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16-QAM Quantum Receiver with Hybrid Structure Outperforming the Standard Quantum Limit

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Abstract. Quantum receivers which can discriminate phase shift keying (PSK) and pulse position modulation (PPM) signals below the standard quantum limit have been proposed and some have been demonstrated experimentally. But for quadrature amplitude modulation (QAM) signals, few literatures have been reported so far. It is important to reduce the average error probability of QAM signals below the standard quantum limit (SQL), since these modulation have a high spectral efficiency. In this paper, we present a quantum receiver for 16-QAM signals discrimination with hybrid structure, which contains a homodyne receiver and a displacement receiver. By numerical simulation, we prove that the performance of the quantum receiver can outperform the SQL, and it can be improved by an optimized displacement.

1 Introduction

It is known for decades that quantum receivers can discriminate different signals below the standard quantum limit (SQL) and achieve even lower error probability limit called Helstrom bound [1]. For phase shift keying (PSK) and pulse position modulation (PPM) signals discrimination, many kinds of quantum receivers have been proposed theoretically and some of them have been demonstrated experimentally [2-11]. Dolinar receiver was proposed to discriminate binary phase shift keying (BPSK) signals. Its performance can achieve Helstrom limit theoretically [3]. Later, Bondurant proposed two type Bondurant receiver to discriminate quadrature phase shift keying (QPSK) [4]. For PPM signals, a conditionally nulling receiver was proposed by Dolinar, which is able to achieve very nearly the Helstrom limit [9].

However, few literatures have been paid attention to QAM signals discrimination. The QAM signals have a high spectral efficiency in optical communication [12]. So it is very meaningful to reduce the average error probability of QAM signals below the SQL. Inspired by Bondurant [4] and Müller [5, 11], we present an original 16-QAM quantum receiver with a hybrid structure, which contains a homodyne receiver and a displacement receiver. Numerical simulation has shown that it can outperform the SQL. What's more, its performance can be even better, if we optimize the displacement, especially when the signals are weak. By using these quantum receivers, the communication distance and capacity for 16-QAM signals are promising to be improved.

2 Schemes and Performances

2.1 Classical receiver system

In an optical communication protocol, a sender encodes information by coherent light. For 16-QAM signals, information are encoded into the amplitude and phase. The 16-QAM signals include 16 alphabet $|\alpha_{uv}\rangle$, $u,v \in \{-3,-1,1,3\}$, where

$$\alpha_{uv} = u\,|\,\alpha\,| + j\,v\,|\,\alpha\,|. \tag{1}$$

Where $j=\sqrt{-1}$ is imaginary unit. The real part and imaginary part correspond to the two quadrature amplitudes x and p respectively. It is well known that a conventional detection scheme is a heterodyne receiver, which measure the two quadrature amplitudes simultaneously. In ideal condition without any noise and interference, the probability density function is given as follows [1]:

$$P(x,p\,|\,u,v) = \frac{1}{\pi}\exp\left[-\left(x-u|\alpha|\right)^2-\left(p-v|\alpha|\right)^2\right] \tag{2}$$

The detection domain $D_{u',v'}$, where signal $|\alpha_{uv}\rangle$ is detected as $|\alpha_{u'v'}\rangle$, can be represent as

$$\begin{aligned} D_{u',v'} = \{(x,p)\,|\,D_L(u') < x \le D_U(u'), \\ D_L(v') < p \le D_U(v')\}. \end{aligned} \tag{3}$$

Where the notation D_L and D_U are defined as

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$$D_L(k)=\begin{cases}-\infty, & k=-3\\ k-1, & k=-1,1,3\end{cases},$$
$$D_U(k)=\begin{cases}\infty, & k=3\\ k+1, & k=-3,-1,1\end{cases}. \tag{4}$$

Then, we can calculate the average error probability by the formula

$$P_e=1-\frac{1}{16}\sum_{u,v}\iint_{D_{u,v}}P(x,p\,|\,u,v)\,\mathrm{d}x\mathrm{d}p\,. \tag{5}$$

2.2 16-QAM Helstrom limit

In the quantum detection and estimation theory, the detectors are described by a set of positive operator-valued measure (POVM). The optimal quantum detectors can be solved by semi-definite programming (SDP) [1]. Instead of solving 16 POVM matrices, we compute only one matrix by solving the dual problem [13].

2.3 Hybrid receiver system

In order to discriminate the 16-QAM signals below SQL, we designed a hybrid structure scheme, shown in the Fig. 1 (*a*). From Fig. 1 (*a*), we can see the phase space configuration of 16-QAM signals and our receiver scheme. At first, the incoming signal is split into two equivalent portions by a beam splitter (BS) with reflect ratio $r^2=0.5$. Then one beam is directly fed into a homodyne detector (HD), whilst the other one is detected using a displacement receiver, which is controlled using a feed-forward and feedback strategy. The HD determines the p quadrature of the incoming signal and it is described by the POVM elements

$$\hat{\Pi}_i^{HD}=\int_{D_i}\mathrm{d}p\,|p\rangle\langle p|,i=1,2,3,4. \tag{6}$$

Where

$$D_1=\{p\,|\,p>2|r\cdot\alpha|\},D_2=\{p\,|\,0<p\le 2|r\cdot\alpha|\},$$
$$D_3=\{p\,|\,0\ge p>-2|r\cdot\alpha|\},D_4=\{p\,|\,p\le -2|r\cdot\alpha\,|\}. \tag{7}$$

For signal $|\alpha_{uv}\rangle$, the correct output corresponds to the detection domain D_i with $\mathrm{Im}(r\cdot\alpha_{uv})\in D_i$. We write this domain as $D(\alpha_{uv})$. And the probability of observing correct outcome for this signal is

$$\begin{aligned}P_c^{\mathrm{HD}}(r\cdot\alpha_{uv})&=\int_{D(\alpha_{uv})}|\langle p|r\cdot\alpha_{uv}\rangle|^2\,\mathrm{d}p\\&=\frac{1}{2}\mathrm{erf}\left(\sqrt{2}p\right)\Big|_{p_L(D)}^{p_U(D)}.\end{aligned} \tag{8}$$

Where $p_U(D_i)$ and $p_L(D_i)$ are the upper and lower bound for D_i .

Supposing HD yield correct results, then 16 hypotheses are reduced to only 4 hypotheses H_i (i = 1, 2, 3, 4) as shown in Fig. 1 (*b*). In order to discriminate these 4 signals, we use a displacement receiver (DR) which contains a displacement operator and an ideal single photon detector (SPD). It sequentially nulls signals 1 → 2 → 3 → 4 just like Bondurant receiver [4]. At first, the receiver choose the hypnosis H_1 and null the first signal by a displacement $D(\gamma)$. The imaginary part of γ is determined by HD detection result. If the hypnosis is correct, then after displacement $D(\gamma)$, the signal fed into SPD becomes vacuum state and no photon can be detected. So we regard the signal 1 as the correct result. Otherwise, every time a photon counting event happens, the receiver immediately changes the displacement to null the next signal and keeps counting. Suppose N is the photon count number in whole symbol interval, if $N\le 3$, we choose the hypothesis H_{N+1}; if $N>3$, choose H_4.

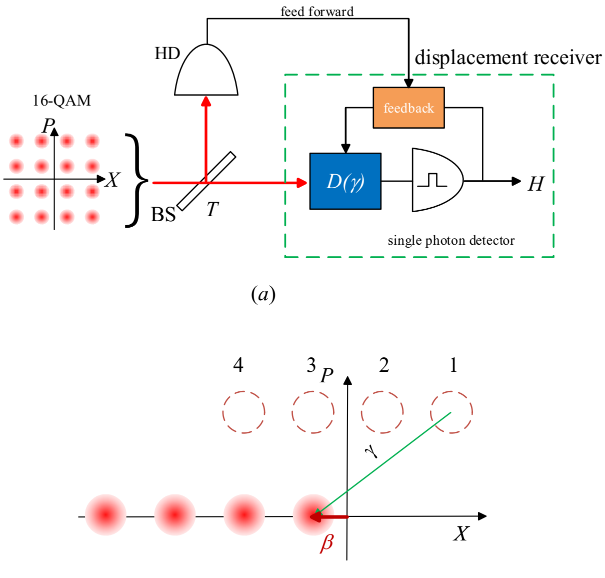

Figure 1. (*a*) Hybrid structure for 16-QAM signals. The incoming signal is equally divided into two parts by beam splitter (BS). One part is detected by a homodyne detector (HD) and the result is feed forward to the displacement receiver. (*b*) Displacement strategy for displacement receiver. When using the exactly nulling, choose β = 0. When using the optimal displacement, choose β by numerical optimization.

More specifically, if signal 1 is sent and no photon click event happens, then we get the correct answer with probability

$$P(1|1)=\exp\left(|\alpha_1-\gamma_1|^2\right). \tag{9}$$

If signal 2 is sent, we get the correct answer when only one photon click event happens, which occurs with probability

$$\begin{aligned}P(2|2)=&\int_0^T\mathrm{d}t_1\frac{|\alpha_2-\gamma_1|^2}{T}\exp\left(-|\alpha_2-\gamma_1|^2\frac{t_1}{T}\right)\\&\times\exp\left(-|\alpha_2-\gamma_2|^2\frac{T-t_1}{T}\right).\end{aligned} \tag{10}$$

Where T is signal pulse interval, and $\alpha_i\left(i=1,2,3,4\right)$ is the selected signal amplitude. And t_i stands for the i-th photon arrival time. If signal 3 is sent, we get the correct answer when two photon click event happen exactly, which occurs with probability

$$P(3|3)=\int_0^T \mathrm{d}t_1 \int_{t_1}^T \mathrm{d}t_2 \frac{\left|\alpha_3-\gamma_1\right|^2}{T}\exp\left(-\left|\alpha_3-\gamma_1\right|^2\frac{t_1}{T}\right)\frac{\left|\alpha_3-\gamma_2\right|^2}{T} \times\exp\left(-\left|\alpha_3-\gamma_2\right|^2\frac{t_2-t_1}{T}\right)\exp\left(-\left|\alpha_3-\gamma_3\right|^2\frac{T-t_2}{T}\right). \tag{11}$$

If signal 4 is sent, we get the correct answer when there are more than or equal to 3 photon click events happen, which occurs with probability

$$P(4|4)=\int_0^T \mathrm{d}t_1 \int_{t_1}^T \mathrm{d}t_2 \int_{t_2}^T \mathrm{d}t_3 \frac{\left|\alpha_4-\gamma_1\right|^2}{T}\exp\left(-\left|\alpha_4-\gamma_1\right|^2\frac{t_1}{T}\right) \times\frac{\left|\alpha_4-\gamma_2\right|^2}{T}\exp\left(-\left|\alpha_4-\gamma_2\right|^2\frac{t_2-t_1}{T}\right) \times\frac{\left|\alpha_4-\gamma_3\right|^2}{T}\exp\left(-\left|\alpha_4-\gamma_3\right|^2\frac{t_3-t_2}{T}\right). \tag{12}$$

Thus, for given signal α_{uv}, the correct probability for the displacement receiver is $P_c^{\mathrm{DR}}(t\cdot\alpha_{uv})=P(j\,|\,j)$, where t is transparent ratio, $t^2=1-r^2=0.5$, and j is the index of the signal which is shown in Fig. 1(b). The total average probability of the hybrid receiver giving the correct outcome is

$$P_c=\frac{1}{16}\sum_{u,v}P_c^{\mathrm{HD}}\left(r\cdot\alpha_{uv}\right)P_c^{\mathrm{DR}}\left(t\cdot\alpha_{uv}\right) \tag{13}$$

In this case, we use exact nulling strategy namely $\gamma_i=\alpha_i\left(i=1,2,3,4\right)$. There are at most 3 photon click events when using exact nulling strategy. Because no photon will be detected once the signal is displaced to vacuum state.

2.4 Optimal displacement

In the above, we set the exact nulling for the displacement receiver. However, exact nulling is not a good idea when signal is weak [5, 11]. By adding an additional displacement β just like generalized Kennedy receiver, namely $\gamma_i=\alpha_i+\beta\left(i=1,2,3,4\right)$, which is shown in Fig. 1(b), the error probability can be further reduced. The best value of β is given by

$$\beta^*=\underset{\beta}{\operatorname{argmax}}\{P_c(\beta)\}. \tag{14}$$

Which can be determined by numerical optimization.

2.5 Simulation and Results

We simulate our receiver under two different configurations, exact nulling (Type I) and optimal displacement (Type II). In the Type II configuration, we use numerical optimization to determine the best value of β to gain the smallest average error probability. Results are shown in Fig. 2. As in Fig. 2 (a), the optimal displacement decays with the average photon number increasing. When the signal is strong enough, the additional displacement β can be ignored, so the performance of the Type I receiver approaches to the performance of the type II receiver as shown in Fig. 2(b). In this case, type I receiver (blue) and type II receiver (green) can both outperform SQL (red). However, when the signal is weak, the performance of the type I is above the SQL, but the type II receiver can still outperform SQL within a wider range of average photon number.

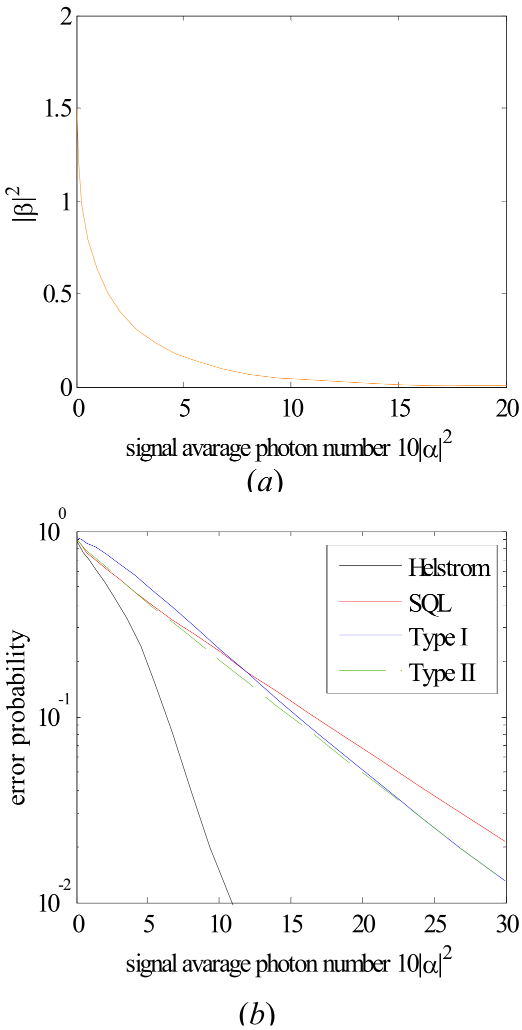

Figure 2. (a) Optimal displacement parameter $|\beta|^2$ as a function of signal energy. (b) Error probability for hybrid structure receiver using exact nulling (type I, blue) and optimal displacement (type II, green) compare to SQL (red) and Helstrom limit (black).

3 Conclusion

In this paper, two type receivers have been proposed for 16-QAM signals discrimination, and ran numerical simulation to show that the hybrid structure quantum receivers can be also applied to 16-QAM signals. And the performance of these receivers outperforms the SQL in bright light regime. Finally, comparing two type receiver, we conclude that the performance can be improved by optimal displacement, especially when the signal is weak. Since these receivers can outperform the SQL, it is promising to apply these receivers to increase the distance and capacity in optical communication.

References

1. C. W. Helstrom, *Quantum detection and estimation theory*(Academic press, 1976).
2. R. S. Kennedy, MIT Research Laboratory of Electronics Quarterly Progress Report**,** **110**, 219 (1973).
3. S. Dolinar, MIT Research Laboratory of Electronics Quarterly Progress Report**,** **111**, 115 (1973).
4. R. S. Bondurant, Opt Lett**,** **18**, 1896 (1993).
5. C. R. Müller, M. A. Usuga, C. Wittmann, M. Takeoka, C. Marquardt, U. L. Andersen, G. Leuchs, New J Phys**,** **14**, 83009 (2012).
6. R. L. Cook, P. J. Martin, J. M. Geremia, Nature**,** **446**, 774 (2007).
7. F. E. Becerra, J. Fan, G. Baumgartner, J. Goldhar, J. T. Kosloski, A. Migdall, Nat Photonics**,** **7**, 147 (2013).
8. K. Li, Y. Zuo, B. Zhu, IEEE Photonic Tech L**,** **25**, 2182 (2013).
9. S. J. Dolinar Jr, The Telecommunications and Data Acquisition Progress Report**,** **42**, 72 (1982).
10. J. Chen, J. L. Habif, Z. Dutton, R. Lazarus, S. Guha, Nat Photonics**,** **6**, 374 (2012).
11. C. R. Müller, G. Leuchs, C. Marquardt, in CLEO: QELS_Fundamental Science, Optical Society of America, 2014, p. M3A.
12. P. J. Winzer, Lightwave Technology, Journal of**,** **30**, 3824 (2012).
13. Y. C. Eldar, A. Megretski, G. C. Verghese, Information Theory, IEEE Transactions on**,** **49**, 1007 (2003).