Measurement Uncertainty Evaluation of Digital Modulation Quality Parameters: Magnitude Error and Phase Error

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Abstract. In digital modulation quality parameters traceability, the Error Vector Magnitude, Magnitude Error and Phase Error must be traced, and the measurement uncertainty of the above parameters needs to be assessed. Although the calibration specification *JJF1128-2004 Calibration Specification for Vector Signal Analyzers* is published domestically, the measurement uncertainty evaluation is unreasonable, the parameters selected is incorrect, and not all error terms are selected in measurement uncertainty evaluation. This article lists formula about magnitude error and phase error, than presents the measurement uncertainty evaluation processes for magnitude error and phase errors.

1   Preface

Shanghai Institute of Measurement and Testing Technology (SIMT), which is also the National Center of Measurement and Testing for East China, undertakes the task of measurement traceability in East China. When tracing the measurement of digital modulation quality parameters, the most important traceability parameters are Error Vector Magnitude (EVM), magnitude error and phase error. Relevant calibration specifications have been already published, such as *JJF1128-2004 Calibration Specification for Vector Signal Analyzers*, which specifies the traceability method of EVM, magnitude error and phase error. But in *JJF1128-2004 Calibration Specification for Vector Signal Analyzers*, the uncertainty assessment methods are not so reasonable, not all the uncertainty components are included, and the assessment method and procedure are also questionable. This article discusses and assesses the source of measurement uncertainty of magnitude error and phase Error, and shows the result of National Comparison of Digital Modulation Quality Parameter organized by China Radio Metrology Technology Committee ([1]). The value of this assessment result is basically same as the result assessed by National Institute of Metrology (NIM).

2   Measuring method

According to *JJF1128-2004 Calibration Specification for Vector Signal Analyzers*, digital modulation quality parameters are calibrated by direct comparison method. Schematic diagram is shown in Figure 1.

Connect calibrated Digital Signal Generator (DSG) to a Vector Signal Analyzer (VSA) or a Spectrum Analyzer with the analyzing function of vector signal. The time base output of Vector Signal Generator (VSG)

should be connected to the time base input of VSA, and the time base of VSA should be set as external time base. Set the output frequency of the calibrated DSG, and generally the output level is set to -10 dBm. Select the required standard format or the customized modulation parameters, such as modulation, filter type, filter factor α and symbol rate. Set the input frequency of the VSA equal to the one of calibrated DSG, and input level is -10 dBm or select **Level Auto Range** button to regulate the input level of VSA automatically. Select the required modulation standard format or set the modulation system, filter type, filter factor α and symbol rate which is consistent with the VSG. Read the digital modulation quality parameters from VSA.
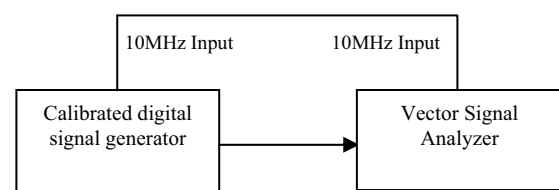

Figure 1. Schematic diagram of digital modulation quality parameter calibration

3   Definition of digital modulation quality parameter

For various vector modulation, the signal magnitude and phase can both be represented by the points on the constellation diagram. However, due to the effects such as non-ideal performance of vector modem hardware and the precision of vector modulation algorithm, there are errors occurring between signal vector in real and ideal situation. Vector modulation error defines the error between real constellation points and corresponding

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reference constellation points in every symbol period. It is also called Digital Modulation Quality Parameter. The major parameters are magnitude error (MagErr), phase error (PhaseErr) and EVM, shown in Figure 2. Those parameters are usually represented by root mean square (RMS) ([2-3]).
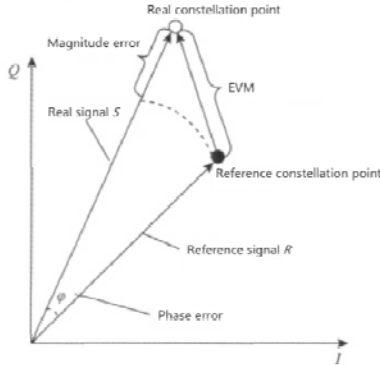

Figure 2. Definition of vector modulation error

As Eq. (1)~(3), the subscript rms is expressed as the root mean square value.

*MagErr*rms is expressed as:

$$MagErr_{\mathrm{rms}} = \sqrt{\frac{\sum_{i=0}^{i=N-1}\left(|S_i|-|R_i|\right)^2}{\sum_{i=0}^{i=N-1}|R_i|^2}} \times 100\% \qquad (1)$$

*PhaseErr*rms is expressed as:

$$PhaseErr_{\mathrm{rms}} = \sqrt{\frac{1}{N}\sum_{i=0}^{i=N-1}\left(\arg S_i - \arg R_i\right)} \qquad (2)$$

*EVM*rms is expressed as:

$$EVM_{\mathrm{rms}} = \sqrt{\frac{\sum_{i=0}^{i=N-1}|S_i - R_i|^2}{\sum_{i=0}^{i=N-1}|R_i|^2}} \times 100\% \qquad (3)$$

In Eq. (1)~(3), *S* is real signal vector, and *R* is reference signal vector.

4 Evaluation of measurement uncertainty of magnitude error

EVM will be first introduced before the evaluation of measurement uncertainty of digital modulation quality parameters are introduced.

An ideal vector modulation signal expression is shown in the Eq. (4).

$$V(t) = I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t) \qquad (4)$$

Assume *I/Q* gain imbalance degree *g* (*g*=1 means complete balance), phase error *φ*, and interference signal *D*sin(2*πf_it*) are exist. Phase noise *θ(t)* is a random variable which variance is *σP*. *n(t)* is Gauss white noise with a variance of *σG*.

Then Eq. (4) turns to Eq. (5):

$$V(t) = I(t)\cos\left[2\pi f_c t + \frac{\phi}{2} + \theta(t)\right] - gQ(t)\sin\left[2\pi f_c t - \frac{\phi}{2} + \theta(t)\right]$$
$$+ n(t) + D\sin(2\pi f_i t) \qquad (5)$$

It is concluded that *EVM* and the error factor can be obtained by the derivation ([3]).

$$EVM_{\mathrm{rms}} = \left[2-(1+g)\cos\left(\frac{\varphi}{2}\right)\sqrt{\frac{1}{\frac{(g^2+g^2\sigma_p^2+\sigma_p^2+1)}{2}}} + ISR\right]^{\frac{1}{2}} \qquad (6)$$

$$ISR = \frac{\sigma_G^2 + \dfrac{D^2}{2}}{I^2(t)+Q^2(t)} \qquad (7)$$

In Eq. (7):

I/Q gain imbalance degree *g* is a linear value, not a logarithm value.

Unit of phase error *φ* is radian.

ISR is the power ratio of interfering signals and useful signals, and is also a linear value.

If *φ*=0, *EVM* in Eq. (6) equals *MagErr*rms.

$$MagErr_{\mathrm{rms}} = \left[2-(1+g)\sqrt{\frac{1}{\frac{(g^2+g^2\sigma_p^2+\sigma_p^2+1)}{2}}} + ISR\right]^{\frac{1}{2}} \qquad (8)$$

Eq. (8) is the expression of measurement uncertainty of magnitude error in digital modulation quality parameters traceability.

4.1 Uncertainty source

In *JJF1128-2004 Calibration Specification for Vector Signal Analyzers*, an example of measurement uncertainty evaluation is also given. For magnitude error and phase error, its uncertainty source is the error of vector signal generated by the VSG, resolution of VSA and measurement repeatability. When the VSG is to be calibrated, the VSA is the standard. The connection graph is the same as Figure 2, which means calibration of digital modulation quality parameters of VSG and digital modulation quality parameters of VSA is mutual cycle.

From Eq. (8) it can be concluded, when using the DSG to calibrate the digital modulation magnitude error of the VSA, the magnitude error is related to the following factors: *I/Q* gain imbalance (including inband flatness) *g*, signal-noise ratio *ISR*, phase noise, *I/Q* origin offset, resolution and measurement repeatability ([2, 4, 5, 6]).

4.2 Uncertainty analysis

The evaluation process of the measurement uncertainty component is introduced below. For the Eq. (8) is complex, in the calculation of the uncertainty of the component, consider the impact of individual components, and finally combine the components.

4.2.1 Standard measurement uncertainty components produced by I/Q gain imbalance g

I/Q gain imbalance *g* includes inband flatness, which means *g* is related to signal bandwidth. Bandwidth of the signal becomes wider, then *I/Q* gain imbalance increaser, and magnitude error also increaser.

Only consider the effect of *I/Q* gain imbalance on magnitude error, it can be obtained from Eq. (8).

$$MagErr_{rms} = \sqrt{2 - (1+g)\sqrt{\frac{2}{1+g^2}}} \quad (9)$$

According to the technical specification, *I/Q* gain imbalance is 1.01. It can be obtained from Eq. (9).

$$a_1 = 0.5\%,$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.

$$u_1 = 0.29\%$$

4.2.2 Standard measurement uncertainty components produced by signal-noise ratio

Signal-noise ratio is the ratio between signal power and noise power. The greater the SNR is, the stronger the receiver's receptivity.

Only consider the effect of *SNR* on magnitude error, it can be obtained from Eq. (7).

$$ISR = 1/SNR \quad (10)$$

Put Eq. (10) into Eq. (8), it can be obtained.

$$MagErr_{rms} = \sqrt{2 - 2\sqrt{\frac{1}{1+ISR}}} \quad (11)$$

In fact, SNR can reach more than 55dB in VSA. It can be obtained from Eq. (11).

$$a_2 = 0.21\%$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.

$$u_2 = 0.13\%$$

4.2.3 Standard measurement uncertainty components produced by phase noise

Phase noise is related to the power spectral density of the single side. With reference to the technical specification, the phase noise parameter of the VSA is roughly as follows:

-80dBc/10Hz, -96dBc/100Hz, -105dBc/1kHz,
-127dBc/10kHz, -126dBc/100kHz,
-144dBc/1MHz, -146dBc/10MHz.

Then:
N/(10Hz~100Hz)=(-80-96)/2+10lg(100-10)=-68dB,
Similarly,

N/(100Hz~1kHz)=-71dB,
N/(1Hz~10kHz)=-76dB,
N/(10kHz~100kHz)=-77dB,
N/(100kHz~1MHz)=-75dB,
N/(1MHz~10MHz)=-75dB,

Formula of phase jitter is

$$phase = \sqrt{10^{\frac{N}{10}} \times 2} \quad (12)$$

Then:

$$phase/(10Hz\text{-}100Hz)=5.6\times10^{-4}$$
$$phase/(100Hz\text{-}1kHz)=4.0\times10^{-4}$$
$$phase/(1kHz\text{-}10kHz)=2.2\times10^{-4}$$
$$phase/(10kHz\text{-}100kHz)=2.0\times10^{-4}$$
$$phase/(100kHz\text{-}1MHz)=2.5\times10^{-4}$$

$$phase/(1MHz\text{-}10MHz)=2.5\times10^{-4}$$

Then:

$$\sigma_P = phase/(10Hz \sim 10MHz)$$
$$=8.3\times10^{-4} \text{ rad}$$

Ignoring other factors, it can be obtained by the effect of phase noise on EVM.

$$MagErr_{rms} = \sqrt{2 - 2\sqrt{\frac{1}{1+\sigma_p^2}}} \quad (13)$$

Put the calculation result of σ_P into Eq. (13), then $a_3=0.083\%$. Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.

$$u_3 = 0.05\%$$

4.2.4 Standard measurement uncertainty components produced by I/Q origin offset

Demodulator imbalance will cause the carrier leakage, which generates an interference signal. It interferences signal vector and deviates it from the correct position, then origin drift. Therefore, origin offset is carrier leakage signal components of demodulated signal.

Ignoring other factors, it can be obtained by the effect of interference signal on magnitude error. It can be obtained from Eq. (8).

$$MagErr_{rms} = \sqrt{2 - 2\sqrt{\frac{1}{1+ISR}}} \quad (14)$$

ISR is *I/Q* origin offset. According to the technical specification of VSA, *I/Q* origin offset is -60dB. It can be obtained from Eq. (14).

$$a_4 = 0.1\%$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.

$$u_4 = 0.06\%$$

4.2.5 Standard measurement uncertainty components produced by resolution

When the vector signal is demodulated by the VSA, the minimum resolution is 0.01.

$$a_5 = 0.005\%$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.

$$u_5 = 0.003\%$$

4.2.6 Standard measurement uncertainty components produced by measurement repeatability

Measure the magnitude error for 10 times when standard value is 4.5%.

Times	Measurement Value
1	4.51%
2	4.48%
3	4.51%
4	4.49%
5	4.48%
6	4.49%
7	4.52%

8	4.52%
9	4.52%
10	4.49%

According to the Bessel formula:
$$u_6=s= 0.013\%$$

4.3 Calculation of the combined standard uncertainty

4.3.1 List of uncertainty

No.	The uncertainty component					
	The sources of the uncertainty	Type	Symbol and value	Distribution	The coverage factor	The symbol and value of standard uncertainty
1	*I/Q* gain imbalance	B	a_1=0.5%	Uniform	$\sqrt{3}$	u_1=0.29%
2	signal-noise ratio	B	a_2=0.22%	Uniform	$\sqrt{3}$	u_2=0.13%
3	phase noise	B	a_3=0.08%	Uniform	$\sqrt{3}$	u_3=0.05%
4	*I/Q* origin offset	B	a_4=0.1%	Uniform	$\sqrt{3}$	u_4=0.06%
5	resolution	B	a_4=0.005%	Uniform	$\sqrt{3}$	u_5=0.003%
6	measurement repeatability	A	/	/	/	u_6=0.013%

4.3.2 The synthesis of the standard uncertainty

Each Standard uncertainty component is independent from each other with no correlation, then consider the larger value of resolution and measurement repeatability involved in the synthesis of the standard uncertainty.

$$u = \sqrt{u_1^2 +u_2^2 +u_3^2 +u_4^2 +u_6^2} = 0.328\%$$

4.4 The expanded uncertainty

The coverage factor is 2.
$$U=0.66\%(k=2)$$

5 Evaluation of measurement uncertainty of phase error

5.1 Uncertainty source

Phase error is related to the following factors: *I/Q* gain imbalance, *I/Q* origin offset, resolution and measurement repeatability.

5.2 Uncertainty analysis

5.2.1 Standard measurement uncertainty components produced by I/Q gain imbalance

The formula of the effect of *I/Q* gain imbalance on phase error is:

$$PhaseErr = 45 - \frac{180}{\pi} arctg\left(\frac{1}{g}\right) \quad (15)$$

According to the previous hypothesis, g=1.01. Error is 1%.
$$a_1=0.29°$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.
$$u_1=0.17°$$

5.2.2 Standard measurement uncertainty components produced by I/Q origin offset

Figure 3 as an example, the effect of *I/Q* origin offset on the measurement results is introduced. According to the analysis, the magnitude of the error vector is perpendicular to X axis, and the *I/Q* origin is shifted in X axis, which has the greatest influence on the phase error.
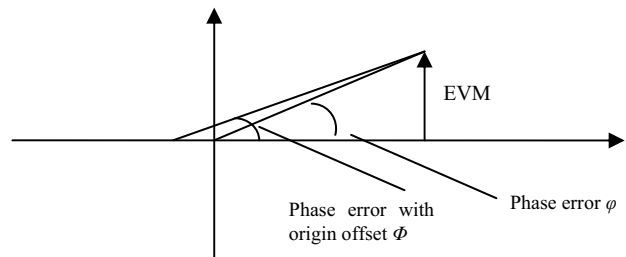
Figure 3. Phase error with origin offset

Assuming the origin offset is -60dB, which is 0.1%. According to Figure 3, the influence of the phase error is approximately equal to the origin offset value, which is in radian. It needs to multiply $\frac{180°}{\pi}$,then turns to degree.
$$a_2=0.06°$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.
$$u_2=0.04°$$

5.2.3 Standard measurement uncertainty components produced by resolution

Resolution is 0.01°.
$$a_3=0.005°,$$

Assuming it's a uniform distribution, the coverage factor is $\sqrt{3}$.
$$u_3=0.003°$$

5.2.4 Standard measurement uncertainty components produced by measurement repeatability

Measure 10 times.

Times	Measurement Value
1	3.49°
2	3.50°
3	3.48°
4	3.49°
5	3.50°
6	3.49°

7	3.50°
8	3.51°
9	3.51°
10	3.51°

Then

$$u_4 = s = 0.010°$$

5.3 Calculation of the combined standard uncertainty

5.3.1 List of uncertainty

	The uncertainty component				
No.	The sources of the uncertainty	Type	Symbol and value	Distribution	The coverage factor	The symbol and value of standard uncertainty
1	*I/Q* gain imbalance	B	$a_1=0.29°$	Uniform	$\sqrt{3}$	$u_1=0.17°$
2	*I/Q* origin offset	B	$a_2=0.06°$	Uniform	$\sqrt{3}$	$u_2=0.033°$
3	resolution	B	$a_3=0.005$	Uniform	$\sqrt{3}$	$u_3=0.003°$
4	measurement repeatability	A	/	/	/	$u_4=0.010°$

5.3.2 The synthesis of the standard uncertainty

Each Standard uncertainty component is independent from each other with no correlation, then consider the larger value of resolution and measurement repeatability involved in the synthesis of the standard uncertainty.

$$u = \sqrt{u_1^2 + u_2^2 + u_4^2} = 0.174°$$

5.4 The expanded uncertainty

The coverage factor is 2.

$$U = 0.35°(k=2)$$

6 Conclusion

In *JJF1128-2004 Calibration Specification for Vector Signal Analyzers*, the measurement uncertainty evaluation of magnitude error is unreasonable and the measurement uncertainty evaluation formula of phase error is not perfect. In this article, the uncertainty source of the magnitude error and phase error are comprehensively analyzed. In line with the principle of non repetition and no missing, measurement uncertainty of magnitude error and phase error are assessed. This article is uncertainty evaluation content when participating in the National Comparison of Digital Modulation Quality Parameters organized by China Radio Metrology Technology Committee. Table 1 and Table 2 are part of the result of National Comparison of Digital Modulation Quality Parameters.

Table 1. Magnitude error comparison result (1800MHz, 16QAM modulation, symbol rate 4Mbit/s, filter RRC, filter coefficient 0.22)

Company	Measurement value	Uncertainty (*k*=2)
National Institute of Metrology,China(Comparison leading laboratory)	4.49%	0.61%
Ministry of Industry and Information Technology Communication Measurement Center	4.5%	1.1%
China Aerospace Science&Industry Corporation 203	4.51%	0.45%
Measurement and testing center of research laboratory equipment of General Armament Department	4.5%	1.0%
Keysight Technologies,Inc.	4.52%	0.82%
Rodhe&Schwarz(China)Technology Co.,Ltd	4.5%	1.0%
Beijing Oriental Institute of Measurement and Test	4.5%	1.0%
National Instruments Corporation	4.55%	0.70%
Shanghai Institute of Measurement and Testing Technology	4.51%	0.66%
Jiangsu Institute of Metrology	4.5%	1.0%
Guangdong Provincial Institute of Metrology	4.54%	0.62%
The Fifth Electronics Research Institute of Ministry of Industry and Information Technology	4.47%	0.08%

Table 2. Phase error comparison result (1800MHz, 16QAM modulation, symbol rate 4Mbit/s, filter RRC, filter coefficient 0.22)

Company	Measurement value	Uncertainty (*k*=2)
National Institute of Metrology,China(Comparison leading laboratory)	3.50°	0.28°
Ministry of Industry and Information Technology Communication Measurement Center	3.51°	0.61°
China Aerospace Science&Industry Corporation 203	3.51°	0.28°
Measurement and testing center of research laboratory equipment of General Armament Department	3.50°	0.62°
Keysight Technologies,Inc.	3.52°	0.57°
Rodhe&Schwarz(China)Technology Co.,Ltd	3.50°	0.62°
Beijing Oriental Institute of Measurement and Test	3.5°	0.5°
National Instruments Corporation	3.52°	0.20°
Shanghai Institute of Measurement and Testing Technology	3.50°	0.35°
Jiangsu Institute of Metrology	3.5°	0.6°
Guangdong Provincial Institute of Metrology	3.53°	0.72°
The Fifth Electronics Research Institute of Ministry of Industry and Information Technology	3.48°	0.28°

From table 1 and table 2, the evaluation of measurement uncertainty is basically same as the result assessed by Comparison leading laboratory (NIM).

Acknowledgements

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