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THE IMPLEMENTATION OF VENDOR MANAGED INVENTORY IN THE SUPPLY CHAIN WITH SIMPLE PROBABILISTIC INVENTORY MODEL

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ABSTRACT

Numerous studies show that the implementation of Vendor Managed Inventory (VMI) benefits all members of the supply chain. This research develops model to prove the benefits obtained from implementing VMI to supplier-buyer partnership analytically. The model considers a two-level supply chain which consists of a single supplier and a single buyer. The analytical model is developed to supply chain inventory with probabilistic demand which follows normal distribution. The model also incorporates lead time as a decision variable and investigates the impacts of inventory management before and after the implementation of the VMI. The result shows that the analytical model has the ability to reduce the supply chain expected cost, improve the service level and increase the inventory replenishment. Numerical examples are given to prove them.

Keywords: inventory, supply chain, economic, probabilistic

INTRODUCTION

Industry players are increasingly aware that to provide a cheap, qualified and quick products they need the role of all players. The players are suppliers, manufacturers, distributors, retails and corporate supporters such as logistics. The companies establish a network that work together to create and deliver products to the end user. The formed network is called supply chain (Pujawan, 2005). Supply chain is a dynamic system that includes all activities like inventory control, manufacturing, distribution, warehousing, and customer service to deliver product from raw material supplier to the end customer (Pasandideh et al., 2010). All activities need to be coordinated and integrated by supply chain management (SCM). SCM gives profitability and cost reduction for whole supply chain member in facing global competition (Tyana and Wee, 2003). SCM coordinates the enterprises along the logistic network and focus in finding the best strategy (Simchi-Levi et al.,2003).

In the traditional supply chain, the buyers determine the timing and size of orders itself based on the information they have. Supplier will respond the buyer's request without finding out more why the buyer ordered that volume. To meet the buyer's demand, the supplier predicts the raw materials and products needed in inventory based on the buyer past orders. This condition forces the supplier to store more inventory to overcome the buyer uncertainty orders. One possible case that often occurs in the traditional supply chain is the buyer's demand changes suddenly. Something that supplier did not anticipate. This condition causes the supplier to change the production schedule. Frequent changes in production schedules lead to schedule nervousness (Pujawan, 2005).

Many companies use the vendor managed inventory (VMI) to solve this problem. In the VMI model, the buyer no longer decides what, when, and how the goods will be purchased from the supplier. The buyer only gives the information about the customer demand and the remaining inventory. They are useful for suppliers to decide the timing and amounts of inventory replenishment (Disney and Towell, 2003). And of course the buyer should also provide information about the minimum and maximum inventory they expect (Pujawan, 2005).

Numerous studies show that the implementation of VMI benefits all members of the supply chain. VMI gives the competitive advantages to the retailers such as the higher product availability and service level, lower ordering cost and inventory monitoring (Waller et al., 1999). The benefit of applying VMI for the supplier are reducing the bullwhip effect (Disney and Towell, 2003) and synchronizing the inventory replenishment planning (Cetinkaya and Lee, 2000).

This research develops a model to prove the benefits obtained from implementing WMI to supplier-buyer parthership analytically. The model is developed for two level supply chain which consist of a single supplier and a single buyer. This research is the extension of Pasandideh et al.'s research which investigated the application of VMI model in supply chain in the EOQ model with shortage (Pasandideh et al., 2010). This research changes the demand assumption from deterministic to probabilistic.

In the probabilistic model, lead time becomes an important issue. Its management can prevent the stock out, reduce the safety stock, improves the customer service level and the competitive advantage of business (Ouyang et al, 2007). Reducing the lead time usually causes an added cost namely the crashing cost. Research of inventory model which consider lead time as decision variable have developed by many researchers. The probabilistic inventory model which incorporate lead time as decision variable was firstly conducted by (Liao and Shyu, 1991). The lead time usually consist of several component, such as order preparation, order transit, set up time, delivery time and the supplier lead time (Tersine, 1994).

This research develops the analytical model of the VMI implementation to supply chain inventory with probabilistic demand which follows normal distribution. The model incorporates lead time as decision variable and investigates the impacts of inventory management before and after the implementation of VMI.

METHODOLOGY

The developed model is used to prove the benefits obtained by implementing VMI to the supply chain inventory with probabilistic demand. In the traditional supply chain (non VMI), the supplier can not obtain the information of consumer demand directly but through the buyer's ordering policy. On the contrary, in the supply chain with VMI, the supplier obtains the information of consumer demand directly by its information system. As a result, the supplier has a combination of two inventory cost, that is order set up and holding cost (Dong and Xu, 2002).

To prove the benefits obtained from VMI implementation on supply chain with probablistic demand, mathematical models will be established for two supply chain models. They are mathematical model for traditional supply chain (without VMI) and mathematical model for integrated (VMI) supply chain. The mathematical models are established by using the following assumptions and notations. The next step is modeling the total cost of both the traditional (Non VMI) and the integrated (VMI) supply chain.

The mathematical models are developed based on the following assumptions :

a. The supply chain consists of a single supplier and a single buyer with a single item which is considered.
b. Demand during planning horizon is a simple probabilistic with average demand *D,* deviation standart *S* and normal distribution.
c. The ordering lot size $Q*$ is constant for every ordering time, the products will come simultaneusly with leadtime L, and the order is done when the inventory level attains to the reorder point *r*.
d. The price of product *p* is constant.
e. The service level η or the probability of inventory shortage is known and determined by the company.
f. The crashing cost is not allowed in this model.

The following notations will be used to develop the model :

TC_{noVMI}	Total expected cost of the traditional (non VMI) supply chain
TC_{VMI}	Total expected cost of the integrated (VMI) supply chain
Q_{noVMI}	The order quantity of the traditional (non VMI) supply chain
Q_{VMI}	The order quantity of the integrated (VMI) supply chain
A_S	The supplier's ordering cost per unit
A_B	The buyer's ordering cost per unit
D	The buyer's demand rate
S	The standard deviation of demand
h_B	Product holding cost per unit held in buyer's store in a period
α_{noVMI}	The probability of inventory shortage in the traditional (non VMI) supply chain
α_{VMI}	The probability of inventory shortage in the integrated (VMI) supply chain
BI_{noVMI}	The buyer's expected inventory cost of the traditional (non VMI) supply chain
BI_{VMI}	The buyer's expected inventory cost of the integrated (VMI) supply chain
SI_{noVMI}	The supplier's expected inventory cost of the traditional (non VMI) supply chain
SI_{VMI}	The supplier's expected inventory cost of the integrated (VMI) supply chain
L_{noVMI}	Lead time of the traditional (non VMI) supply chain
L_{VMI}	Lead time of the integrated (VMI) supply chain
$\hat{\pi}$	The shortage cost per unit per time
Z_α	The safety factor
$f(Z_\alpha)$	The standard normal probability density function
$\psi(Z_\alpha)$	The cumulative distribution function

The case of traditional (Non VMI) Supply Chain

In the traditional supply chain with probabilistic demand, the buyer places an every Q number of demands. The average cycle time is $\frac{Q}{D}$ respectively. The safety stock (SS) is the expected net inventory level which formulated by $SS = Z_\alpha S\sqrt{L}$. The expected net inventory level is $Q + SS$ immediately after the arrival of procurement. The average inventory is about $\frac{Q}{2} + SS$ over the cycle. At the end cycle, the expected demand shortage is given by N with $N = S\sqrt{L}\beta$ and $\beta = f(Z_\alpha) - Z_\alpha\psi(Z_\alpha)$ (Hadley and Within, 1963).

The buyer's inventory cost in the non VMI supply chain comprises of the ordering cost, holding cost and the shortage cost which can be expressed as Eqn. (1).

$$BI_{noVMI} = \frac{\mathrm{A_B}D}{Q_{noVMI}} + \mathrm{h_B}\left(\frac{Q_{noVMI}}{2} + Z_\alpha\, S\,\sqrt{L_{noVMI}}\right) + \frac{\hat{\pi}D\, S\,\sqrt{L_{noVMI}}\,\beta}{Q_{noVMI}} \tag{1}$$

The supplier's inventory cost in the non VMI supply chain comprises of the ordering cost which can be expressed as Eqn. (2).

$$SI_{noVMI} = \frac{\mathrm{A_S}D}{Q_{noVMI}} \tag{2}$$

The total inventory cost of traditional supply chain is the summation of the buyer's inventory cost and the supplier's inventory cost indicated by the following Eqn. (3).

$$TC_{noVMI} = \frac{\mathrm{A_B}D}{Q_{noVMI}} + \mathrm{h_B}\left(\frac{Q_{noVMI}}{2} + Z_\alpha\, S\,\sqrt{L_{noVMI}}\right) + \frac{\hat{\pi}D\, S\,\sqrt{L_{noVMI}}\,\beta}{Q_{noVMI}} + \frac{\mathrm{A_S}D}{Q_{noVMI}} \tag{3}$$

The buyer's inventory cost in Eqn. (1) is a function of both Q_{noVMI} and L_{noVMI}. The optimal values of both Q_{noVMI} and L_{noVMI} can be obtained by partially differentiating of Eqn. (1). The result from differentiating Eqn. (1) with respect to Q_{noVMI} and setting it zero is as follows.

$$\frac{\partial BI_{noVMI}}{\partial Q_{noVMI}} = 0 \tag{4}$$

$$\frac{\mathrm{A_B}D}{Q_{noVMI}} + \mathrm{h_B}\left(\frac{Q_{noVMI}}{2} + Z_\alpha\, S\,\sqrt{L_{noVMI}}\right) + \frac{\hat{\pi}D\, S\,\sqrt{L_{noVMI}}\,\beta}{Q_{noVMI}} = 0 \tag{5}$$

$$Q^2{}_{noVMI} = \frac{2\mathrm{D}}{\mathrm{h_B}}\left([\mathrm{A_B} + \hat{\pi}DS\,\beta\sqrt{L_{noVMI}}\,]\right) \tag{6}$$

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$$Q_{noVMI} = \sqrt{\frac{2\text{D}}{\text{h}_\text{B}}\left([\text{A}_\text{B} + \hat{\pi}DS\,\beta\sqrt{L_{noVMI}}\,]\right)} \tag{7}$$

Differentiating Eqn. (1) with respect to L_{noVMI} and setting it zero is as follows.

$$\frac{\partial BI_{noVMI}}{\partial L_{noVMI}} = 0 \tag{8}$$

$$\frac{\text{h}_\text{B}Z_\alpha S}{2\sqrt{L_{noVMI}}} + \frac{\hat{\pi}DS\,\beta}{2\sqrt{L_{noVMI}}\,Q_{noVMI}} = 0 \tag{9}$$

$$\text{h}_\text{B}Z_\alpha SQ_{noVMI} = -\hat{\pi}DS\beta \tag{10}$$

$$Q_{noVMI} = -\frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha} \tag{11}$$

By substituting Eqn. (11) into Eqn. (7), we can obtain:

$$\frac{2\text{D}[\text{A}_\text{B}+\hat{\pi}S\sqrt{L_{noVMI}}\,\beta]}{\text{h}_\text{B}} = \left[-\frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha}\right]^2 \tag{12}$$

$$2D\hat{\pi}S\sqrt{L_{noVMI}}\,\beta = \frac{1}{\text{h}_\text{B}Z_\alpha^2}(\hat{\pi}D\beta)^2 - 2\text{DA}_\text{B} \tag{13}$$

$$\sqrt{L^*_{noVMI}} = \left(\frac{\hat{\pi}D\beta}{2S\text{h}_\text{B}Z_\alpha^2}\right) - \left(\frac{\text{A}_\text{B}}{\hat{\pi}S\beta}\right) \tag{14}$$

Substituting Eqn. (14) into Eqn. (7), we can obtain the optimal order quantity of the non VMI supply chain as follows:

$$Q^*_{noVMI} = \sqrt{\frac{2\text{D}}{\text{h}_\text{B}}[\text{A}_\text{B} + \hat{\pi}S\beta\,\left(\frac{\hat{\pi}D\beta}{2S\text{h}_\text{B}Z_\alpha^2} - \frac{A_B}{\hat{\pi}S\beta}\right)} \tag{15}$$

$$Q^*_{noVMI} = \sqrt{\frac{2\text{D}}{\text{h}_\text{B}}\left[\text{A}_\text{B} + \left(\frac{\hat{\pi}^2D\beta^2}{2\text{h}_\text{B}Z_\alpha^2} - \text{A}_\text{B}\right)\right]} \tag{16}$$

$$Q^*_{noVMI} = \frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha} \tag{17}$$

Furthermore, by substituting Eqn.(14) and Eqn. (17) into Eqn. (3), the expected total inventory cost of the traditional supply chain becomes as Eqn. (18):

$$TC^*_{noVMI} = \frac{3D\hat{\pi}\beta}{2\text{Z}_\alpha} + \frac{\text{h}_\text{B}\text{Z}_\alpha}{\hat{\pi}\beta}(\text{A}_\text{S} - \text{A}_\text{B}) \tag{18}$$

The case of the integrated (VMI) supply chain

As a result of the implementation of VMI, the buyer's inventory cost switches to supplier. Thus, the buyer's cost becomes zero. The inventory cost of both supplier and buyer also the total inventory cost of the integrated (VMI) supply chain are calculated as follows.

$$BI_{VMI} = 0 \tag{19}$$

$$SI_{VMI} = \frac{\text{A}_\text{B}D}{Q_{VMI}} + \text{h}_\text{B}\left(\frac{Q_{VMI}}{2} + Z_\alpha\,S\sqrt{L_{VMI}}\right) + \frac{\hat{\pi}DS\sqrt{L_{VMI}}\,\beta}{Q_{VMI}} + \frac{\text{A}_\text{S}D}{Q_{VMI}} \tag{20}$$

$$TC_{VMI} = \frac{\text{A}_\text{B}D}{Q_{VMI}} + \text{h}_\text{B}\left(\frac{Q_{Q_{VMI}}}{2} + Z_\alpha\,S\sqrt{L_{VMI}}\right) + \frac{\hat{\pi}D(S\sqrt{L_{VMI}}\,\beta)}{Q_{VMI}} + \frac{\text{A}_\text{S}D}{Q_{VMI}} \tag{21}$$

The optimal values of both Q^*_{VMI} and L^*_{VMI} can be obtained by partially differentiating of Eqn. (21). The result from differentiating Eqn. (21) with respect to Q_{VMI} and setting it zero is as follows.

$$\frac{\partial TC_{VMI}}{\partial Q_{VMI}} = 0 \tag{22}$$

$$Q^2{}_{VMI} = \frac{2\text{D}}{\text{h}_\text{B}}\left(\text{A}_\text{B} + \text{A}_\text{S} + \hat{\pi}S\,\beta\sqrt{L_{VMI}}\,\right) \tag{23}$$

$$Q_{VMI} = \sqrt{\frac{2\text{D}}{\text{h}_\text{B}}\left(\text{A}_\text{B} + \text{A}_\text{S} + \hat{\pi}S\,\beta\sqrt{L_{VMI}}\,\right)} \tag{24}$$

The optimal value of L^*_{VMI} can be obtained by partially differentiating of Eqn. (21) with respect to L_{VMI}, and the result is as follows.

$$\frac{\partial TC_{VMI}}{\partial L_{VMI}} = 0 \tag{25}$$

$$\frac{\text{h}_\text{B}Z_\alpha SQ_{VMI} + \hat{\pi}DS\,\beta}{2\sqrt{L_{VMI}}\,Q_{VMI}} = 0 \tag{26}$$

$$Q_{VMI} = -\frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha} \tag{27}$$

Inserting Eqn. (27) to Eqn. (24), we will obtain :

$$\frac{2\text{D}[\text{A}_\text{B}+A_S+\hat{\pi}S\sqrt{L_{VMI}}\,\beta]}{\text{h}_\text{B}} = \left[\frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha}\right]^2 \tag{28}$$

$$2D\hat{\pi}S\sqrt{L_{VMI}}\,\beta = \frac{1}{\text{h}_\text{B}Z_\alpha^2}(\hat{\pi}D\beta)^2 - 2\text{D}(\text{A}_\text{B} + \text{A}_\text{S}) \tag{29}$$

$$\sqrt{L^*_{VMI}} = \left(\frac{\hat{\pi}D\beta}{2S\text{h}_\text{B}Z_\alpha^2}\right) - \left(\frac{\text{A}_\text{B}+\text{A}_\text{S}}{\hat{\pi}S\beta}\right) \tag{30}$$

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The optimal order quantity of the VMI supply chain (Q^*_{VMI}) is obtained by inserting Eqn. (30) to Eqn. (27), as follows.

$$Q^*_{VMI} = \sqrt{\frac{2\text{D}}{\text{h}_\text{B}}\left(\text{A}_\text{B} + \text{A}_\text{S} + \frac{\hat{\pi}D\beta}{2S\text{h}_\text{B}Z_\alpha^2} - \text{A}_\text{B} - \text{A}_\text{S}\right)} \tag{31}$$

$$Q^*_{VMI} = \frac{\hat{\pi}D\beta}{\text{h}_\text{B}Z_\alpha} \tag{32}$$

The expected total inventory cost of the VMI supply chain is obtained by inserting Eqn. (30) and Eqn. (32) to Eqn. (21) and the result is as Eqn.(33).

$$TC^*_{VMI} = \frac{3D\hat{\pi}\beta}{2\text{Z}_\alpha} - \frac{\text{h}_\text{B}\text{Z}_\alpha}{\hat{\pi}\beta}(\text{A}_\text{S} + \text{A}_\text{B}) \tag{33}$$

RESULTS AND DISCUSSION

The comparison of two supply chain models is carried out to variables such as the optimal order quantity, lead time, service level, and the expected total cost of supply chain.

The comparison between the order quantity Q^*_{noVMI} with Q^*_{VMI}

Developing inventory models on probabilistic demand and lead time as decision variable result in the same value of order quantity between both of supply chain. We can conclude as Eqn. (34).

$$Q^*_{noVMI} = Q^*_{VMI} = \frac{D\hat{\pi}\beta}{\mathrm{h_B Z_\alpha}} = Q^* \quad (34)$$

The comparison between the lead time L^*_{noVMI} with L^*_{VMI}

The developed assumption is L^*_{noVMI} greater equal than L^*_{VMI} . Let $L^*_{noVMI} \geq L^*_{VMI}$.

$$L^*_{noVMI} \geq L^*_{VMI} \quad (35)$$

$$\left[\left(\frac{\hat{\pi}D\beta}{2S\mathrm{h_B}Z_\alpha^2}\right) - \left(\frac{\mathrm{A_B}}{\hat{\pi}S\,\beta}\right)\right]^2 \geq \left[\left(\frac{\hat{\pi}D\,\beta}{2S\mathrm{h_B}Z_\alpha^2}\right) - \left(\frac{\mathrm{A_B+A_S}}{\hat{\pi}S\,\beta}\right)\right]^2 \quad (36)$$

$$\left[\left(\frac{\hat{\pi}D\beta}{2S\mathrm{h_B}Z_\alpha^2}\right) - \left(\frac{\mathrm{A_B}}{\hat{\pi}S\,\beta}\right)\right]^2 \geq \left[\left(\frac{\hat{\pi}D\,\beta}{2S\mathrm{h_B}Z_\alpha^2}\right) - \left(\frac{\mathrm{A_B}}{\hat{\pi}S\,\beta}\right) - \left(\frac{\mathrm{A_S}}{\hat{\pi}S\,\beta}\right)\right]^2 \quad (37)$$

let $\left(\frac{\hat{\pi}D\beta}{2S\mathrm{h_B}Z_\alpha^2}\right) - \left(\frac{\mathrm{A_B}}{\hat{\pi}S\,\beta}\right) = \theta$, therefore Eqn. (37) becomes Eqn. (38).

$$\theta^2 \geq \left[\theta - \left(\frac{\mathrm{A_S}}{\hat{\pi}S\,\beta}\right)\right]^2 \quad (38)$$

Eqn. (38) shows that the value of L^*_{noVMI} is greater than L^*_{VMI} which proves that the above developed assumption is fulfilled. This condition shows that the implementation VMI can reduce lead time. The shorter lead time impact to reducing the value of safety stock SS^*_{VMI} and the reorder point r^*_{VMI}, so we can generalize that $SS^*_{noVMI} \geq SS^*_{VMI}$ and $r^*_{noVMI} \geq r^*_{VMI}$. The short lead time of VMI supply chain will make the inventory replenishment more frequently. This brings benefits to the buyer in anticipation of uncertain demand and improves customer service level.

The comparison between the expected total inventory TC^*_{noVMI} with TC^*_{VMI}

let $TC^*_{noVMI} \geq TC^*_{VMI}$ to compare the expected total inventory costs of two supply chains.

$$TC^*_{noVMI} \geq TC^*_{VMI} \quad (39)$$

$$\frac{3D\hat{\pi}\beta}{2\mathrm{Z}_\alpha} + \frac{\mathrm{h_B Z_\alpha A_S}}{\hat{\pi}\beta} - \frac{\mathrm{h_B Z_\alpha A_B}}{\hat{\pi}\beta} \geq \frac{3D\hat{\pi}\beta}{2\mathrm{Z}_\alpha} - \frac{\mathrm{h_B Z_\alpha A_S}}{\hat{\pi}\beta} - \frac{\mathrm{h_B Z_\alpha A_B}}{\hat{\pi}\beta} \quad (40)$$

$$\text{Let } \lambda = \frac{3D\hat{\pi}\beta}{2\mathrm{Z}_\alpha} - \frac{\mathrm{h_B Z_\alpha A_B}}{\hat{\pi}\beta} \quad (41)$$

by inserting Eqn. (41) to Eqn. (40), the Eqn. (40) becomes Eqn. (42).

$$\lambda + \frac{2\mathrm{h_B Z_\alpha A_S}}{\hat{\pi}\beta} \geq \lambda \quad (42)$$

The Eqn. (42) indicates that value of TC^*_{noVMI} is greater than value of TC^*_{VMI}. The difference value of both is about $\frac{\mathbf{2h_B Z_\alpha A_S}}{\hat{\pi}\beta}$. This indicates that the assumption above is met analytically and proves that VMI supply chain can reduce the expected total cost.

The comparison between the probability of inventory shortage α^*_{noVMI} with α^*_{VMI}

The comparison of the probability of stock out between two supply chain is used to determine the service level provided by them. Based on (Jha and Shanker, 2009), the equation of service level constrain is as Eqn. (43).

$$\frac{S\sqrt{L}\beta}{Q} = \alpha \quad (43)$$

The developed assumption is α^*_{noVMI} greater equal than nilai α^*_{VMI}.

$$\alpha^*_{noVMI} \geq \alpha^*_{VMI} \quad (44)$$

$$\frac{\mathrm{S}\sqrt{\mathrm{L^*_{noVMI}}}\,\beta}{\mathrm{Q^*_{noVMI}}} \geq \frac{\mathrm{S}\sqrt{\mathrm{L^*_{VMI}}}\,\beta}{\mathrm{Q^*_{VMI}}} \quad (45)$$

Since $Q^*_{noVMI} = Q^*_{VMI} = Q^*$, so Eqn. (45) can be wrote in Eqn. (46).

$$\frac{S\beta}{Q^*}\sqrt{L^*_{noVMI}} \geq \frac{S\beta}{Q^*}\sqrt{L^*_{VMI}} \quad (46)$$

$$\frac{\hat{\pi}D\beta^2}{2Q^*\mathrm{h_B}Z_\alpha^2} - \frac{\mathrm{A_B}}{Q^*\hat{\pi}} \geq \frac{\hat{\pi}D\beta^2}{2Q^*\mathrm{h_B}Z_\alpha^2} - \frac{\mathrm{A_B}}{Q^*\hat{\pi}} - \frac{\mathrm{A_S}}{Q^*\hat{\pi}} \quad (47)$$

$$\text{Let } \gamma = \frac{\hat{\pi}D\beta^2}{2Q^*\mathrm{h_B}Z_\alpha^2} - \frac{\mathrm{A_B}}{Q^*\hat{\pi}} \quad (48)$$

Eqn. (48) can be simplified into Eqn. (49).

$$\gamma \geq \gamma - \frac{\mathrm{A_S}}{Q^*\hat{\pi}} \quad (49)$$

The value of α^*_{noVMI} is greater than α^*_{VMI} in Eqn. (49). The above assumption in Eqn. (44) is proved. This condition shows that the VMI supply chain gives the value of probability of inventory shortage smaller than traditional supply chain. It means that the VMI supply chain provides the service level better than the Non VMI supply chain. The difference of service level value of both is about $\frac{\mathbf{A_s}}{Q^*\hat{\pi}}$.

NUMERICAL EXAMPLES

This section has three numerical examples to justify and demonstrate the proposed method.

Table-1. The initial data

The Parameters	Example 1	Example 2	Example 3
$\mathrm{A_B}$	30	21	45
$\mathrm{A_S}$	40	75	150
$\mathrm{h_B}$	85	85	85
D	9000	9000	9000
S	900	900	900
$\hat{\pi}$	90	90	90

Table-2. The result obtained for example 1

The variables	Non VMI supply chain	VMI supply chain
Q^*	395	395
L^*	0.016234	0.014444
α^*	0.002527	0.002248
TC^*	50,529.13	48,704.11

Table-3. The result obtained for example 2

The variables	Non VMI supply chain	VMI supply chain
Q^*	395	395
L^*	0.016651	0.013337
α^*	0.002592	0.002076
TC^*	51,532.88	48,110.98

Table-4. The result obtained for example 3

The variables	Non VMI supply chain	VMI supply chain
Q^*	395	395
L^*	0.01555	0.009524
α^*	0.00242	0.001482
TC^*	52,696.33	45,852.53

Table 2 shows that the value of non VMI supply chain variables with α=0.05 are $Q^*_{noVMI} = 395$, $L^*_{noVMI} = 0.016234$, the actual $\alpha^*_{noVMI} = 0.002527$, and the total cost $TC^*_{noVMI} = 50{,}529.13$. While the value of VMI supply chain variables are $Q^*_{VMI} = 395$, $L^*_{VMI} = 0.014444$, the actual $\alpha^*_{VMI} = 0.002248$, and the total expected cost $TC^*_{VMI} = 48{,}704.11$. Both models have the same value of optimal order quantity. But for the other three variables such as lead time, the actual probability of inventory shortage, and the total cost, VMI supply chain has a better value. By implementing the VMI program, the total expected cost can be reduced as 3.75%. This savings is also obtained for the example 2 and 3. The value of total expected cost in table-3 is decreased from 51,532,88 to 48,110.98 with the decreasing value as 7.11%. While the decreasing value of total expected cost is 12.99% for example 3 in table-4.

We can conclude from all table result, the optimal order quantity of non VMI supply chain is equal to the optimal order quantity of VMI one ($Q^*_{noVMI} = Q^*_{VMI}$). This condition occurs in the probabilistic inventory model which considering lead time as a decision variable. The optimal lead time of non VMI supply chain is longer than the optimal lead time of integrated (VMI) one ($L^*_{noVMI} > L^*_{VMI}$). While the actual of inventory shortage probability of Non VMI supply chain is greater than the optimal lead time of integrated (VMI) one ($\alpha^*_{noVMI} > \alpha^*_{VMI}$).

The decreasing of the total expected cost in the VMI supply chain is caused by the decreasing buyer's holding cost and transferring buyer's ordering cost to supplier. Subsequently, the decreasing lead time reduces safety stock and reorder point in the integrated (VMI) supply chain. The shorter lead time can increase inventory replenishment which can ensure the product availability in buyer's store. Finally, it can improve the product service level.

The application of VMI program in the probabilistic inventory model whereas lead time is treated as a decision variable is able to reduce the actual value of α. Both of the supply chain has the smaller actual value of α than the initial one ($\alpha = 0.05$). In brief, implementation of VMI program in supply chain which has condition such as probabilistic demand, a single supplier and a single buyer and lead time as a decision variable has ability reducing the total expected cost, lead time and improving the service level.

CONCLUSIONS

This paper contributes an analytical model that proves benefits obtained from implementing VMI in the supply chain. The model considers a two-level supply chain which consists of a single supplier and a single buyer. The analytical model investigates the impacts of inventory management before and after the implementation of the VMI.

This model is extension of Pasandideh et al's model by changing the deterministic assumption into probabilistic assumption with normal distribution. In this model, lead time is treated as decision variable together with the order quantity. This model has a broader application of the real system compared with the previous one since the demand in the real system is uncertain.

The model results show that the application VMI to the supply chain can reduce the total expected cost of supply chain, lead time and improve the service level. The study of changing the above assumptions will be a possible topic for further research.

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