Bayesian denoising of white light interference signal in rough surface measurement

Zou Wendong¹, Zhu Lihua, Wang Weihong, Chen Chungeng
Nanchang Hangkong University, Key Laboratory of Nondestructive Test (Ministry of Education), Nanchang, Jiangxi, 330063)

Abstract—The measurement and quantitative analysis of three-dimensional microtopography of failure surface has important values for understanding the initiation and development of failure process. In this paper, we used scanning white-light interferometry (SWLI) for the characterization of metallic failure surface, but for it's large height variation, non-homogeneous and weak spectral reflectivity, the detected interferometric signals would be severely disturbed by the noises, consequently causing large reconstruction error. For this reason, we used a novel denoising method based on Bayesian estimation to process the extremely weak white light interference signal, theoretical and algorithmic derivations were given out as well. Finally, the effect of signals denoising was evaluated and a quantitative comparison with other typical approaches showed that the Bayesian estimation method had obvious advantage in the signal-to-noise ratio improvement and made much more reduction in the mean squared error, along with a better smoothness and stability.

1 Introduction

Characteristic information on the material’s surface is of great value which provides a stationary solution for deep study of it's formation processes and mechanical behaviors [1]. Generally, scanning electron microscopy (SEM) [2] is used to get the microstructure on the surface of an object because of high precision and large depth of field. However, in a number of specific applications, the uneven distribution of reflectivity on the object’s surface causes the serious loss of fringe visibility and it happens frequently that the reflectivity on some of the regional surface is extremely low, so that the detection becomes much more difficult, leading to a lower signal-to-noise ratio of the interference signal and a larger variation range of the optical path difference. Therefore, the denoising processing with the testing signals on the object’s surface plays a positive role, which can not only enhance the signal contrast, but highlight the characteristics of the useful signal also, so as to have a great improvement on the signal-to-noise ratio and make a big reduction on the mean squared error. According to the relevant literature [3-5], the study on the problem of signal detection mainly adopts such testing theories as stochastic resonance, time domain averaging, sampling integral, spectrum analysis, chaos theory, short-time Fourier and wavelet transform. This paper aims to discuss the method of signal detection based on Bayesian estimation [6], which is in combination with preprocessing and postprocessing, to make estimation of the white light interference signal collected, and thus improve the accuracy of the estimated parameter. Bayesian estimation, on the whole, is served as a method of model parameter estimation, whose basic idea is to set up a model equation, and then calculate the distribution of posterior probability, determining the maximum probability point of signal parameters, finally drawing a more accurate experimental result. That how to establish the model equation of white light interference signal on the object’s surface, placed as the key component of Bayesian estimation, is the most difficult to design, and it would cause great error of signal reconstruction if unsuitably modeling.

In this article, scanning white-light interferometry is applied to the measurement of microcosmic morphology on the object’s surface. What is more, MATLAB as a tool for simulation is in full use to detect the white light interference signal in a single pixel on the surface of objects through the processing with the sequences of white-light interferograms. Moreover, a new denoising method based on Bayesian estimation is proposed to cope with the white light interference signal in the interest of suppressing the noise in the inter-

2. Theory

2.1 Rough surface measurement based on scanning white-light interferometry

Scanning white-light interferometry [7] is a very important method in optical measurement, which eliminates the error of phase ambiguities and reduces the limitation of measurement range on one hand and possesses such characters as simple and small measuring structure, large...
measuring range and high precision on the other hand, in contrast to the way using laser interferometry. Consequently, it is applicable for the signal detection on the complex measured surface, which has been widely used in those areas, such as optical fiber sensing technology, optical fiber dispersion measurement, non-contact measurement of surface topography, film thickness measurement and interference localization, so having a certain application value. To analyze the microstructure features on the surface of the objects, this article puts forward to adopt a structure of Linnik interference system \(^9\) on the foundation of scanning white-light interferometry, among which the white-light interferogram is captured by the digital CCD plane array and the spatial orientation scanning is driven by the piezoelectric ceramic (PZT) \(^9\). The outcome of the experiment indicates that this detection device is provided with the transverse optical resolution of 1.18 \(\mu\)m, the longitudinal resolution of 10 nm and the maximum measuring range reaching to 120 \(\mu\)m.

In practice, however, due to the impact caused by the surface properties of beam splitter, the white light interferometer works with an optical path difference not being equal to zero, and thus the interference fringes of each wavelength will be stagger gradually with the increase of optical path difference and interference series, which makes the contrast of fringes decrease little by little and ultimately the interference fringes will disappear completely to a certain degree. As the figure 1 shows, it is a typical ideal white light interference signal, while the actual white light interference signal in a single pixel on the 30CrMnSiA alloy fracture surface, with the contrast of fringes approximately 0.04, is depicted in figure 2, whose curve presents a severe deformation and fluctuation with the zero-order stripe no longer being the center and bilateral symmetry. And a frame of white-light interferogram with its pixel array of \(400 \times 400\) on the surface of the test sample is shown in figure 3. Through comparing with the ideal white light interference signal, it is not difficult to reveal that the actual white light interference signal is severely lost within the intense noise. In consequence, the process of de-noising the white light interference detection signal on the surface of the objects is particularly significant and worthy of further analysis \(^10-11\).

\[ P(x^i | y^i) \sim f(y^i | x^i)P(x^i) \]

\( P(x^i | y^i) \) is the a posteriori probability of a possible denoised white light interference signal value \(x^i\) given the collected data of white light interference signal \(y^i\). The likelihood function \(f(y^i | x^i)\) depicts the probability of the

![Figure 1. Typical ideal white light interference signal in a single pixel along scanning axis.](image1)

![Figure 2. Actual white light interference signal in a single pixel on the metal surface along scanning position.](image2)

![Figure 3. A frame of white-light interferogram on the metal surface.](image3)

### 2.2. Bayesian surface estimation

In this paper, the method on the base of Bayesian estimation, associated with preprocessing and postprocessing, is used to eliminate the noise in the interference signal on the surface, in allusion to the core idea of making the parameter estimation of the white light interference signal that has been collected already and accordingly improving its signal-to-noise ratio and reducing its mean squared error, which contributes to make the waveform of the interference signal more genuine, stable and reliable.

Assume a set \(U\) of pixel sites in a single white-light interferogram and a pixel index \(i \in U\). In addition, make assumptions that the white light interference signal for each pixel on the measured surface is \(y^i\) and the possible denoised white light interference signal for each pixel is to be \(x^i\) correspondingly. Given as the mode of the conditional probability density \(P(x^i | y^i)\), we make a posterior estimation to look for the denoised white light interference signal \(x^i\). For each pixel \(i\), according to the Bayesian paradigm, it can be expressed by observations \(y^i\) given a possible value of white light interference signal \(x^i\). The a priori probability \(P(x^i)\) carries the prior knowledge on the surface.

Make \(\delta\) represent the union of a pixel and its immediate neighborhood. And as a matter of convenience, we index the elements of \(\delta\) with 0, ..., \(n\), in which \(\delta_0\) denotes the
measured pixel of the white-light interferograms. Then the formula (1) can be written as:
\[ P(\delta | y^\delta) \sim f(\delta | x^\delta)P(\delta) \]  
(2)
where, \( x^\delta \) and \( y^\delta \) are combinations of the possible denoised values and all signal observations in \( \delta \), respectively.

In order to get the a posteriori probability of the possible denoised interference signal for the measured pixel, the likelihoods \( f(y^i | x^i) \) at neighboring pixels in \( \delta \) are assumed to be distributed independently and identically. And thus the formula (2) can be converted into:
\[ P(\delta | y^\delta) \sim \prod_{i \in \delta} f(y^i | x^i)P(\delta) \]  
(3)
In the application for this article, we need to look for the a priori probability \( P(\delta) \), which helps to express the prior knowledge on the object’s surface. Thereby, we may consider:
\[ P(\delta) = \begin{cases} \| 2\varepsilon & \text{if } x^\delta_1, \ldots, x^\delta_n \in [-\varepsilon, x^\delta_0 + \varepsilon] \\ 0 & \text{otherwise} \end{cases} \]  
(4)
where \( \varepsilon \) is a constant and the possible denoised values \( x^\delta \) is assumed to follow uniform distribution.

Plugging the a priori probability into the formula (3), we can write:
\[ P(\delta | y^\delta) \sim \prod_{i \in \delta} \sum x^\delta_{i} f(y^i | x^i) / 2\varepsilon \]  
(5)
By using the risk minimization Bayesian estimation to obtain the possible values, an equation will be given by
\[ x^\delta = \arg \max_{x^\delta} P(\delta | y^\delta) \]  
(6a)
\[ = \arg \max_{x^\delta} \prod_{i \in \delta} \sum x^\delta_{i} f(y^i | x^i) / 2\varepsilon \]  
(6b)
Suppose that the noise, which has nothing to do with the detecting interference signal on the object’s surface, is considered as Gaussian white noise with the mean of zero and the variance of \( \sigma^2 \) and then we will obtain a computable formula as follows
\[ x^\delta = \arg \max_{x^\delta} \prod_{i \in \delta} \sum x^\delta_{i} \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ \frac{-(y^i - x^\delta_i)^2}{2\sigma^2} \right] / 2\varepsilon \]  
(7)
Consider that the possible denoised values \( x^i \) at the neighboring pixels in \( \delta \) are closely equal to the measured pixel value, that is, the formula (7) is equivalent to
\[ x^\delta = \arg \max_{x^\delta} \prod_{i \in \delta} \exp \left[ \frac{-(y^i - x^\delta)^2}{2\sigma^2} \right] \]  
(8a)
\[ = \arg \min_{x^\delta} \prod_{i \in \delta} (y^i - x^\delta)^2 \]  
(8b)
Aiming to attain the possible value \( x^\delta_i \), the algorithm offered above thus far depends on the following parameters: \( n, \delta \). With these determined parameters, our final execution is tested to be practical and thus accomplishes our anticipative goal. Without doubt, for other kinds of noise modeling, the idea in this paper is suitable as well.

### 3. Numerical analysis and results

#### 3.1 Denoising process of white light interference signal using Bayesian estimation

Taking an application example, this paper which built upon the foundation of scanning white-light interferometry is involved to acquire the white light interference signal on the 30CrMnSiA alloy fracture surface. And we obtains the denoised white light interference signal from the calculable algorithm in the formula (8b), noting that the measured pixel is \( U(381,209) \) for all simulations, and its accuracy can be further increased with the choice of the suitable value \( n \). As is illustrated in figure 4, it pictures the denoised interference signal by choosing different \( n \) with all other factors being equal, and the histograms and normal curves of white light interference signals are showed in figure 5.

For more detailed and direct display of the denoising performance, as tabulated in table 1, it shows the results of SNR(signal-to-noise ratio) and MSE(mean squared error), from which it is obvious that the performances of measured white light interference signal reduce significantly with the increase of the value of neighborhood pixels \( n \).
Above all, we are able to screen an intuitive $n = 4$ that better matches the Bayesian denoising algorithm.

Table 1. SNR and MSE results with the choice of different $n$ based on Bayesian estimation for the white light interference signal on the metal surface.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n=4$</th>
<th>$n=8$</th>
<th>$n=12$</th>
<th>$n=20$</th>
<th>$n=24$</th>
<th>$n=28$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR (dB)</td>
<td>48.404</td>
<td>45.683</td>
<td>45.103</td>
<td>43.876</td>
<td>43.223</td>
<td>42.924</td>
</tr>
<tr>
<td>MSE</td>
<td>0.939</td>
<td>1.757</td>
<td>2.008</td>
<td>2.666</td>
<td>3.096</td>
<td>3.316</td>
</tr>
</tbody>
</table>

3.2 White light interference signal denoising with different approaches

With a view to verify the superiority of the Bayesian denoising method, as is visualized in figure 6, different denoising approaches are employed to deal with the white light interference signal on the surface of 30CrMnSiA fracture, which provides a direct comparison among the different methods. Note that a scale of $3 \times 3$ pixels is chosen for the simulation and throughout the Bayesian denoising, $n = 4$ is used which in other words means that $\delta$ includes the 4 nearest neighbors of the measured pixel.

Meanwhile, as presented in table 2, the results of SNR and MSE are used to have a good visual effect on the denoising quality of interference signal on the metal surface, and the histograms and normal curves of white light interference signal described in figure 7 are showed to graphically summarize and display the distribution of the signal dataset which works for the more precise analysis of signal details and useful informations.

Correspondingly, the white light interference signal spectrums described in figure 8 are showed to graphically summarize and display the frequency distribution of the signal dataset which works for the more precise analysis of signal details and useful informations, with the amplitude of three main spectrum peaks from left to right listed in table 3.

Accompanying figure 5, figure 6, table 2 and table 3, some helpful conclusions are drawn:

1. The denoising processing with the detecting white light interference signal on the surface of metal not merely make the signal waveform more smooth and stable, but also evidently wipe off the outliers in the interference signal,
whereas our method can conserve much more detailed information and useful informations occurred in the edge. By seriously analyzing and researching in figure 5, we could observe clearly that the position with maximum modulation of white light interference signal that is also the position with zero light path difference is located at the scanning position $Z=17.75 \mu m$, which helps us detect the surface height at that tested pixel and realize the surface height construction from the white-light interferograms.

2. By comparing and analyzing, we are left in no doubt that our method has more or less improvements than other denoising algorithms, which leads to a better signal-to-noise ratio and a smaller mean squared error. The difference between the maximum and minimum in PSNR is close to 9.2 dB, while it is about 6.9 in MSE.

3. From the results of spectrum data, it is easily seen that the amplitude of denoised signals by the method of median filtering, mean filtering and Fourier transform decreases obviously. And compared with wavelet transform, the spectrum signal processed with bayesian estimation is most similar to the original shape, that is, our method greatly saves the spectrum characteristics of the original signal. Among these methods, the denoising effect of Fourier transform is relatively poorer, which seriously affects the frequency resolution of weak white light interference signals.

For purpose of further studying the three-dimensional shape on the metal surface, the reconstruction image is shown in figure 10 through the use of the algorithm proposed, in which way we are able to have access to more comprehensive information such as quantitative depth, width and direction, etc. we can clearly observe the characteristic information on the metal surface.

4. Conclusion

In this paper, a new methodology for signal denoising is proposed, while the white light interference signal in a single pixel is extracted from the interferogram sequences that are obtained by the scanning white-light interferometry. And this kind of method applied to the detecting white light interference signal on the surface is based on Bayesian estimation where the noise is supposed to be distributed following a Gaussian distribution with the mean of zero. In order to challenge the algorithms based on Bayesian estimation, we choose different neighbor pixels with the choice of a more suitable variance of noise that better matches the Bayesian estimation. In addition, we make a quantitative comparison between our method and the other denoising approaches by the means of MATLAB implementation and the simulation results show that all four approaches are able to evidently suppress noise and decimate outliers, but that the Bayesian estimation could preserve more detail features with a continuous histogram distribution, and lead to a better PSNR by about 5.2 dB to 9.2 dB, which corresponds to a smaller MSE from about 2.2

<table>
<thead>
<tr>
<th>Method</th>
<th>Original signal</th>
<th>Median filtering</th>
<th>Mean filtering</th>
<th>Fourier transform</th>
<th>Wavelet soft-threshold</th>
<th>Bayesian estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak amplitude</td>
<td>10.28</td>
<td>10.27</td>
<td>10.27</td>
<td>10.27</td>
<td>10.28</td>
<td>10.28</td>
</tr>
<tr>
<td></td>
<td>4.390</td>
<td>3.638</td>
<td>3.567</td>
<td>3.462</td>
<td>3.918</td>
<td>4.177</td>
</tr>
<tr>
<td></td>
<td>4.390</td>
<td>3.638</td>
<td>3.567</td>
<td>3.462</td>
<td>3.918</td>
<td>4.177</td>
</tr>
</tbody>
</table>

Figure 8. Spectrums of white light interference signals with different denoising approaches.

Figure 9. Original three-dimensional shape of metal surface

Figure 10. Three-dimensional shape of metal surface denoised by Bayesian estimation
to 6.9. Furthermore, through the way of three-dimensional shape reconstruction on the metal surface, we can not only have access to more comprehensive information of the surface, but also possess an available ways to validate the feasibility of signal detection by adopting the denoising method described in this article, with the analysis result revealing that the reconstruction effect is relatively satisfying and obvious \[^{14}\]. Finally, associated with all the fruit of this research, we can draw a conclusion that the method based on Bayesian estimation represents an efficient mean for denoising the real signals on the surface.

Acknowledgments

This work is supported by National Natural Science Foundation of China under grant No. 51241005, The Industry Project of Science & Technology Pillar Program of Jiangxi Province, China (Grant No.20122BBE500040), Natural Science Foundation of Jiangxi Province, China (Grant No. 2008GZW0011, No.20122BAB202010).

References