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The Recursive Forward Dynamics of Flexible Mechanical Systems

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Abstract. Lie groups and Lie algebras are used to study the recursive dynamics of flexible multi-body systems. First the adjoint transformations and adjoint operators of Lie groups and Lie algebras are discussed. Then the generalized mass matrix of flexible mechanical systems is built on the basis of modal vector. And then the inverse and forward dynamics of flexible mechanical systems are constructed. These two recursive formulas are of high efficient. Finally a four-bar model with flexible body is simulated with above method. The simulation results show that with the method can be solved quickly and efficiently.

1 Introduction

The mechanical systems are becoming larger in scale and more complex. Many kinds of flexible material are used in engineering as members of mechanical systems. The assumption of rigid body dynamics has been unable to explain the dynamics of such systems. It is necessary to consider a wide range of sports deformed parts and components [1]. Kinetic analysis of the elastic deformation is not assumed to affect multiple body systems rigid body motion which is mainly confined to the low-speed linear elastic range. Rigid mechanical systems coupled nonlinear problem has become increasingly prominent. The dynamics of flexible multi-body systems have been rapidly developed [2].

There is a variety of flexible multi-body modeling methods. A motion can be expressed as the synthesis of rigid body motion and the relative flexibility of motion according to flexible multi-body system theory. The classic formula of flexible multi-body systems can be got if the additional relatively flexible movement is built in the body reference frame. Singh establish the equations of tree topology flexible multi-body system dynamics [3]. Shabana use linear and nonlinear finite element method combination of multi-body dynamics theory to establish a flexible body dynamics model [4]. However, these methods have a problem in dealing with the efficiency and integration method [5].

On the basis of the above methods, this paper describes the dynamics with Lie groups and Lie algebras. It establishes the dynamic equations by recursive methods. It leads to a recursive dynamics of flexible mechanical systems.

2 Recursive dynamics of flexible mechanical systems

2.1 Adjoint transformations of Lie groups and Lie algebras

A Lie group is a group and also it is a differentiable manifold expressed by *G*. The tangent space of a Lie group's element is called the Lie algebra for that group. The Lie algebra forms a vector space with a bilinear map which is called the Lie bracket [8].

The rotation matrix is an orthogonal matrix in three dimensional Euclidean space and it is an element of the special Orthogonal group *SO(3)*. The homogeneous transformation is the Lie group referred to as the special Euclidean group *SE(3)*. The Lie algebra of *SO(3)* is *so(3)* which consists of the set of skew symmetric matrices on $R^{3\times 3}$. Similar with it, the Lie algebra of *SE(3)* is *se(3)*. The relation between a Lie group *SO(3)* or *SE(3)* and the associated Lie algebra *so(3)* or *se(3)* is the matrix exponential.

A linear mapping on the Lie algebra is an element of the Lie group. This is called the Adjoint map on *SE(3)* and is denoted by *Ad*, where $Ad_G(h) = GhG^{-1}$, $G \in SE(3)$ and $h \in se(3)$. Note that *h* is the homogeneous matrix.

$$Ad_G(h) = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \begin{bmatrix} \omega_h \\ v_h \end{bmatrix} \tag{1}$$

The linear mapping on the dual space *se(3)* is a dual Adjoint operator denoted by Ad^*. For h^*:

$$Ad_G^*(h^*) = \begin{bmatrix} R^T & R^T[p]^T \\ 0 & R^T \end{bmatrix} \begin{bmatrix} \mathscr{M} \\ \mathscr{F} \end{bmatrix} \tag{2}$$

The Lie algebra can also be used as a linear mapping on itself denoted by *ad*. It has the form of $ad_g(h) = [g,h] = gh - hg$, where $g, h \in se(3)$.

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$$ad_g(h) = \begin{bmatrix} [\omega_g] & 0 \\ [v_g] & [\omega_g] \end{bmatrix} \begin{bmatrix} \omega_h \\ v_h \end{bmatrix} \tag{3}$$

Similarly, the dual operator ad^* is given by $ad_g^*(h^*) = [g, h^*]$ and:

$$ad_g^*(h^*) = \begin{bmatrix} [\omega_g]^T & [v_g]^T \\ 0 & [\omega_g]^T \end{bmatrix} \begin{bmatrix} \mathcal{M} \\ \mathcal{F} \end{bmatrix} \tag{4}$$

In mutibody dynamics, the Adjoint operator *Ad* maps the spatial velocity screws from one reference frame to body frame and the dual Adjoint operator Ad^* maps the spatial force screws. The mapping *ad* and ad^* respectively define the standard cross product operation of the 6-vector velocity screws and force screws.

2.2 Generalized mass matrix of flexible mechanics

Deformation vectors are shown in Figure 1. OXYZ is inertial reference and oxyz is body-fixed reference. The vector of m point in deformable body is u_0 in body-fixed reference and u in inertial reference. The deformation displacement vector according to flexible is u_f .

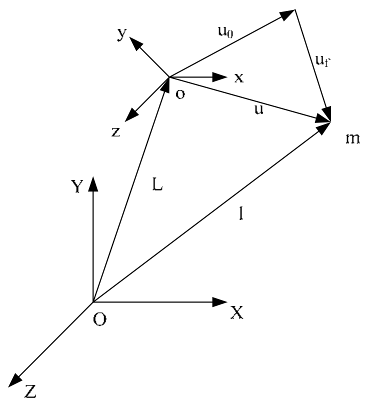

Figure 1. Deformation vectors in inertial reference system and the body-fixed reference

$$u = u_0 + u_f \tag{5}$$

$$u_f = \sum_{i=1}^{n_f} \psi(u_0) q_f(t) = \Psi^T q_f \tag{6}$$

Where $\Psi^T = [\psi_1, \psi_2, \cdots, \psi_{n_f}]$, $q_f^T = [q_{f1}, q_{f2}, \cdots, q_{fn_f}]$, and q_{fi} $(i = 1, 2, \cdots, n_f)$ are the i-th order deformation mode vectors and generalized coordinates respectively. n_f is the order of deformation mode. The vector of point m in the inertial coordinate system can be expressed as:

$$l = L + Ru = L + R(u_0 + \psi^T q_f) \tag{7}$$

R is the transformation matrix of body-fixed reference and inertial reference coordinate system. Take derivative of formula 7.

$$\dot{l} = \dot{L} + \dot{R}u + R\psi^T \dot{q}_f \tag{8}$$

$$\dot{R}u = \sum_{i=1}^{n_\theta} \frac{\partial R}{\partial \theta_i} u \dot{\theta}_i = [\frac{\partial R}{\partial \theta_1} u, \frac{\partial R}{\partial \theta_2} u, \cdots, \frac{\partial R}{\partial \theta_{n_\theta}} u] \dot{\Theta} = N\dot{\Theta} \tag{9}$$

Where $N \equiv [\frac{\partial R}{\partial \theta_1} u, \frac{\partial R}{\partial \theta_2} u, \cdots, \frac{\partial R}{\partial \theta_{n_\theta}} u]$. Equation 8 can be rewrited as follows.

$$\dot{l} = [E_3, N, R\Psi^T] \begin{bmatrix} \dot{L} \\ \dot{\Theta} \\ \dot{q}_f \end{bmatrix} \tag{10}$$

The kinetic energy of an object can be expressed as follows.

$$T = \frac{1}{2} \int_\Omega \dot{r}^T \dot{r} dm = \frac{1}{2} \dot{q}^T M \dot{q} \tag{11}$$

Where generalized mass matrix is M .

$$M \equiv \begin{bmatrix} M_{RR} & M_{R\theta} & M_{Rf} \\ M_{R\theta}^T & M_{\theta\theta} & M_{\theta f} \\ M_{Rf}^T & M_{\theta f}^T & M_{ff} \end{bmatrix}$$

Generalized coordinates is $\dot{q}$.

$$\dot{q} \equiv \begin{bmatrix} \dot{L} \\ \dot{\Theta} \\ \dot{q}_f \end{bmatrix}$$

$M_{RR} = \int_\Omega E_3 dm$, $M_{\theta\theta} = \int_\Omega N^T N dm$, $M_{ff} = \int_\Omega \Psi\Psi^T dm$

$M_{R\theta} = \int_\Omega N dm$, $M_{Rf} = R\int_\Omega \Psi^T dm$,

$M_{\theta f} = \int_\Omega N^T R \Psi^T dm$

Matrix subscript f and R represent flexible and rigid module. M_{ff} represents flexible and flexible coupling module. M_{Rf} represents flexible and rigid coupling module. M_{RR} is rigid space inertia. If the object is a rigid body, the generalized inertia matrix is $M \equiv \begin{bmatrix} M_{RR} & M_{R\theta} \\ M_{R\theta}^T & M_{\theta\theta} \end{bmatrix}$.

2.3 Inverse recursive dynamics of flexible mechanics

Similar with rigid body dynamics, we can simply expand each symbol to coordinate containing modal diformation using the finite element method to express the deformation of the flexible object.

$$\begin{aligned}
&Giving \quad V_0^m = 0, \dot{V}_0^m = 0 \\
&for\ i = 1 \to n \\
&\mathbf{V}^m{}_i = Ad^m{}_{\mathbf{T}_{i-1,i}^{-1}}(\mathbf{V}^m{}_{i-1}) + \mathbf{S}^m{}_i \dot{q}^m \\
&\dot{\mathbf{V}}_i^m = Ad^m{}_{\mathbf{T}_{i-1,i}^{-1}}(\dot{\mathbf{V}}_{i-1}^m) + \mathbf{S}^m{}_i \ddot{q}_i^m + \mathbf{a}^m{}_i \\
&a^m{}_i = ad_{V_i^m}(S^m{}_i \dot{q}_i^m)
\end{aligned} \tag{12}$$

$$\begin{aligned}
&Giving \quad F^m{}_n = 0 \\
&for\ k = 1 \to n \\
&\mathbf{F}^m{}_i = Ad^{m*}_{\mathbf{T}_{i,i+1}^{-1}}(\mathbf{F}^m{}_{i+1}) + \mathbf{J}^m{}_i \dot{\mathbf{V}}_i^m - ad^{m*}_{\mathbf{V}_i}(\mathbf{J}^m{}_i \mathbf{V}_i^m) + K_i^m \vartheta_i \\
&\boldsymbol{\tau}_i = \mathbf{S}_i^{m^{\mathrm{T}}} \mathbf{F}^m{}_i
\end{aligned} \tag{13}$$

Where the superscript m represents a modal symbol.Velocity is synthesis of modal velocity and rigid body velocity $\boldsymbol{V}^m{}_i \equiv \begin{pmatrix} \dot{\eta}_i \\ \boldsymbol{V}_i \end{pmatrix}$. The generalized coordinate and velocity of link k is ϑ_i and $\dot{q}_i^m$. $\vartheta_i \equiv \begin{pmatrix} \eta_i \\ q_i \end{pmatrix} \in \mathbb{R}^{N(k)}$, $\dot{q}_i^m \equiv \begin{pmatrix} \dot{\eta}_i \\ \beta_i \end{pmatrix} \in \mathbb{R}^{N(k)}$.

Modal spatial displacement vector on the node j_i^{th} is defined as as $\Pi^j(i) \in R^{6 \times n_m(k)}$ and the modal matrix of link i is defined as $\Pi(i) \in R^{6 n_s(i) \times n_m(i)}$.

$$\Pi^j(i) \equiv \left[\Pi_1^j(i), \ldots, \Pi_{n_m(i)}^j\right] \quad \Pi(i) \equiv col\{\Pi^j(i)\} \tag{14}$$

The Adjoint operator $Ad^m{}_{T_{i-1,i}^{-1}}$ of flexible mechanical system is the synthesis of original adjoint operator rigidity and modal matrix.

$$Ad^m{}_{\mathbf{T}_{i-1,i}^{-1}} \equiv \begin{pmatrix} 0 & \left[\Pi^t(i+1)\right]^* Ad_{\mathbf{T}_{i-1,i}^{-1}} \phi(t_{k+1}, k) \\ 0 & Ad_{\mathbf{T}_{i-1,i}^{-1}} \end{pmatrix} \in \mathbb{R}^{\bar{N}(i+1) \times \bar{N}(i)} \tag{15}$$

The modal spatial force of link i is $\boldsymbol{F}_i^m \in \mathbb{R}^{\bar{N}(i)}$.

$$F_i^m \equiv \begin{pmatrix} \Pi^*(i) F_i^s \\ F_i \end{pmatrix} \tag{16}$$

And the modal stiffness matrix.

$$K_i^m = \begin{pmatrix} \Pi^*(i) K_i^s \Pi(i) & 0 \\ 0 & 0 \end{pmatrix} \in R^{\bar{N}(i) \times \bar{N}(i)} \tag{17}$$

2.4 Forward recursive dynamics of flexible mechanics

Calculation of articulated body inertia

$$\begin{aligned}
&Giving \quad \hat{J}^m_{n+1} = 0, z^m{}_{n+1} = 0 \\
&for\ i = n \to 1 \\
&\hat{J}_i^m = J^m{}_i + Ad^{m*}_{T_{i,i+1}^{-1}}\left(I - \frac{\hat{J}^m_{i+1} S^m{}_{i+1} S^{mT}_{i+1}}{S^{mT}_{i+1} \hat{J}^m_{i+1} S^m{}_{i+1}}\right) \hat{J}^m_{i+1} Ad^m{}_{T_{i,i+1}^{-1}} \\
&b^m{}_i = -ad^{m*}_{V_i}(J^m{}_i V^m{}_i) \\
&B^m{}_i = Ad^{m*}_{T_{i,i+1}^{-1}}\left[z^m{}_{i+1} + \frac{\hat{J}^m_{i+1} S^m{}_{i+1}\left(\tau_{i+1} - S^{mT}_{i+1}\left(z^m{}_{i+1}\right)\right)}{S^{mT}_{i+1} \hat{J}^m_{i+1} S^m{}_{i+1}}\right] + b^m{}_i \\
&z^m{}_i = \hat{J}_i^m a^m{}_i + B^m{}_i
\end{aligned} \tag{18}$$

Calculation of accerlation

$$\begin{aligned}
&Giving \quad \dot{V}_0^m = 0 \\
&for\ i = 1 \to n \\
&\ddot{q}_i^m = \frac{\tau_i - S_i^{mT}\left[\hat{J}_i^m Ad^m{}_{T_{i-1,i}^{-1}}\left(\dot{V}_{i-1}^m\right) + z^m{}_i\right]}{S_i^{mT} \hat{J}_i^m S^m{}_i} \\
&\dot{V}_i^m = Ad^m{}_{T_{i-1,i}^{-1}}\left(\dot{V}_{i-1}^m\right) + S^m{}_i \ddot{q}_i^m + a^m{}_i
\end{aligned} \tag{19}$$

3 Examples

In a Four-bar mechanism, link 2 is made of Aluminum which is considered as flexible body. The bar 3 and 4 are rigid. The parameters of link 2 are: the mass is 0.57kg, the length is 0.75m, the density is 7600kg/m3, the Young's modulus is 200GPa. The parameters of link 3 and 4 are the same of: the mass are 0.5kg and 9.028kg, the moment inertia are [5.0e-5 0 0;0 5.5e-3 0;0 0 5.5e-3] and [1.8e-3 0 0;0 0.678 0;0 0 0.678].

Figure 2 to 5 give the simulation results of position, velocity and acceleration of link 3. It can be seen that the acceleration of link 3 appears tremble and the flexible impact the position and velocity little.

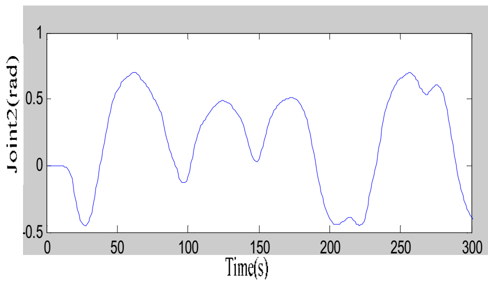

Figure 2. The body position of link 3

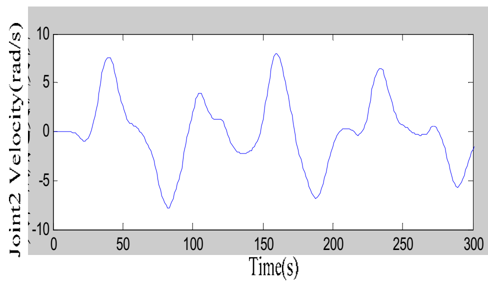

Figure 3. The body velocity of link 3

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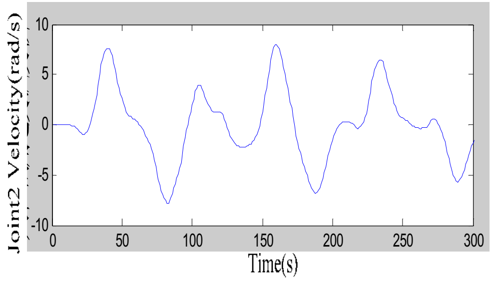

Figure 4. The body velocity of link 3

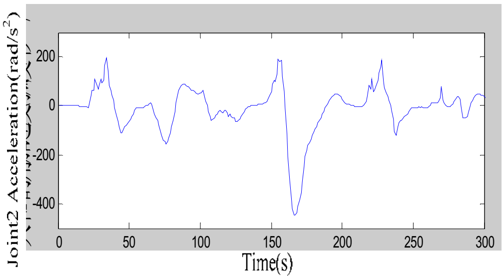

Figure 5. The body acceleration of link 3

4 Conclusions

The recursive dynamics of flexible multi-body systems are discussed. Also Lie groups and Lie algebras are used to model this problem. A four-bar model with flexible body is simulated with above method. The simulation results show that with the method can be solved quickly and efficiently.

From the above derivation, Lie groups and Lie algebras method can solve the flexible problems with higher efficiency. The finite element model is also a recursive method. In the calculation process almost all matrices have joined the modal matrix, which greatly increased the matrix order.

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