m}$  and  $R_{2s}=0.75$ , the minimum of  $P_{th}$  is about 11.95W, the corresponding concentration  $N$  is about  $1.8 \times 10^{20} \text{cm}^{-3}$ .

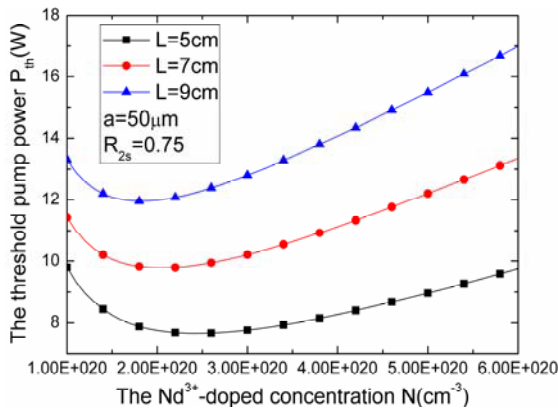


Fig.5 The relationship between  $P_{th}$  and  $N$  with different  $L$

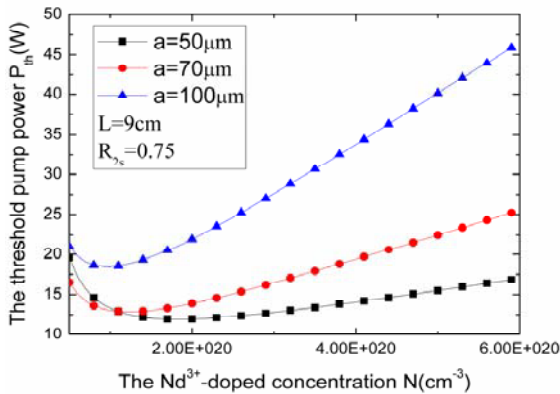


Fig.6 The relationship between  $P_{th}$  and  $N$  with different  $a$

$P_{th}$  changes process with  $L$  is shown in Fig.7. Fig.7 shows that the threshold pump power is lowest with the optimum fiber length, when the fiber length  $L$  is too long or too short, the  $P_{th}$  will increase correspondingly.

Because when the fiber is short, the gain is too small to satisfy the oscillation conditions, so the threshold becomes higher. When the fiber is long, the loss of the fiber becomes big, so the threshold becomes higher.

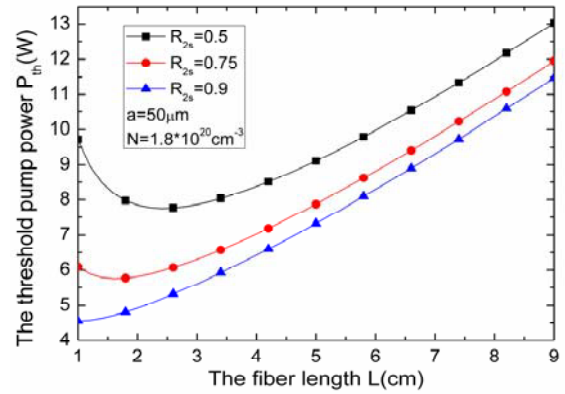


Fig.7 The relationship between  $P_{th}$  and  $L$  with different  $R_{2s}$

$P_{th}$  changes process with fiber core radius  $a$  is shown in

Fig.8. The optimum fiber core radius  $a$  is about 50-55 $\mu\text{m}$  with different  $R_{2s}$  when  $L$  is 9 cm.

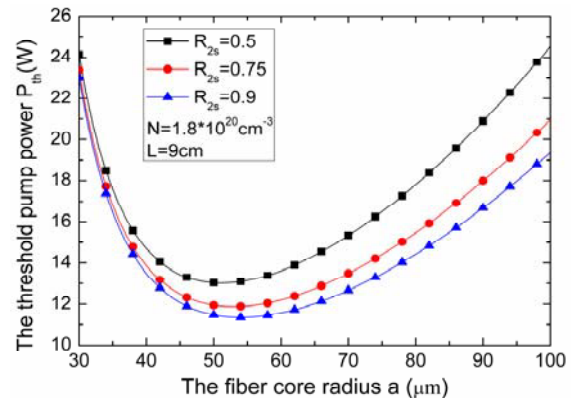


Fig.8 The relationship between  $P_{th}$  and  $a$

## 4 Conclusions

A dual-end-pumped  $\text{Nd}^{3+}$ -doped GG-IAG fiber laser is analyzed in this work. Pump light propagation and threshold characteristics are both researched. Under the requirement of the uniformity of pump power along the fiber [4], the maximal fiber length is 9.3cm. The relationship between threshold pump power  $P_{th}$  and doped concentration  $N$ , fiber length  $L$ , fiber radius  $a$  is researched respectively. Simulation results show that there are optimum value of  $N$ ,  $a$  and  $L$  where  $P_{th}$  is the lowest. When  $a=50\mu\text{m}$ ,  $L=9\text{cm}$ ,  $N=1.8 \times 10^{20} \text{cm}^{-3}$ ,  $R_{1s}=1$ ,  $R_{2s}=0.75$ , the lowest threshold pump power  $P_{th}$  of GG-IAG fiber laser is about 11.95W, the results of the work may be helpful for the later experiments.

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